Beyond rise over run: Activities to invent and connect slope's five faces

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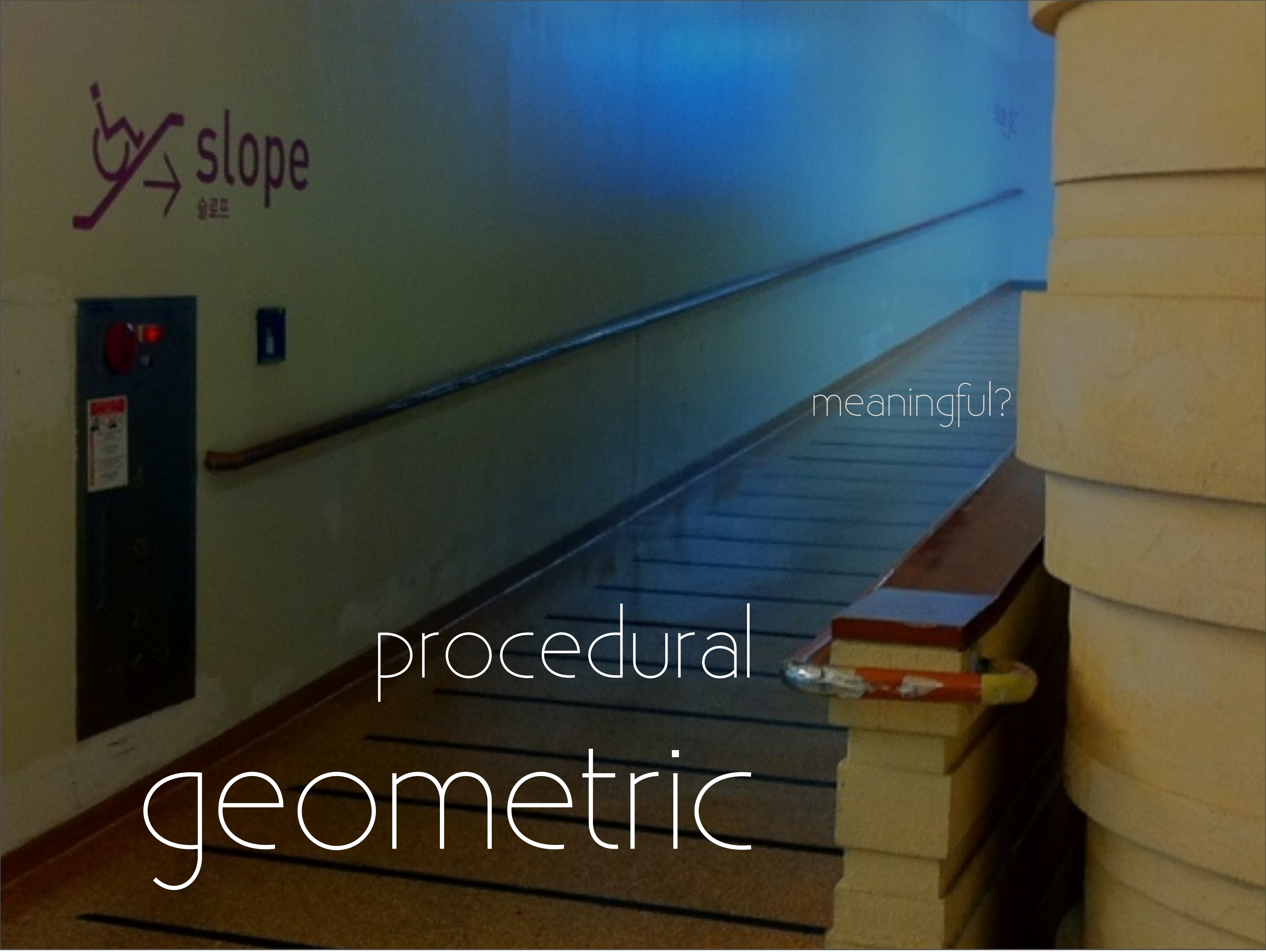
First, let's do some math!

Feel free to write on this.

I'm going to give you a clean copy.

And all handouts are on my webpage:
www.RMEInTheClassroom.com


slope
meaningful?
procedural
geometric


slope
meaningful?

How do students make slope meaningful?

slope

- Parametric coefficient
- Algebraic ratio
- Rate of change
- Geometric ratio
- Physical property

slope

Parametric coefficient	Algebraic ratio	Rate of change	Geometric ratio	Physical property

$$y = {\color{red}a}x + b$$

slope

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

Physical property

$$\frac{y_2 - y_1}{x_2 - x_1}$$

slope

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

Physical property

slope

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

Physical property

$$\frac{\text{rise}}{\text{run}}$$

slope

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

Physical property

"steepness"

slope

- Parametric coefficient
- Algebraic ratio
- Rate of change
- Geometric ratio
- Physical property

Just tell them.

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

Physical property

Algebra Lesson Page

Slope and Rate of Change

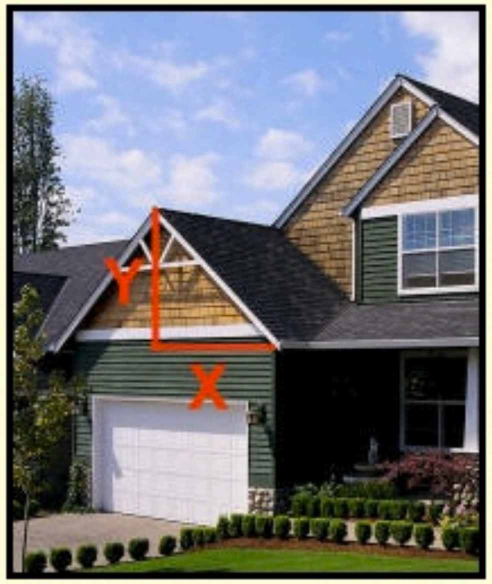
Topic Index | Algebra Index | Regents Exam Prep Center

Slope and Rate of Change

The word **slope** (gradient, incline, pitch) is used to describe the measurement of the steepness of a straight line. The higher the slope, the steeper the line. The slope of a line is a *rate of change*.

$$Slope = \frac{Vertical\ change}{Horizontal\ change} = \frac{Rise}{Run}$$

The building code for using asphalt shingles on roofs states that the minimum pitch must be a rise of 4" for every 12" of horizontal distance (run) covered. Asphalt shingles are not to be used on roofs that have very little steepness. Builders check to see if the pitch (slope) of the roof is $\frac{4}{12}$ or 4:12 or 4 to 12 before using asphalt shingles.



Builders need to know the pitch of a roof to determine which type of shingle will be appropriate for the roof.

Slope is a ratio and can be expressed as:

change in y over change in x. or $\frac{vertical\ change}{horizontal\ change}$ or $\frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{rise}{run}$

http://www.regentsprep.org/regents/math/algebra/AC1/Rate.htm

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

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Physical property

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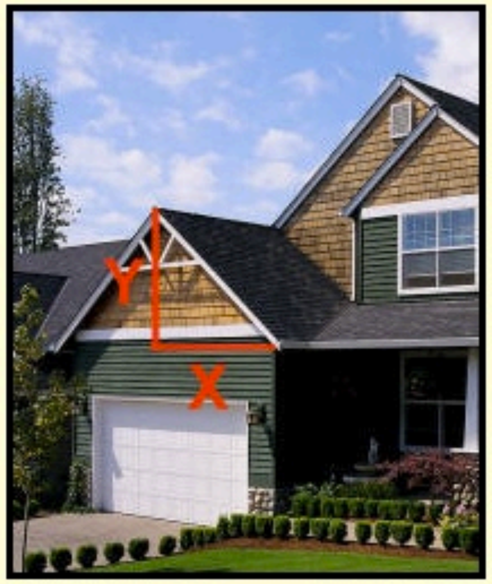

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Physical property

Slop ate of Change

Rate of hange

Geometric ratio

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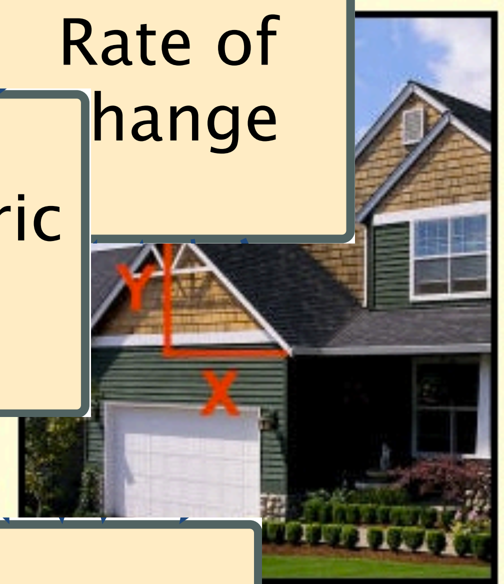

