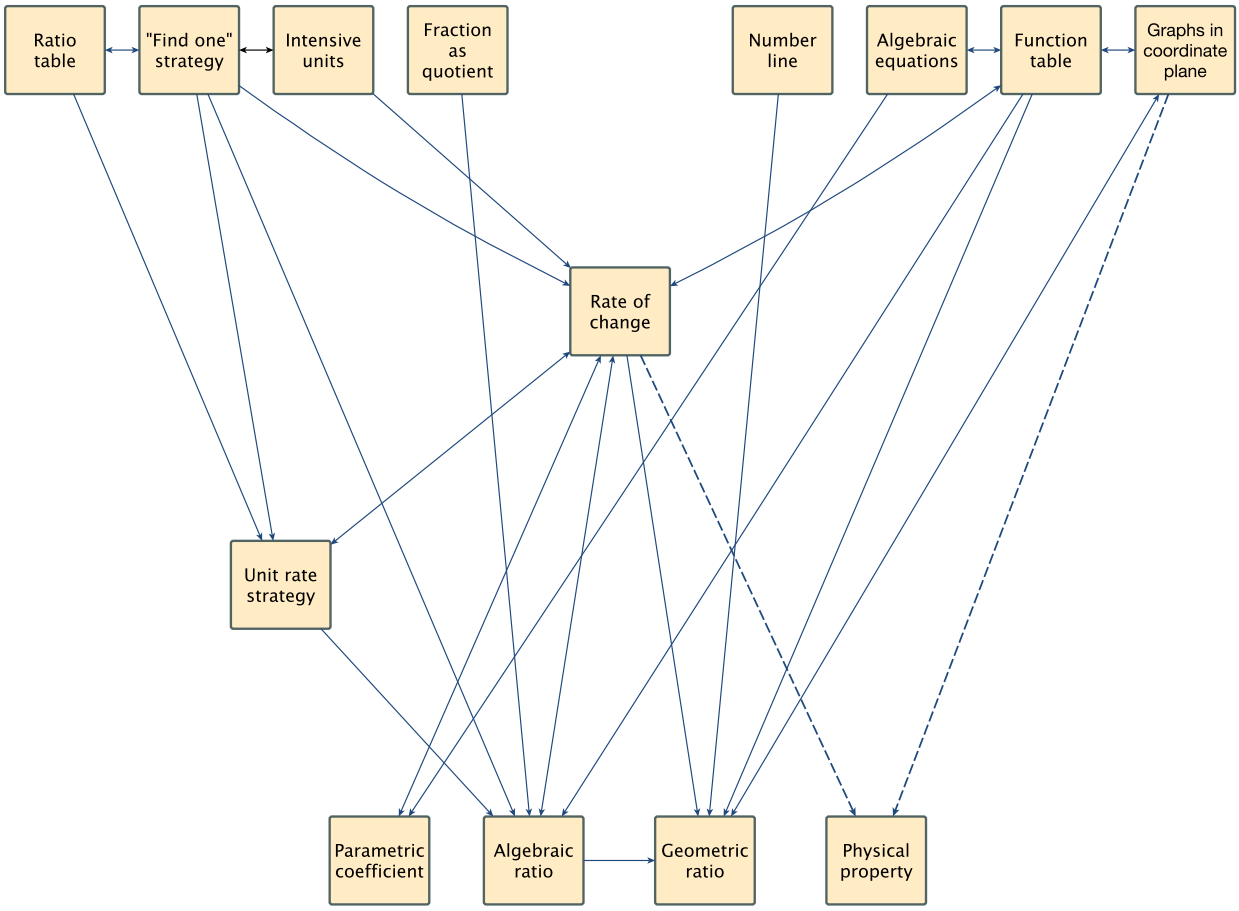
More than rise over run!
Activities to invent and connect the five faces of slope