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Algebraic ratio

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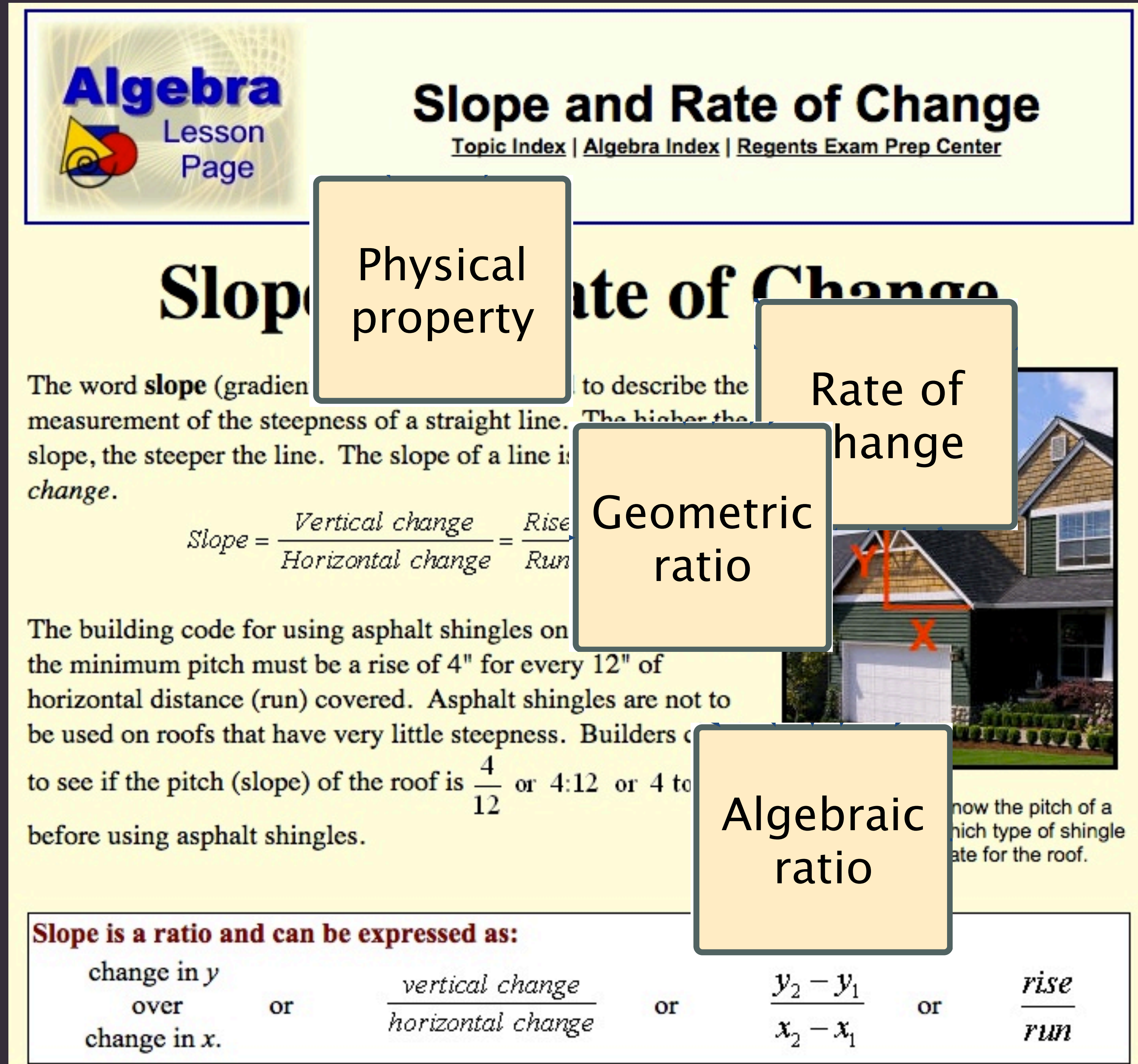


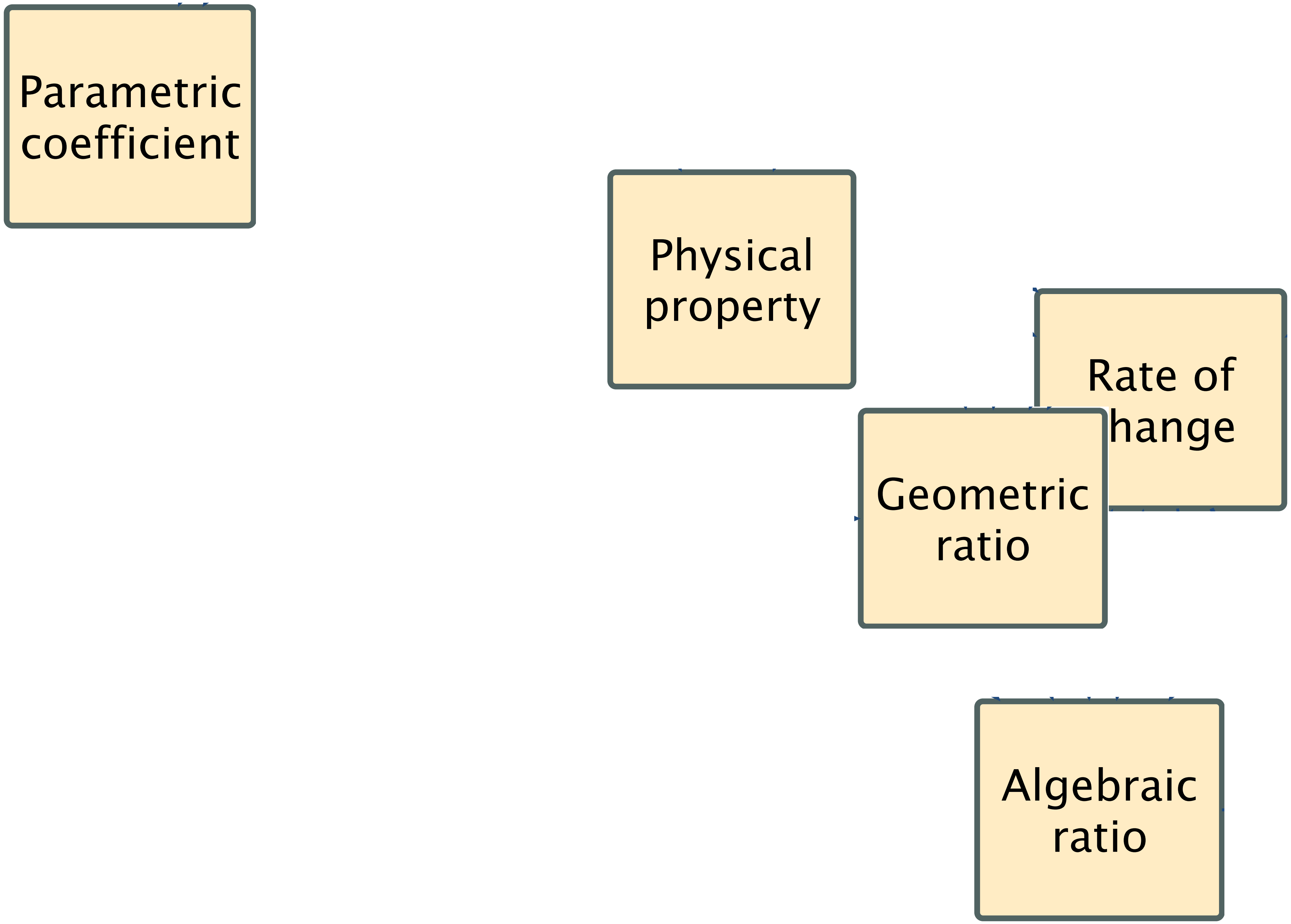
Physical property

Rate of change

Geometric ratio

Algebraic ratio

meaningful?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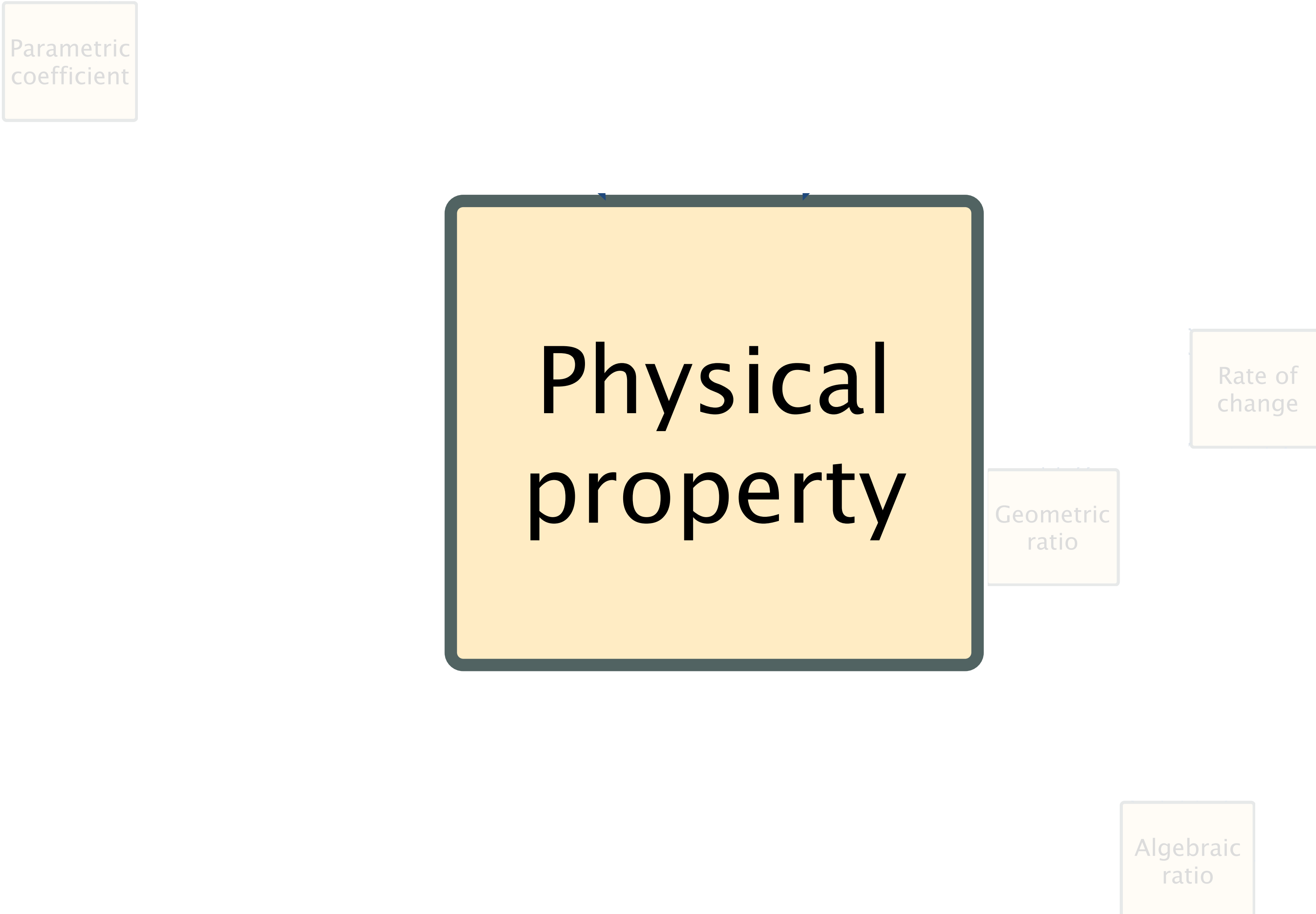
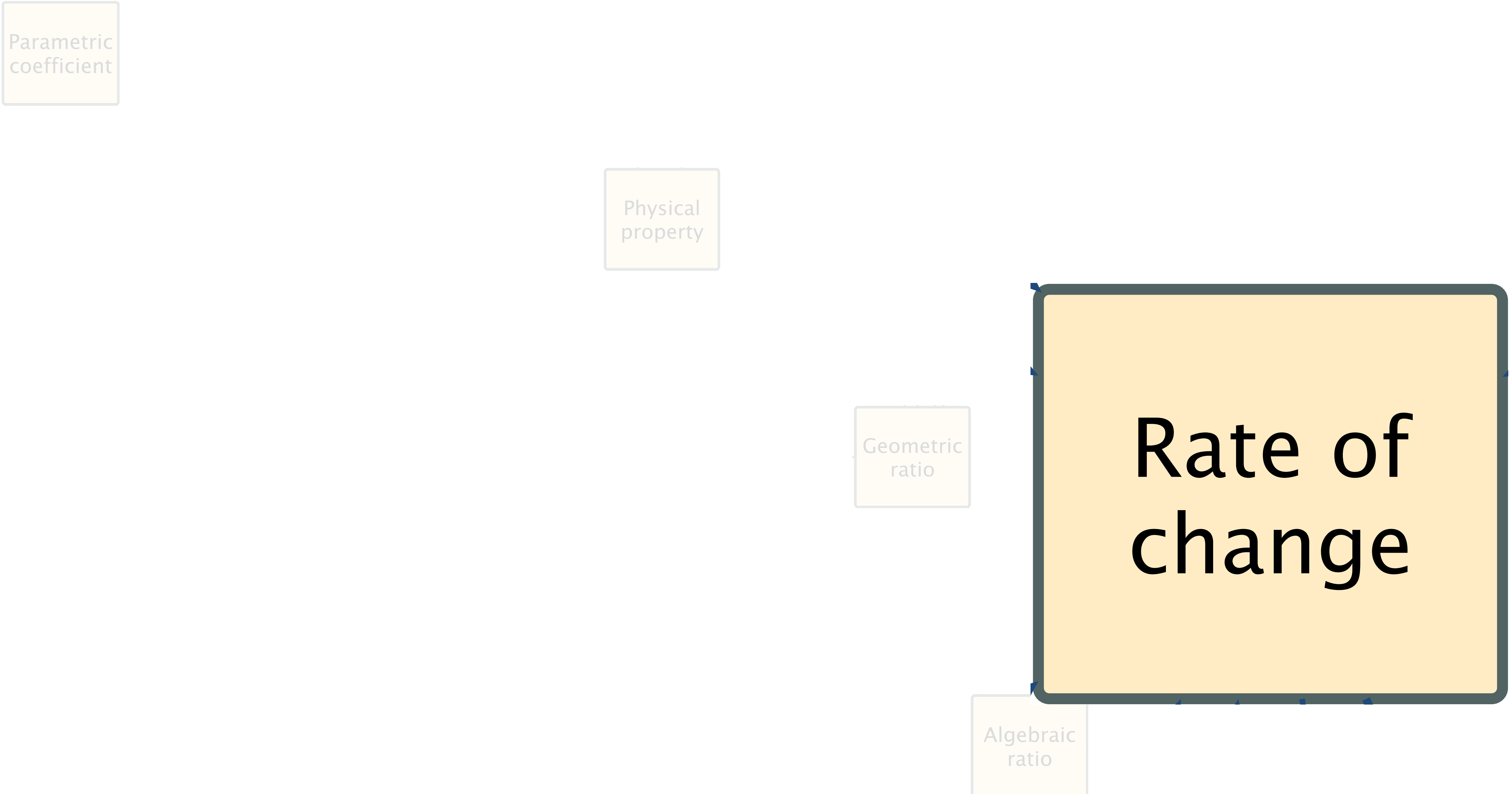


Why not steepness?

- Motivating?
- Robust?
    - y=mx+b



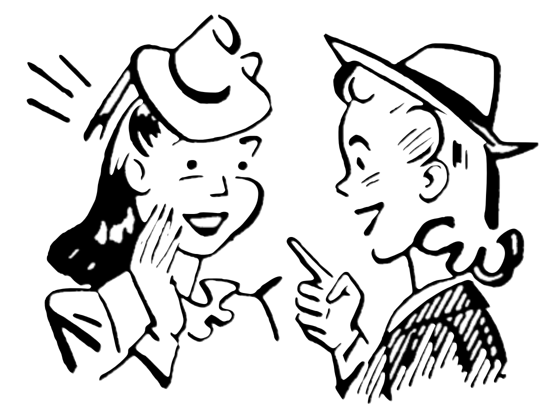
Why rate of change?

- Motivating :: predicting the future
- Robust :: one of five NCTM "key concepts"
  - Starts with proportional reasoning
  - All sub-constructs of slope can be built from there

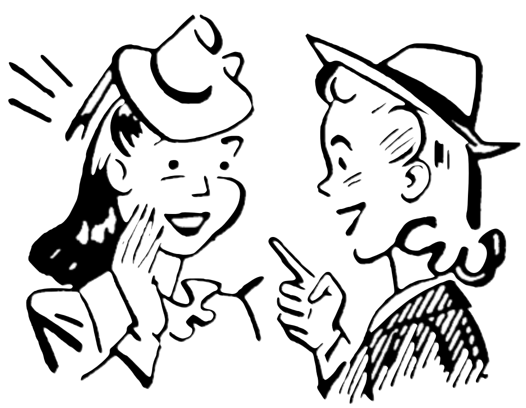
Parametric coefficient

Physical property

Rate of change



Rate of change



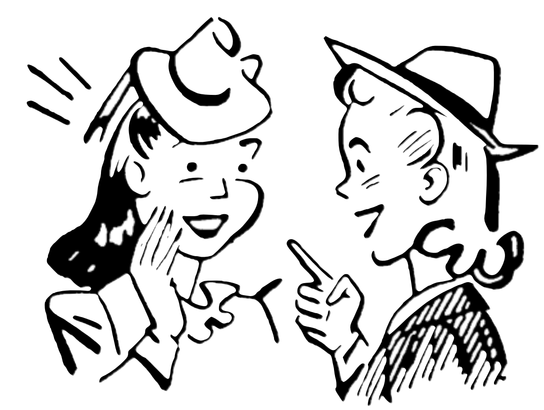
Parametric coefficient

Algebraic ratio

Geometric ratio

Physical property

Rate of change



Parametric coefficient

Algebraic ratio

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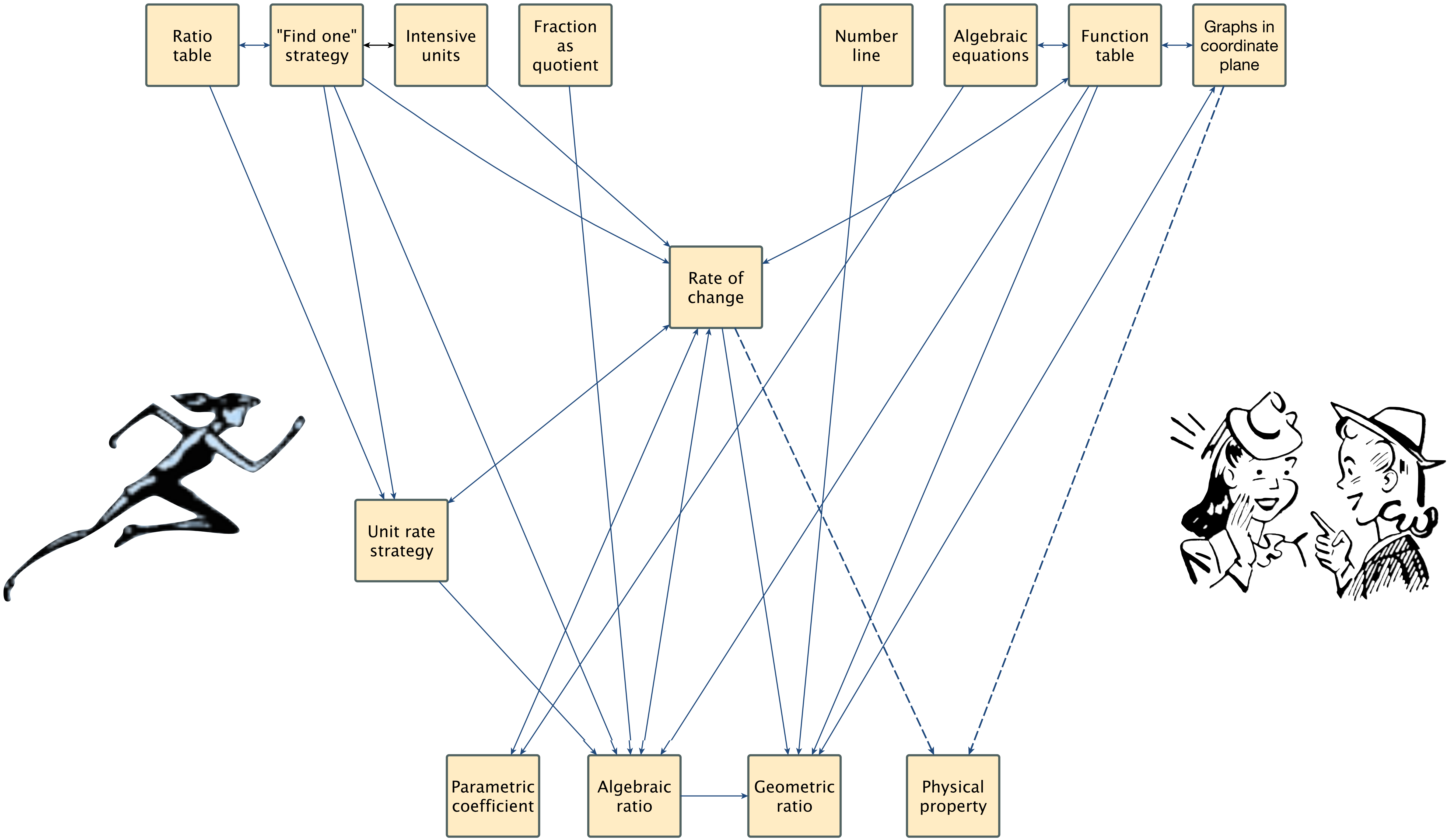
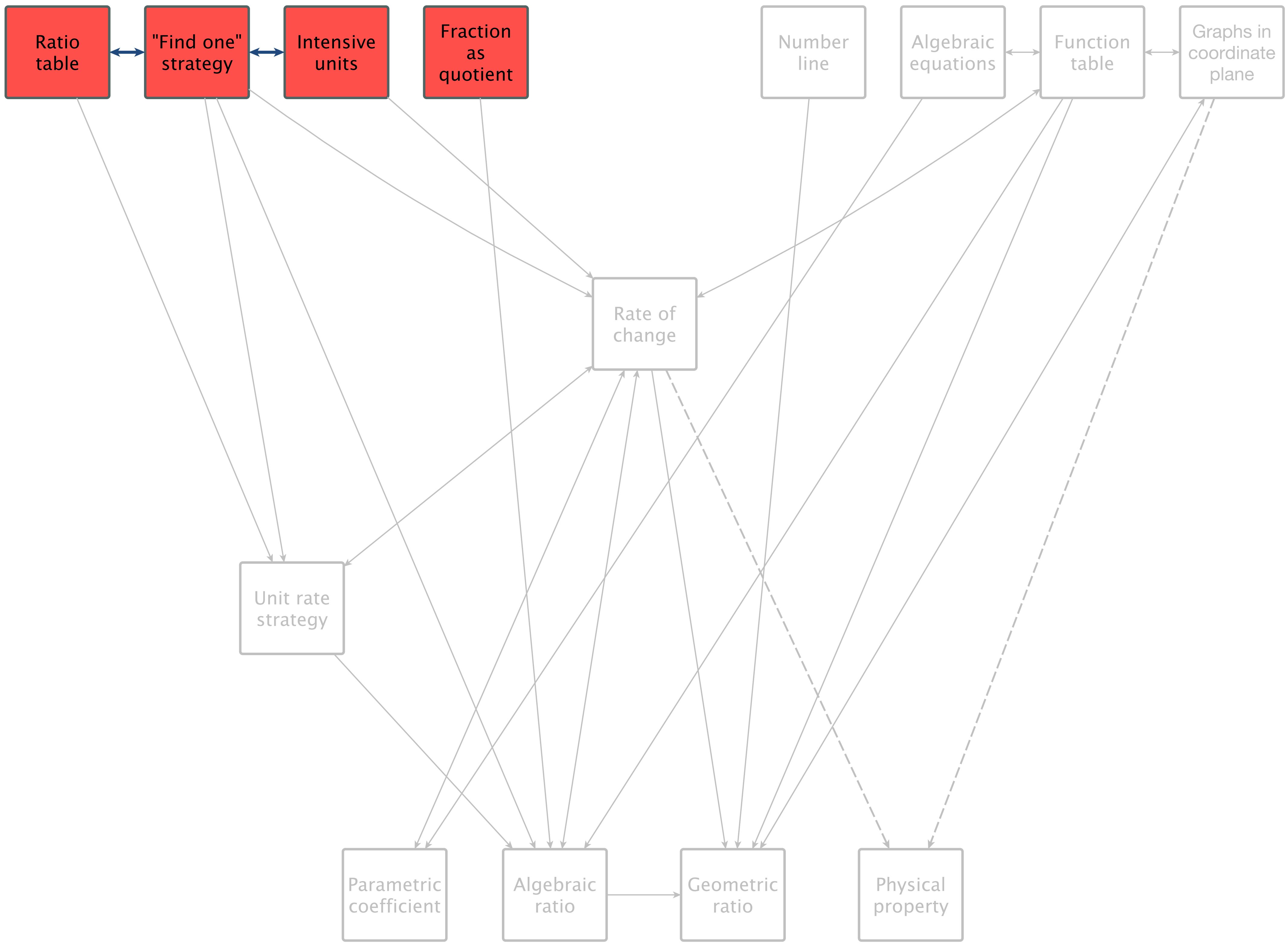
Physical property