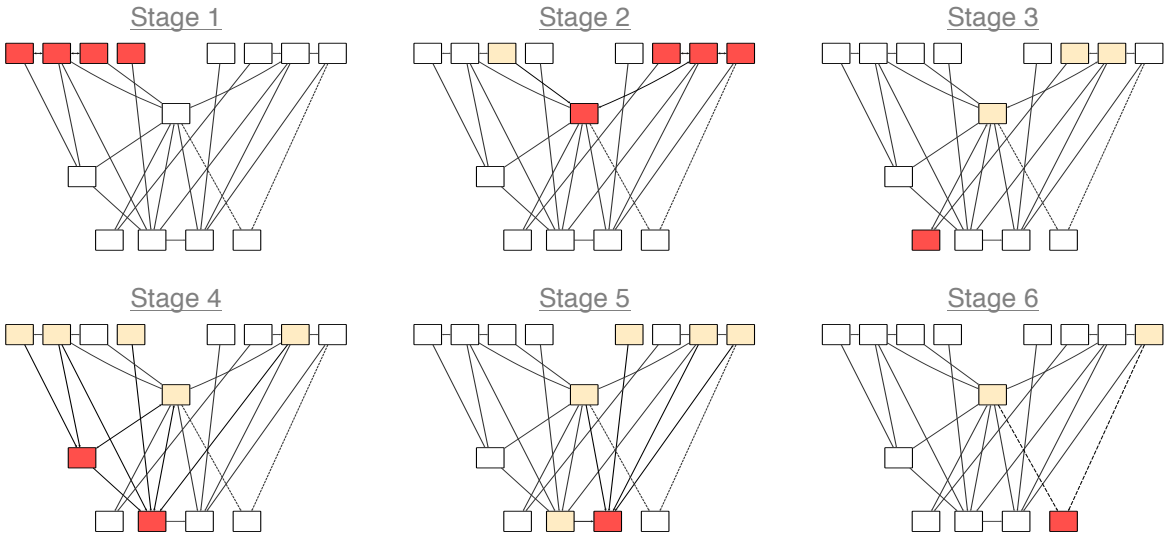
Frederick Peck
Freudenthal Institute US | School of education | University of Colorado

Frederick.Peck@Colorado.edu
www.RMEInTheClassroom.com

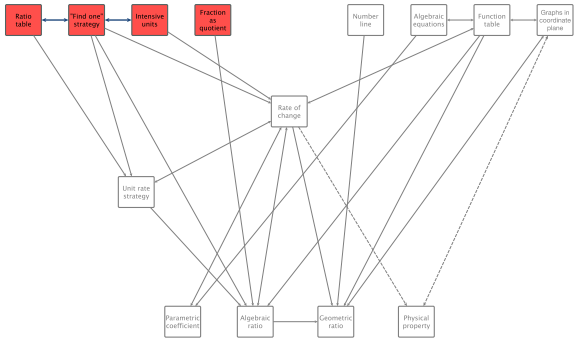
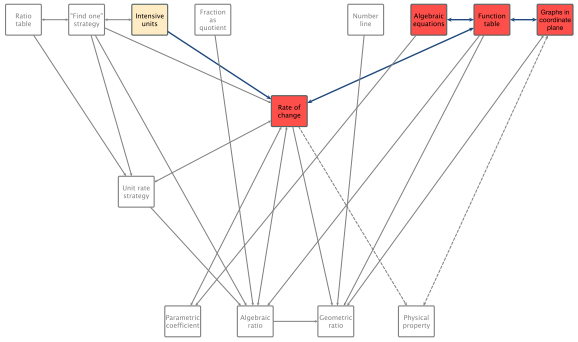
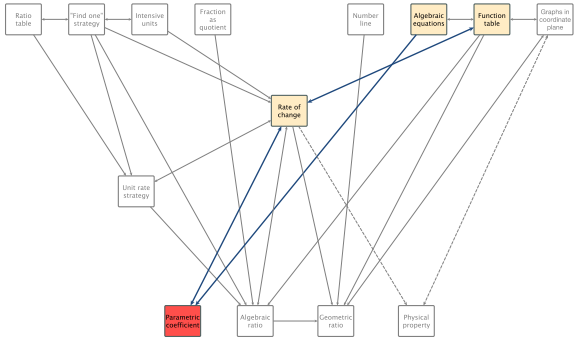
CCTM 2014
Denver, CO
September 26, 2014