cascade of artifacts



stage 1


Ratio table
"Find one" strategy
Intensive units
Fraction as quotient
Number line
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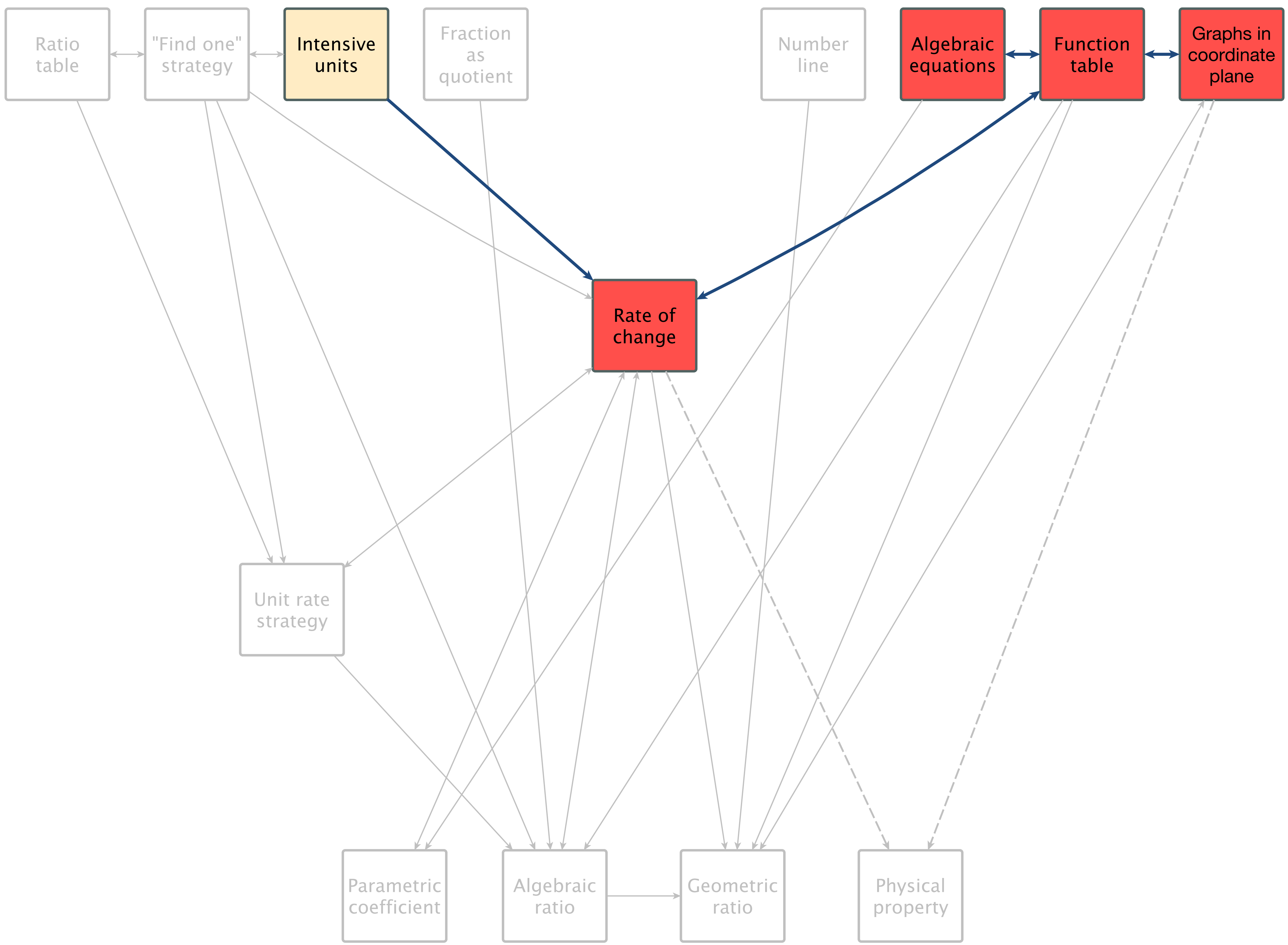
stage 2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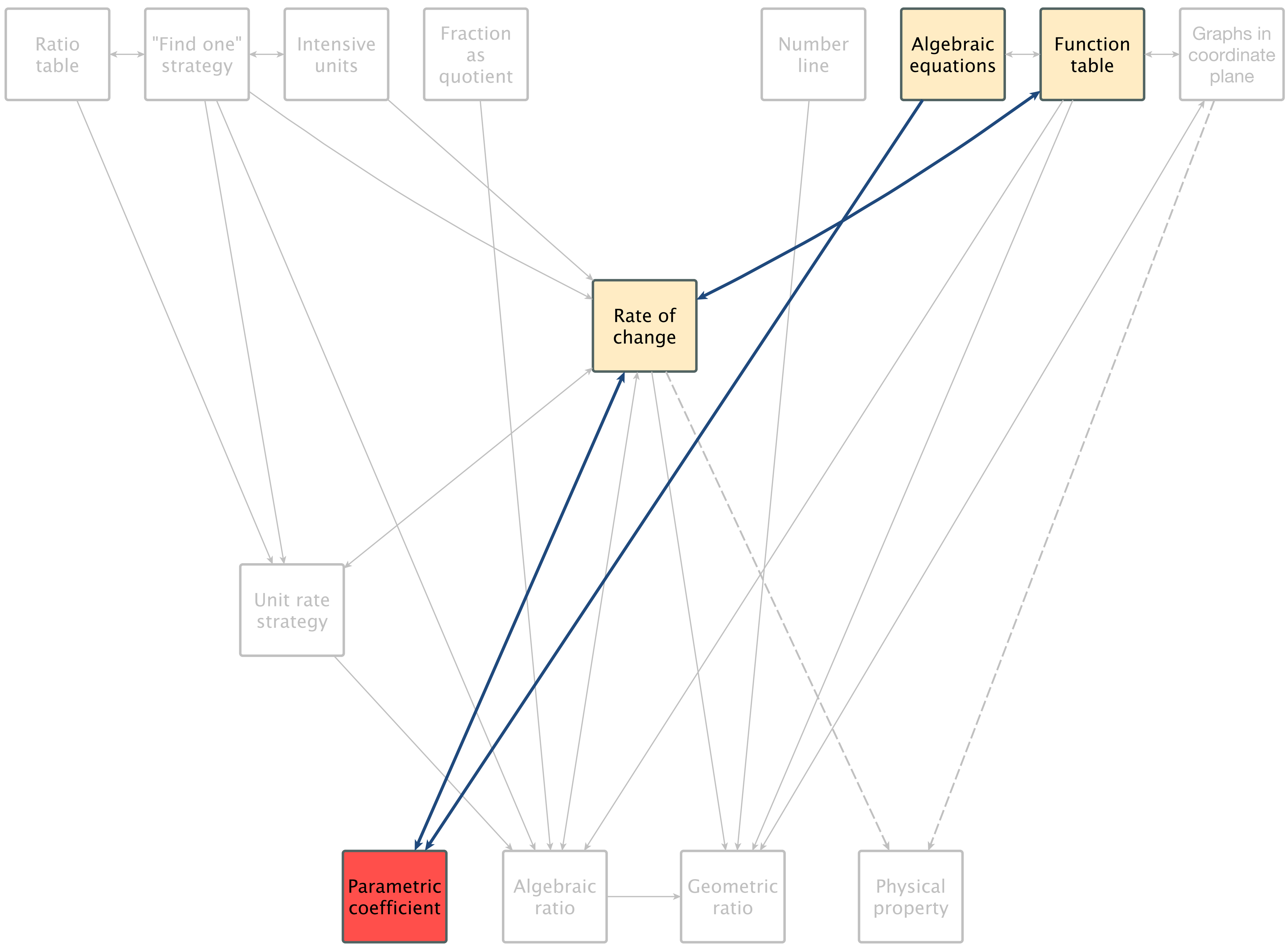
stage 3



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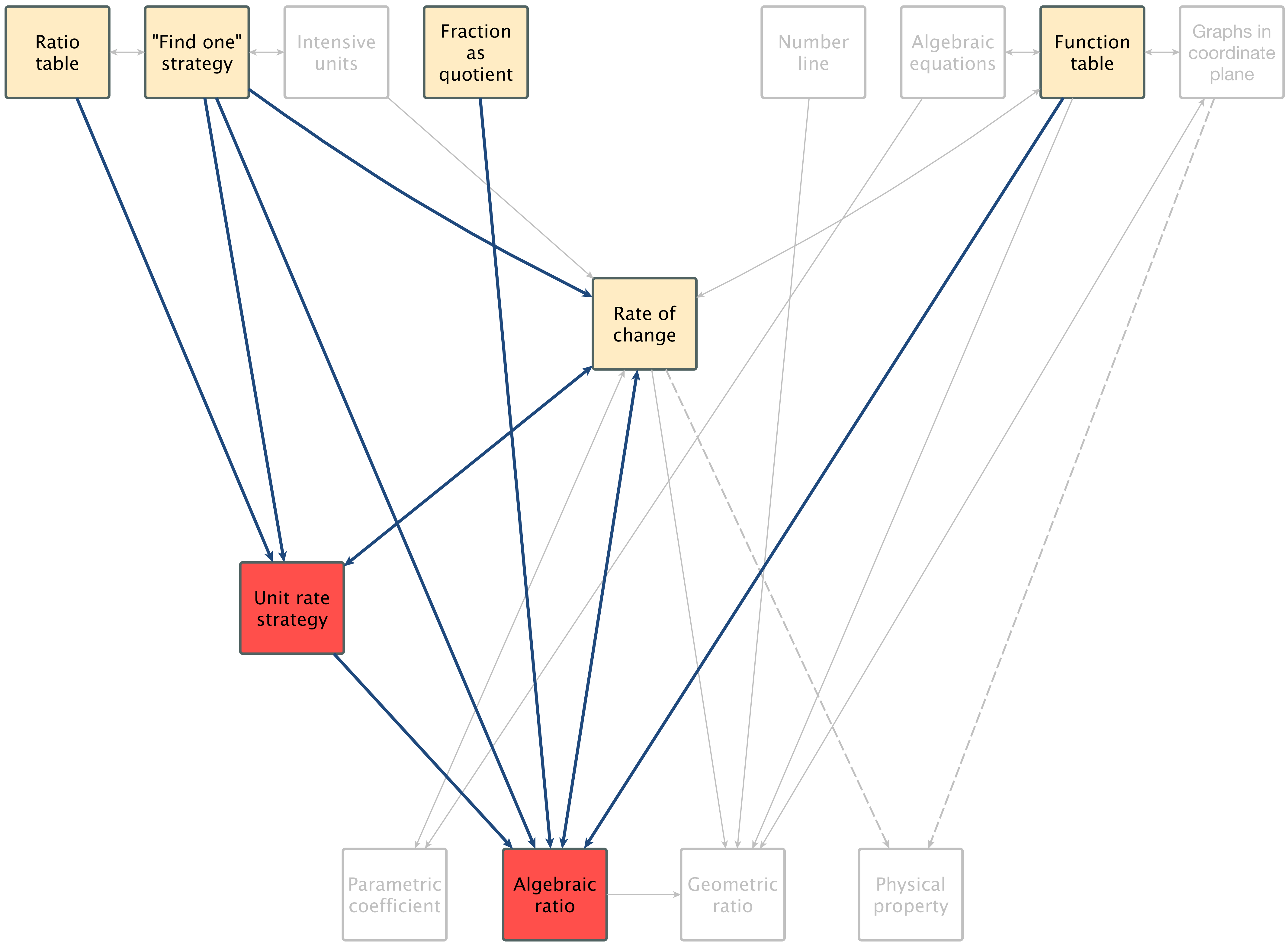
stage 4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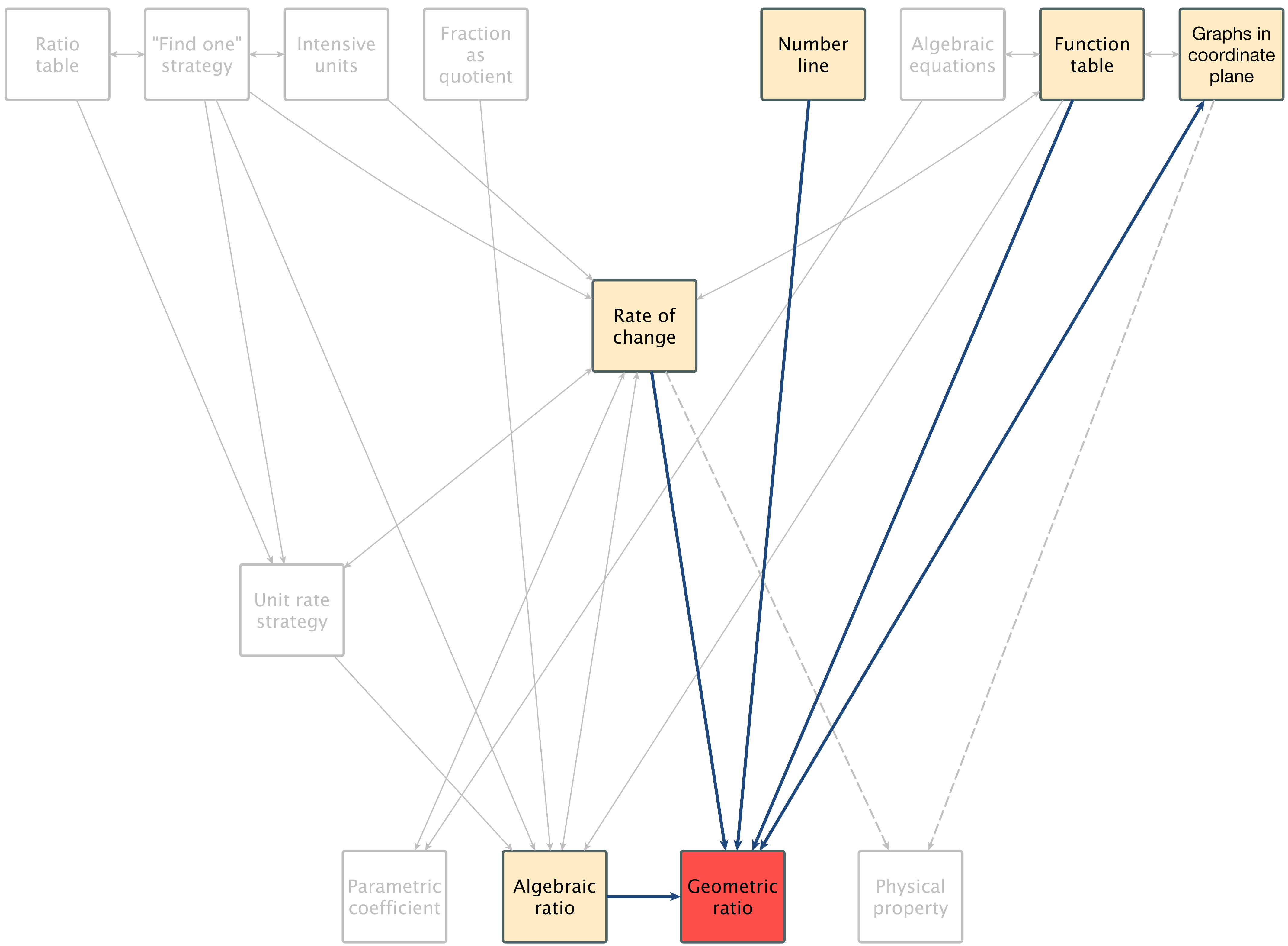
stage 5



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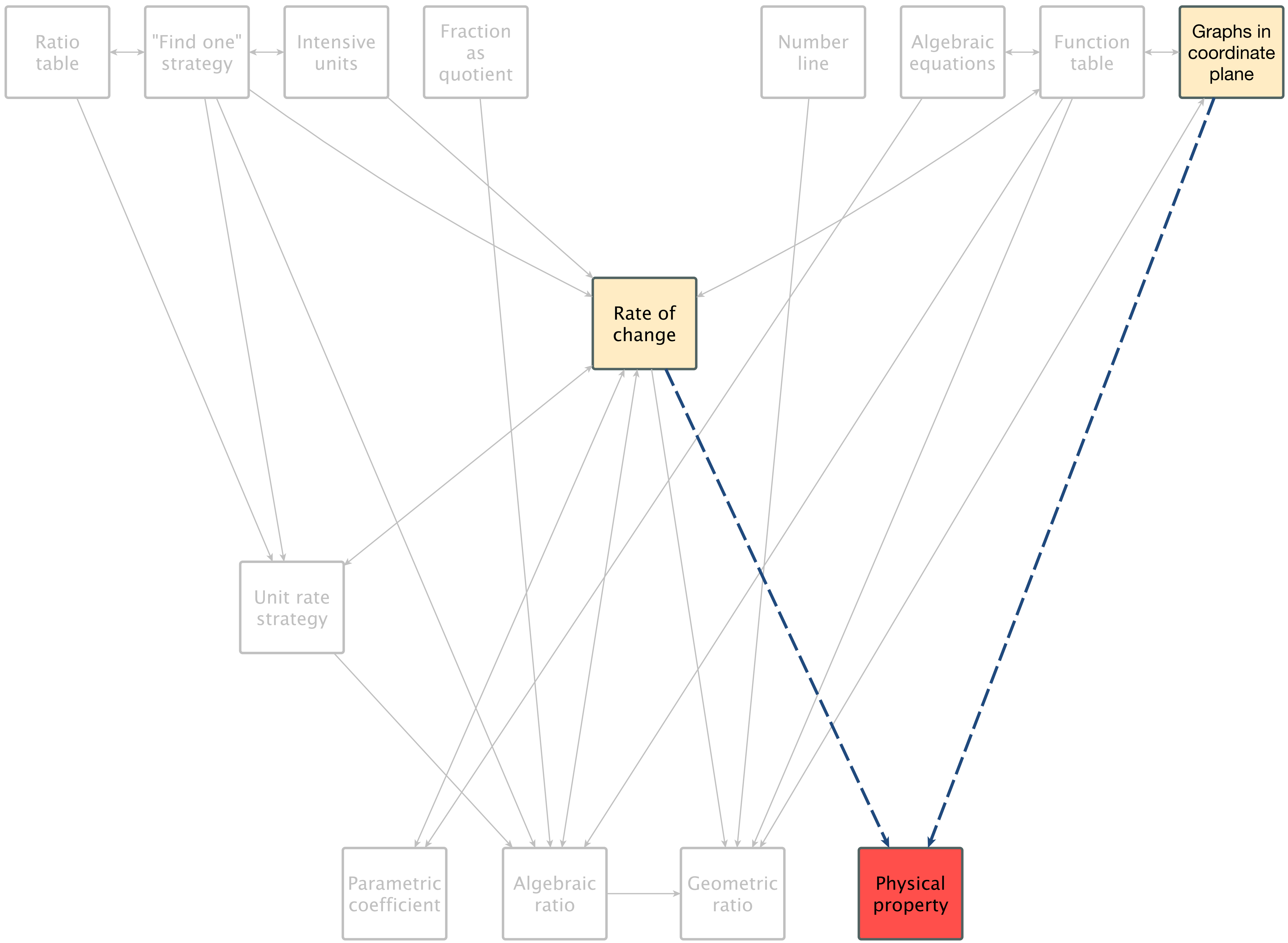
stage 6