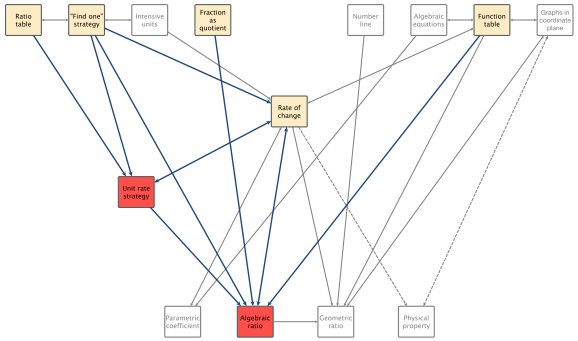
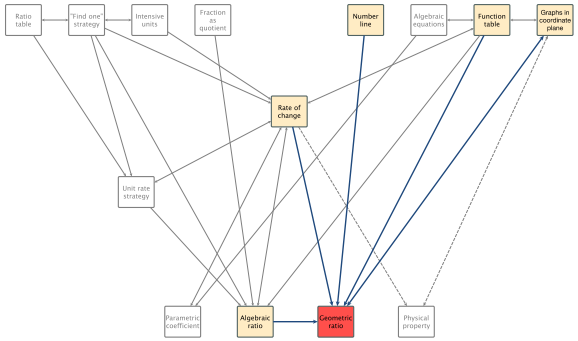
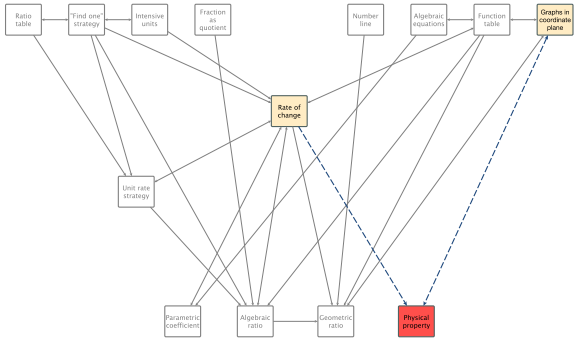
“Cascade of artifacts” for slope

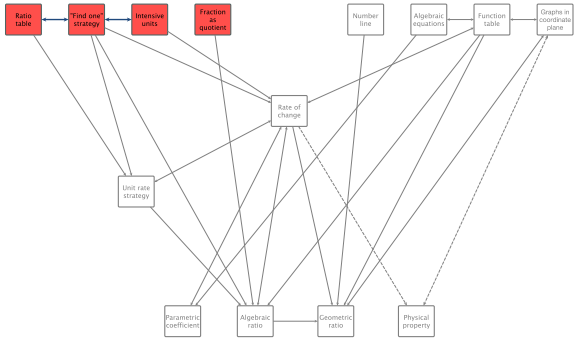




(In the bottom figures, the artifacts that are assembled and coordinated in each stage are shown in yellow, and those that are reinvented at each stage are shown in red)

Stage		Artifacts	Characteristics of tasks
1		Reinvented & made meaningful: • Ratio table • "find one" strategy • Intensive units • Fraction-as-quotient	Tasks that involve the activity of partitive division, including: • finding fair shares • finding unit values with final answers given in intensive units (___ per ___)
2		Assembled and coordinated: • Intensive units Reinvented & made meaningful: • Algebraic equations • Function tables • Graphs in coord. plane • Rate of change	Tasks that involve: • Finding and continuing patterns in geometric figures and tables of values, where there is a "starting value" and the independent variable increases by 1 • Converting between multiple representations of functions (focusing on table rows and points in the plane as *solutions* to two-variable equations)
3		Assembled and coordinated: • Algebraic equations • Rate of change Reinvented & made meaningful: • Parametric coefficient Made more meaningful • Rate of change • Function tables	Making predictions in linear situations, given: • The rate of change and starting value • Multiple data points (e.g., in a table), where the independent variable increases by one.