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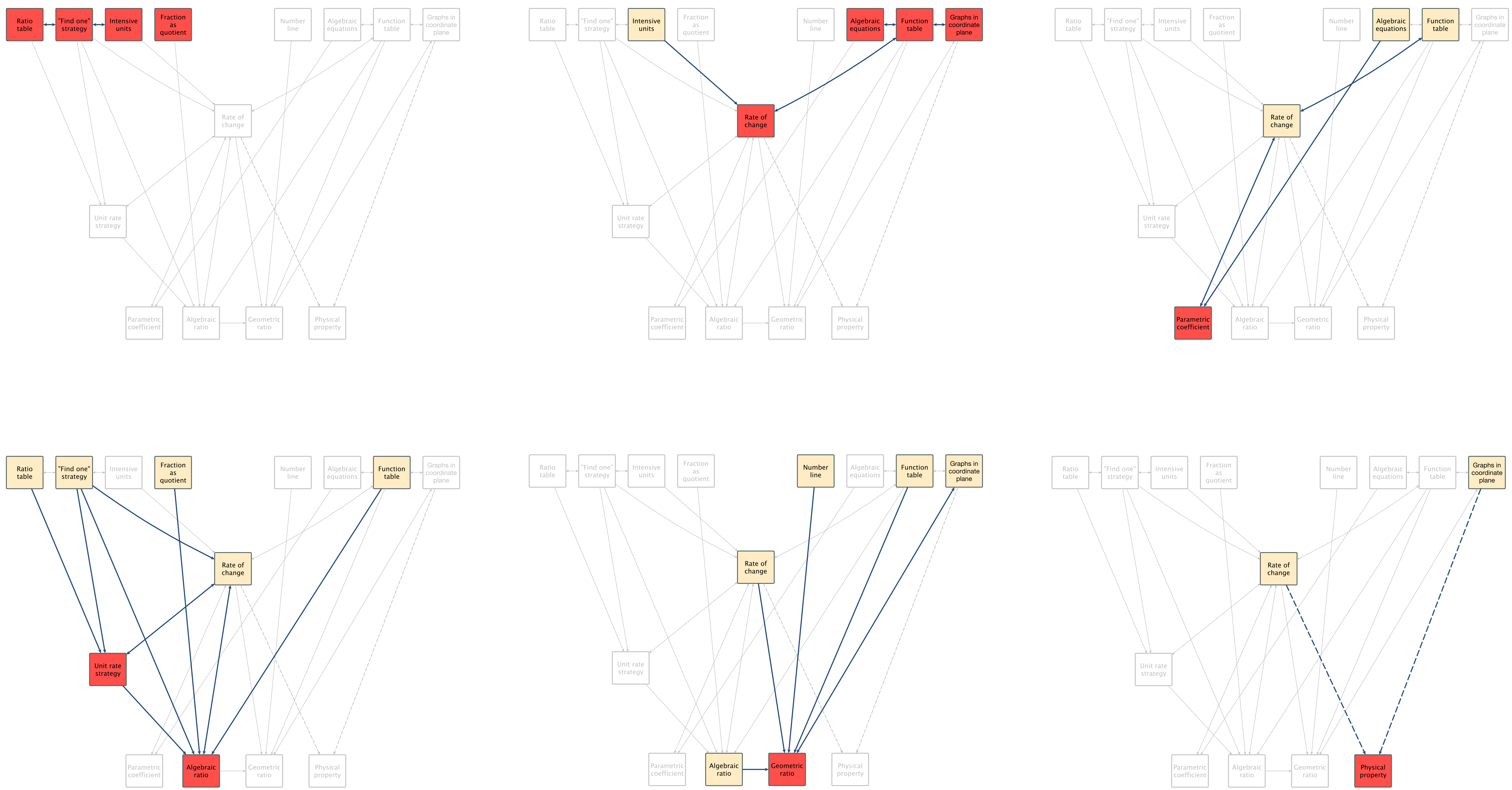
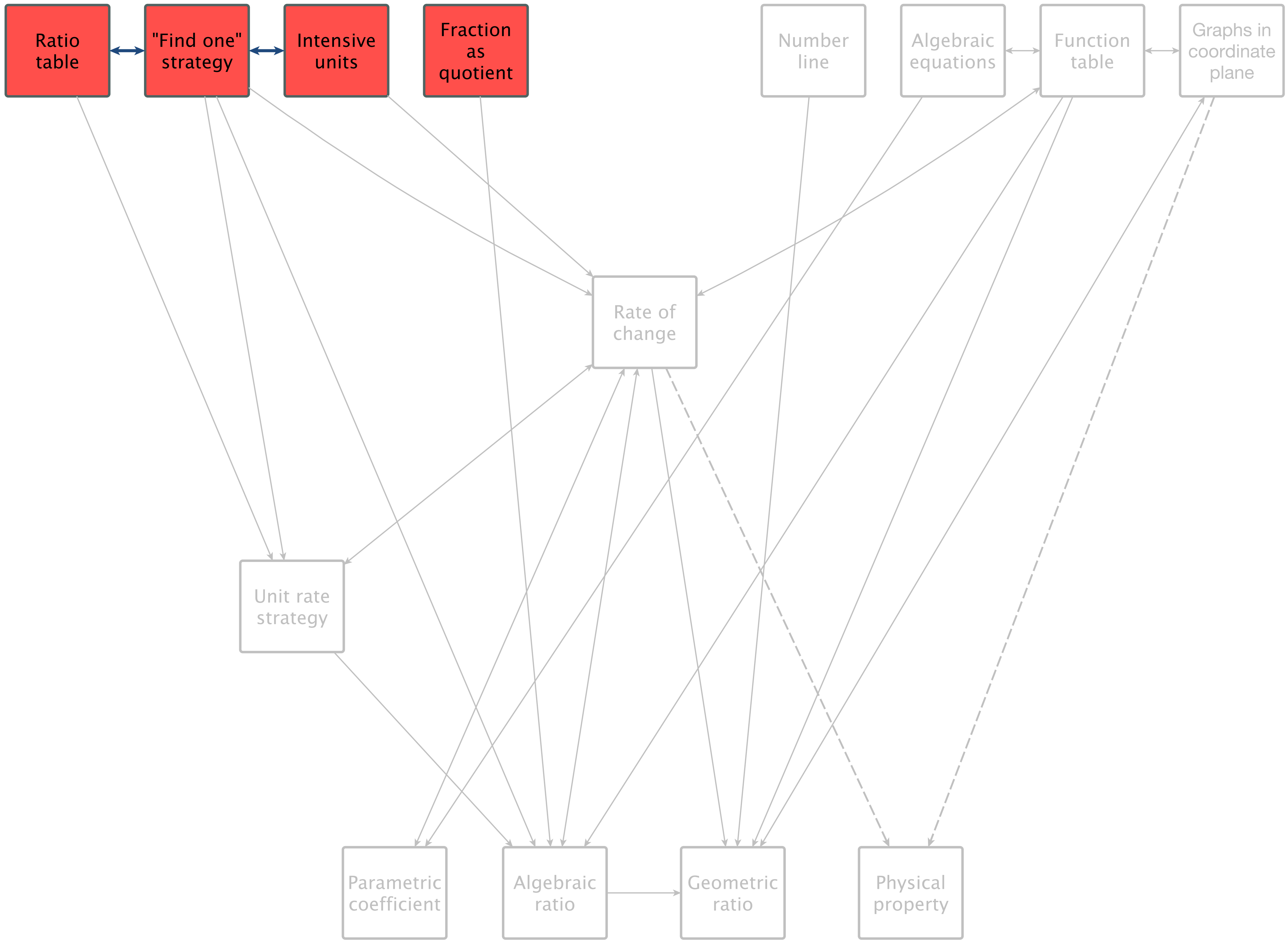
progression of learning



stage 1



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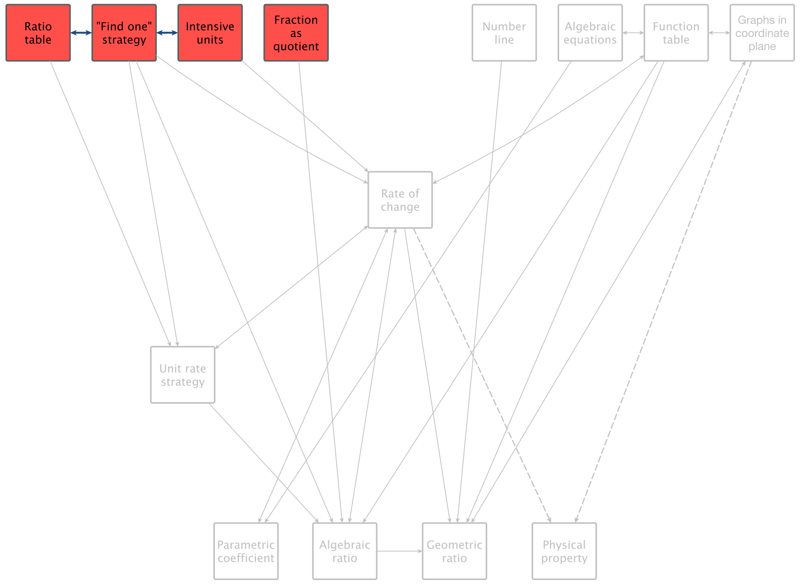


stage 1

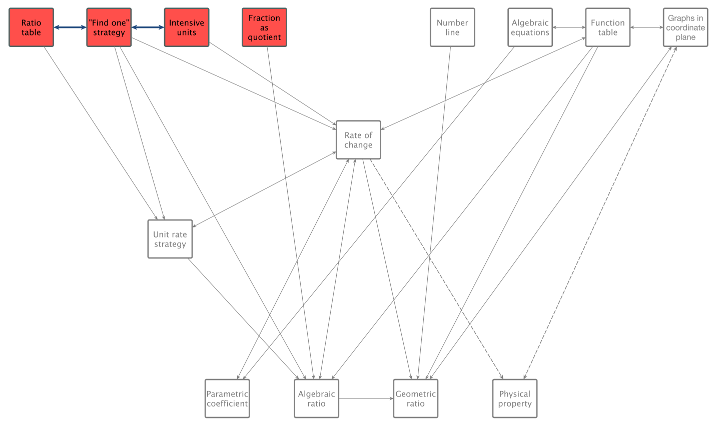
Reinvented & made meaningful

- ratio table
- "find one" strategy
- intensive units (many-to-one)
- fraction-as-quotient