Stage	Artifacts	Characteristics of tasks
4 	Assembled and coordinated: • "Find one" strategy • Ratio table • Fraction as quotient • Function tables • Rate of change Reinvented & made meaningful: • Unit rate strategy • Algebraic ratio Made more meaningful: • Rate of change	Make predictions in linear situations given: • A single data point, for situations where the values of the variables are proportional • Two data points, for situations where there is a starting value. Problem contexts should be chosen to make clear the distinction between changes and values.
5 	Assembled and coordinated: • Number line • Graphs in coord. plane • Rate of change • Algebraic ratio Reinvented & made meaningful: • Geometric ratio Made more meaningful: • Graphs in coord. plane	• Show change on number-line diagrams. • Make predictions in linear situations where there is a starting value, given a graph of a function in a coordinate plane.
6 	Assembled and coordinated: • Rate of change • Graphs in coord. plane Reinvented & made meaningful • Physical property	• Compare rates given two intersecting linear functions graphed in a coordinate plane. • Measure and compare the steepness of objects

Stage	Artifacts	Characteristics of tasks
1 	Reinvented & made meaningful: • Ratio table • "find one" strategy • Intensive units • Fraction-as-quotient	Tasks that involve the activity of partitive division, including: • finding fair shares • finding unit values with answers expressed in intensive units (___ per ___)

Example activities:

1. Three children shared five chocolate bars equally. How much chocolate did each child receive?

State your final answer using units: _____ _____ *per* _____ .

2.

3 pizzas cost 12 dollars

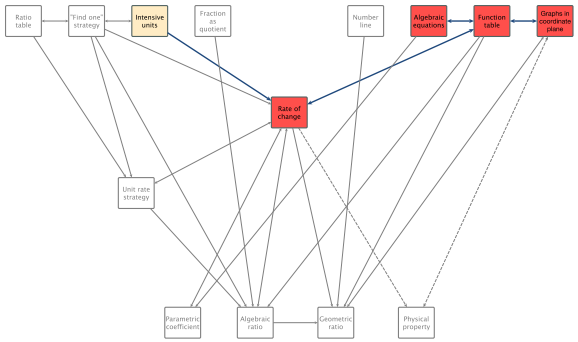
1 pizza costs ____ dollars

_____ _____ *per* _____ .

3 pizzas cost 12 dollars

____ pizzas costs 1 dollar

_____ _____ *per* _____ .

Stage	Artifacts	Characteristics of tasks
2 	Assembled and coordinated: • Intensive units Reinvented & made meaningful: • Algebraic equations • Function tables • Graphs in coord. plane • Rate of change	Tasks that involve: • Finding and continuing patterns in geometric figures and tables of values, where there is a "starting value" and the independent variable increases by 1 • Converting between multiple representations of functions (focusing on table rows and points in the plane as *solutions* to two-variable equations)

Example activities:

Plain cheese (no toppings) ---------- $6.95

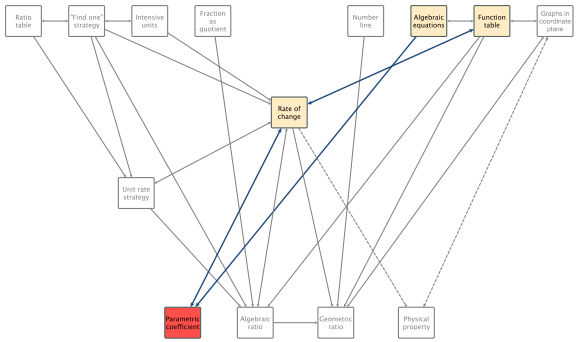
1 topping ---------- $8.20

2 toppings ---------- $9.45

3 toppings ---------- $10.70

Find patterns: What patterns do you notice in the prices?

Find a rule: Write a rule that you can use to predict the total cost of a pizza if you know the number of toppings.

Stage	Artifacts	Characteristics of tasks
3 	Assembled and coordinated: • Algebraic equations • Rate of change Reinvented & made meaningful: • Parametric coefficient Made more meaningful • Rate of change • Function tables	Making predictions in linear situations, given: • The rate of change and starting value • Multiple data points (e.g., in a table), where the independent variable increases by one.

Example activities

1.

Monday, August 04, 2008, 07:00 am PT (10:00 am ET)

Apple already building iPhones at rate of 40 million a year?

By Slash Lane

Apple is reportedly testing the limits of its overseas manufacturing facilities in order to keep up with demand for the new iPhone 3G, with production already cranked nearly sevenfold compared to the first-generation model.

Foxconn, the company's Taiwanese handset and iPod manufacturer, has recently ramped production of the new iPhone to 800,000 units per week, says *TechCrunch*, citing a person "close to Apple with direct knowledge of the numbers."

The build rate is said to be "above current full capacity" for the Foxconn facilities alloted to Apple's handset business, which has led to concerns that quality control may suffer. At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months.

That paces well ahead of analysts' estimates (1, 2, 3) and early reports that suggested Apple's initial iPhone 3G orders spanned only 25 million units through the expected lifespan of the product.

TechCrunch believes Apple's initial order was actually 40 million units over the course of the first twelve months, but is now hearing that "those numbers are being revised upwards sharply."

Apple said it sold 1 million iPhones in the first 72 hours the new iPhone 3G was put on sale, but has not provided an updated sales tally since. The iPhone is currently on sale in 23 countries, with 20 more expected to be added on August 22nd, and another 30 by the end of the calendar year.

(just search Google for "at the current rate" (in quotes))

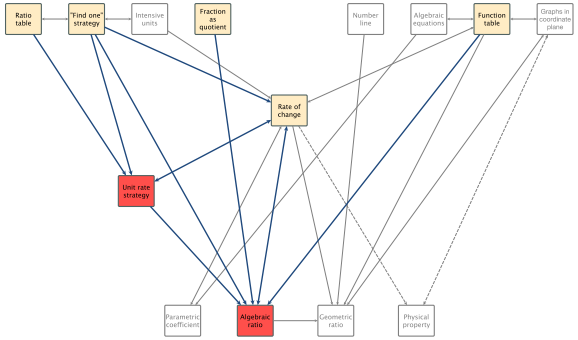
Class discussion:

- "What prediction does the author make?
- "How does the author make this prediction?"
- "Why does multiplication make a prediction?"

Goal is to discipline perception to the role of rates and multiplication to make predictions.

2. The table below shows the cost of shipping used X-box games from CHEEP GAMZ ONLINE. Some of the data is missing. Based on the data in the table, how much would it cost to have 12 games shipped?

Number of games	Total cost
0	
1	
2	20.00
3	26.00
4	32.00
5	38.00
6	44.00

Stage	Artifacts	Characteristics of tasks
4 	Assembled and coordinated: • "Find one" strategy • Ratio table • Fraction as quotient • Function tables • Rate of change Reinvented & made meaningful: • Unit rate strategy • Algebraic ratio Made more meaningful • Rate of change	Make predictions in linear situations given: • A single data point, for situations where the values of the variables are proportional • Two data points, for situations where there is a starting value. Problem contexts should be chosen to make clear the distinction between changes and values.

Example activities:

1. Unit rate strategy

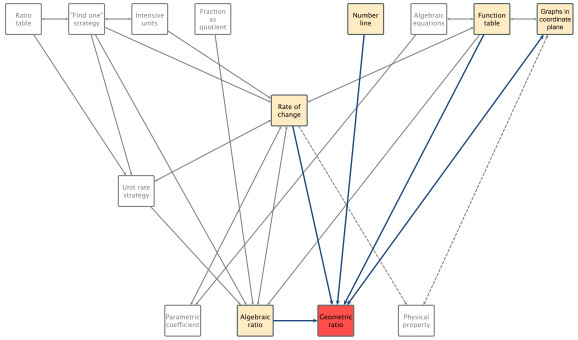
Ms. Magro runs six miles every day. On average it takes her 54 minutes to run six miles. At this rate, how long will it take Ms. Magro to run an 11-mile race?

2. Algebraic ratio

At the end of the summer, the YMCA drains their swimming pool. Raif and Julie are in charge of measuring the height of the pool as it drains.

> **Raif says:** I checked the pool two hours after we started draining it. When I checked, the height of the water was 517 mm.
>
> **Julie says:** I checked the pool seven hours after we started draining it. When I checked, the height of the water was 420 mm.

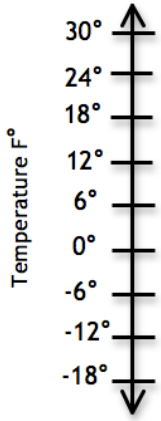
Imagine you checked the height of the pool five hours after the YMCA started draining it. What would the height have been?

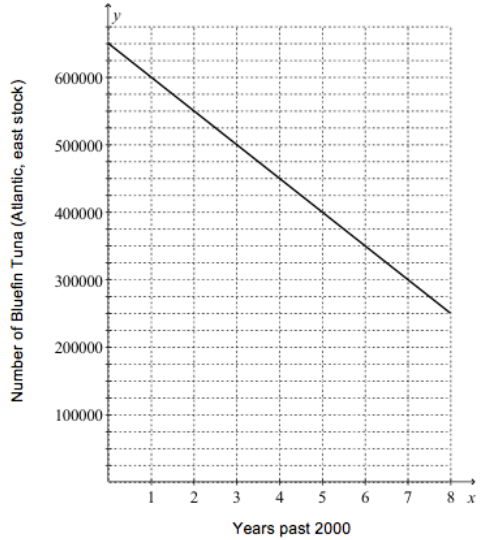
Stage	Artifacts	Characteristics of tasks
5 	Assembled and coordinated: • Number line • Graphs in coord. plane • Rate of change • Algebraic ratio Reinvented & made meaningful: • Geometric ratio Made more meaningful: • Graphs in coord. plane	• Show change on number-line diagrams. • Make predictions in linear situations where there is a starting value, given a graph of a function in a coordinate plane.

Example activities:

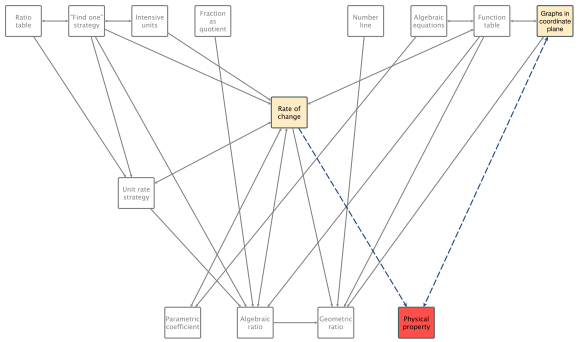
1. The temperature in Alamosa, Colorado rose from -12° to 24°.

   Draw an arrow on the number line below to show this change.



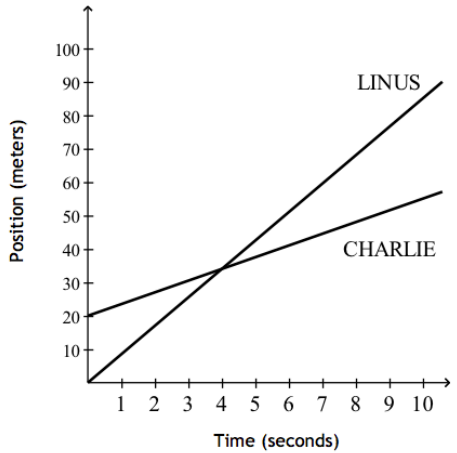
2. 

   - What are two variables that are changing in the graph?
   - Write a sentence that describes how these two variables are changing
   - Find the rate of change in the graph
   - At this rate, when will the stock of Bluefin tuna be depleted?

Stage	Artifacts	Characteristics of tasks
6 	Assembled and coordinated: • Rate of change • Graphs in coord. plane Reinvented & made meaningful • Physical property	• Compare rates given two intersecting linear functions graphed in a coordinate plane. • Measure and compare the steepness of objects

Example activities

Charlie and Linus are running along a straight track. A position vs. time graph for both runners is shown below.



- At the instant, $t = 2$ sec, who is running faster, Charlie or Linus?
- Do Linus and Charlie ever have the same speed? If so, at what time?