Activities

"partitive division" situations

- fair sharing
- find unit values given many-to-many

<table>
<tr><th>Stage</th><th>Artifacts</th><th>Characteristics of tasks</th></tr>
<tr><td>1

</td><td>Reinvented & objectified:
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finding fair shares
finding unit values</td></tr>
</table>

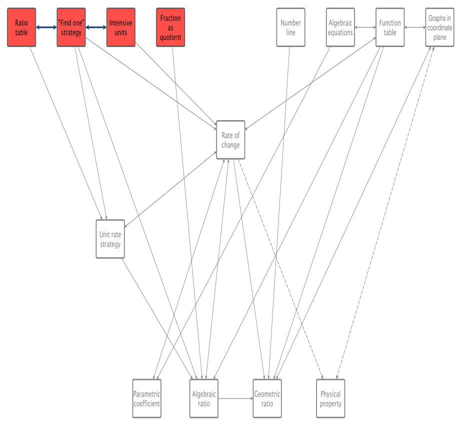
Example activities:

1. Three children shared five chocolate bars equally. How much chocolate did each child receive?

   State your final answer using units: ________ ____________________ *per* ____________________.

2.

3 pizzas cost 12 dollars	3 pizzas cost 12 dollars
1 pizza costs ____ dollars	____ pizzas costs 1 dollar
________ ____________ *per* ____________.	________ ____________ *per* ____________.

Stage	Artifacts	Characteristics of tasks
1 	Reinvented & objectified: • Ratio table • "find one" strategy • Intensive units • Fraction-as-quotient	Tasks that involve the activity of partitive division, including: • finding fair shares • finding unit values

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State your final answer using units: ______ ______________ *per* ______________.

Fraction-as-quotient

Intensive units

2.

3 pizzas cost 12 dollars

1 pizza costs ___ dollars

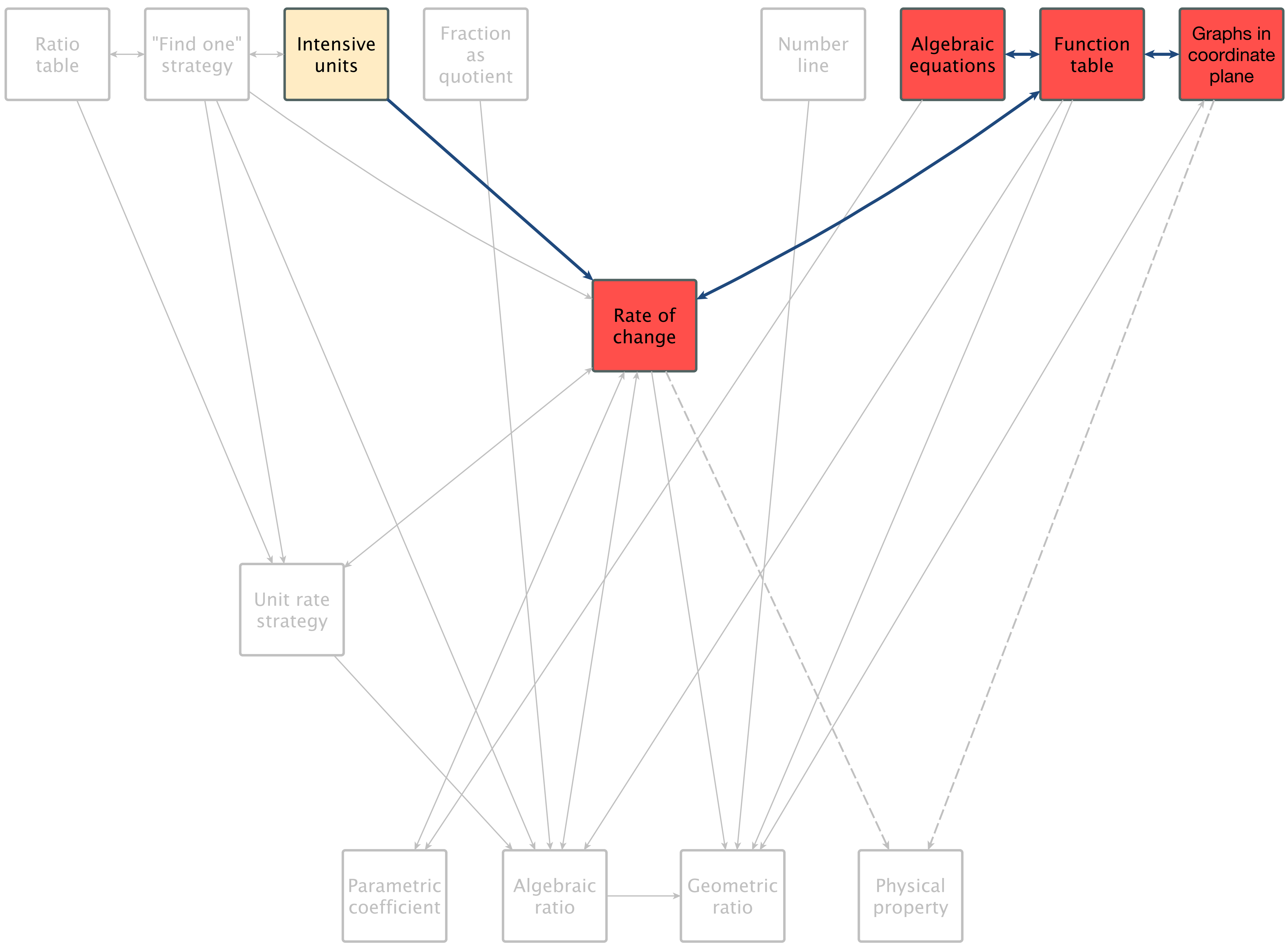
3 pizzas cost 12 dollars

___ pizzas costs 1 dollar

Ratio table

"find one" strategy
intensive units

stage 2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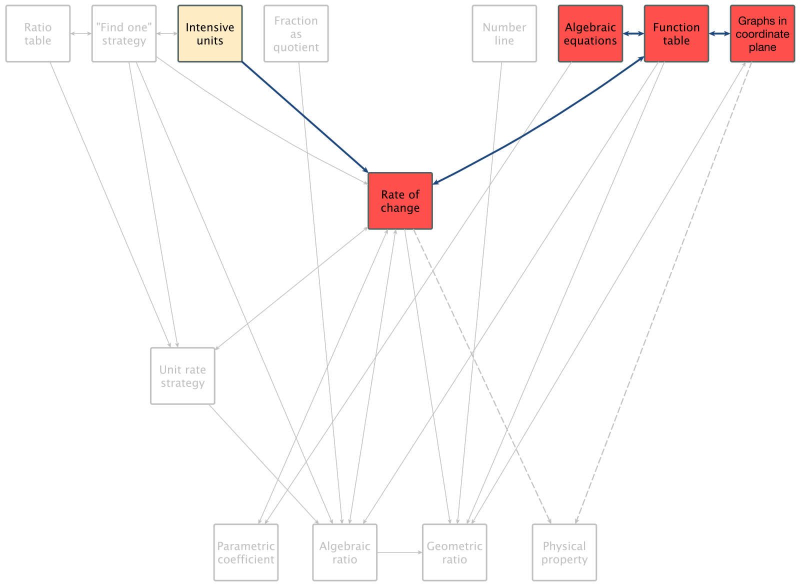
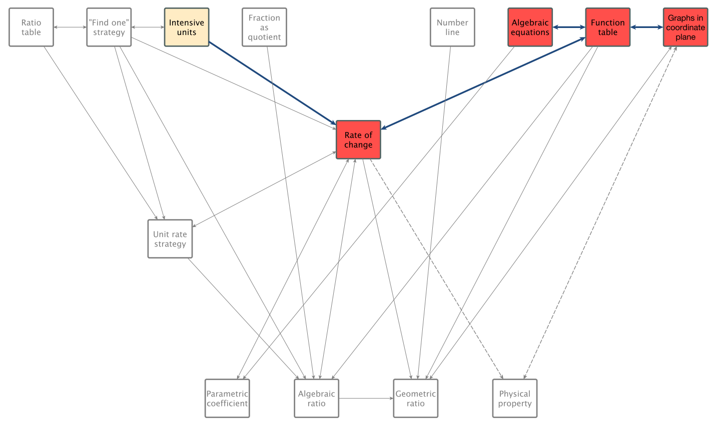


stage 2

Reinvented & made meaningful
- function tables
- algebraic equations
- graphs in coord. plane
- rate of change

Assembled & coordinated
- intensive units

Activities
- find and continue patterns
- convert between multiple representations of functions

Stage	Artifacts	Characteristics of tasks
2 	Assembled and coordinated: • Intensive units Reinvented & objectified: • Algebraic equations • Function tables • Graphs in coord. plane • Rate of change	Tasks that involve: • Finding and continuing patterns in geometric figures and tables of values, where there is a "starting value" and the independent variable increases by 1 • Converting between multiple representations of functions (focusing on table rows and points in the plane as *solutions* to two-variable equations)

Example activities:

Plain cheese (no toppings) ---------------- $6.95

1 topping -------------------------------- $8.20

2 toppings ------------------------------ $9.45

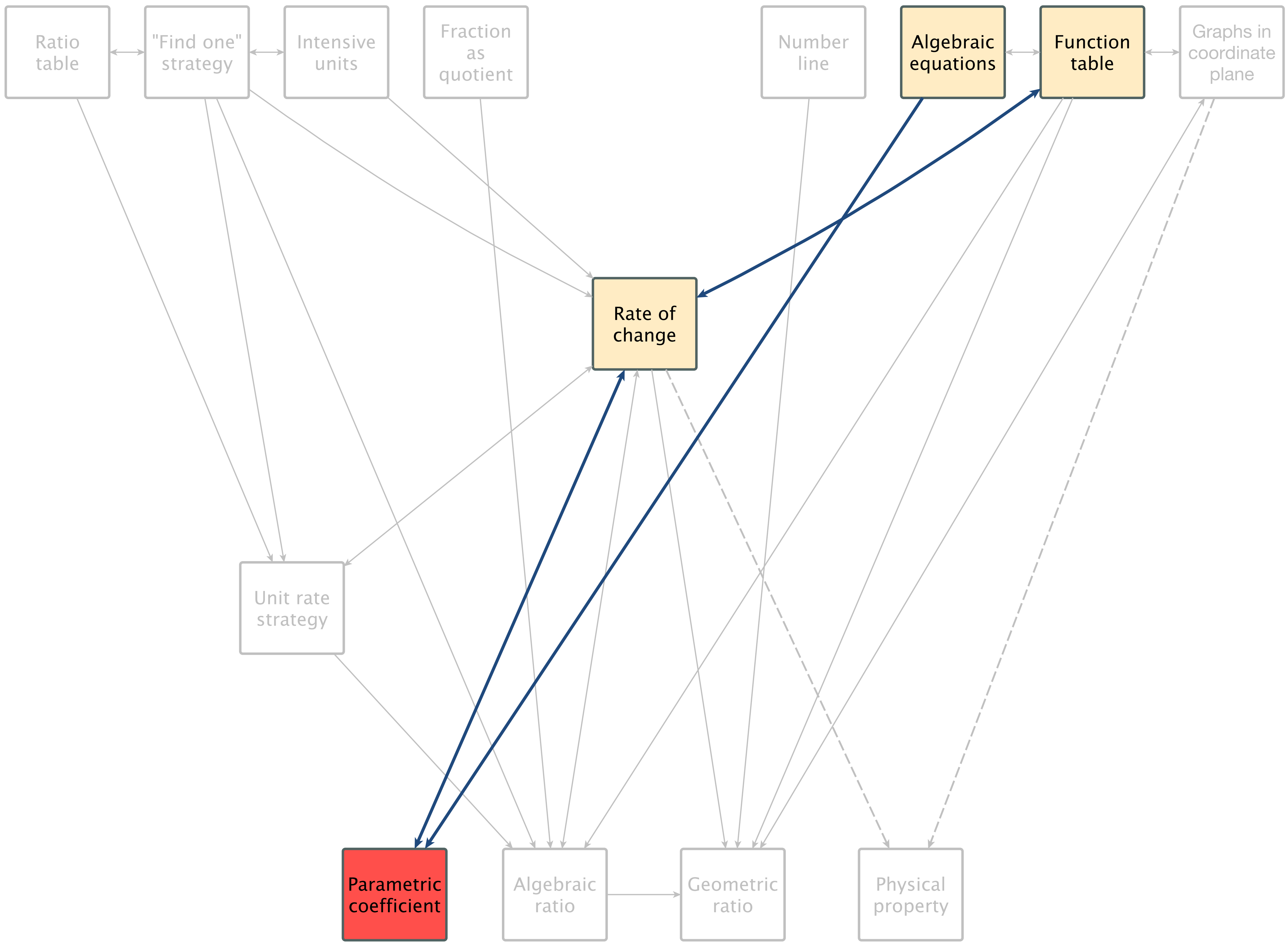
3 toppings ------------------------------ $10.70

Find patterns: What patterns do you notice in the prices?

Find a rule: Write a rule that you can use to predict the total cost of a pizza if you know the number of toppings.

Write the rule as an equation	*Write the rule with an arrow chain*

stage 3



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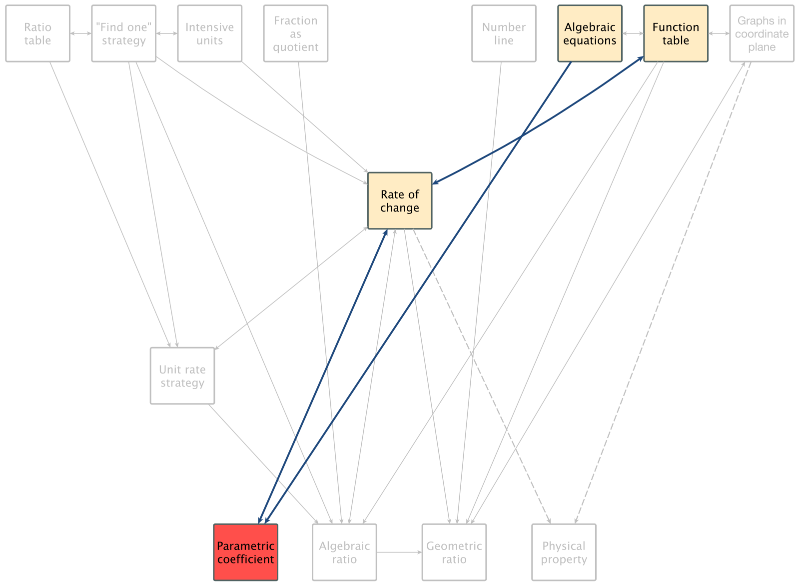


stage 3

Reinvented & made meaningful

- parametric coefficient

Assembled & coordinated

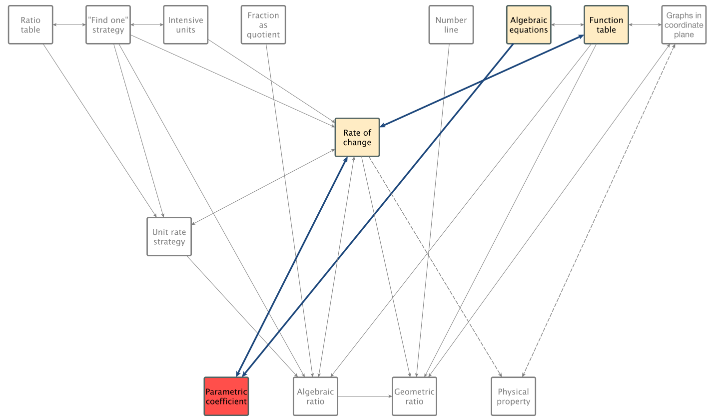
- algebraic equations
- function tables
- rate of change

Activities

make predictions given:

- rate and start
- well-ordered function table ($\Delta x = 1$)

Stage	Artifacts	Characteristics of tasks
3	Assembled and coordinated: • Algebraic equations • Rate of change Reinvented & objectified: • Parametric coefficient Objectified • Rate of change • Function tables	Making predictions in linear situations, given: • The rate of change and starting value • Multiple data points (e.g., in a table), where the independent variable increases by one.



Example activities

1.

Monday, August 04, 2008, 07:00 am PT (10:00 am ET)

Apple already building iPhones at rate of 40 million a year?

By Slash Lane

Apple is reportedly testing the limits of its overseas manufacturing facilities in order to keep up with demand for the new iPhone 3G, with production already cranked nearly sevenfold compared to the first-generation model.

Foxconn, the company's Taiwanese handset and iPod manufacturer, has recently ramped production of the new iPhone to 800,000 units per week, says *TechCrunch*, citing a person "close to Apple with direct knowledge of the numbers."

The build rate is said to be "above current full capacity" for the Foxconn facilities alloted to Apple's handset business, which has led to concerns that quality control may suffer. At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months.

That paces well ahead of analysts' estimates (1, 2, 3) and early reports that suggested Apple's initial iPhone 3G orders spanned only 25 million units through the expected lifespan of the product.

TechCrunch believes Apple's initial order was actually 40 million units over the course of the first twelve months, but is now hearing that "those numbers are being revised upwards sharply."

Apple said it sold 1 million iPhones in the first 72 hours the new iPhone 3G was put on sale, but has not provided an updated sales tally since. The iPhone is currently on sale in 23 countries, with 20 more expected to be added on August 22nd, and another 30 by the end of the calendar year.

(just search Google for "at the current rate" (in quotes))

Class discussion:

- "What prediction does the author make?
- "How does the author make this prediction?"
- "Why does multiplication make a prediction?"

Goal is to discipline perception to the role of rates and multiplication to make predictions.

2. The table below shows the cost of shipping used X-box games from CHEEP GAMZ ONLINE. Some of the data is missing. Based on the data in the table, how much would it cost to have 12 games shipped?

Number of games	Total cost
0	
1	
2	20.00
3	26.00
4	32.00
5	38.00
6	44.00

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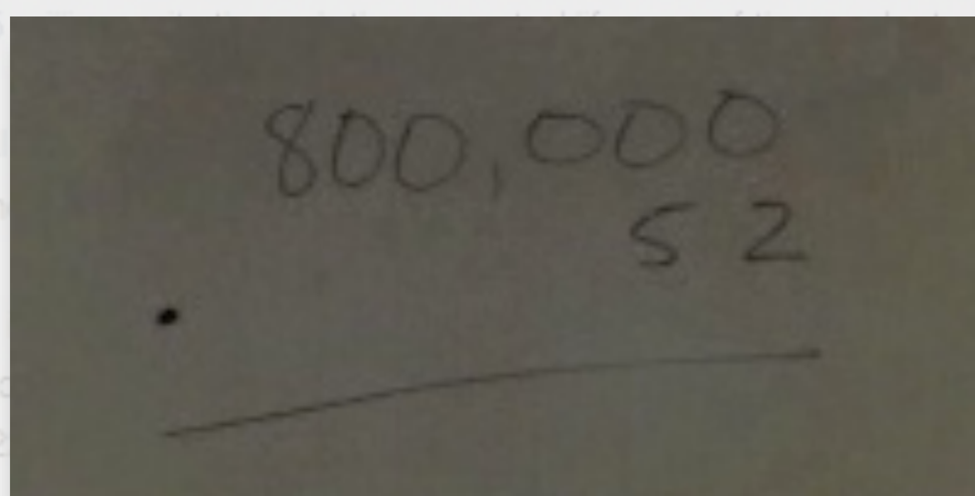
... At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months ...



FAP: Randy why is that [multiplication] going to get us a prediction for the number of iPhones in a year? How does weeks turn into iPhones?

Randy: Because for every week you have, you produce a certain amount of iPhones, so if you multiply it by a certain amount of weeks, the amount of iPhones will go up. [The reason-

FAP: [For every-

Randy: -that might be important is for (investors to know)

NCTM 2014

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Rate as many-per-one

FAP: Randy why ... on] going to get us a pre... r of iPhones in a ye... rn into iPhones?

Randy: Because **for every week you have, you produce a certain amount of iPhones,** so if you multiply it by a certain amount of weeks, the amount of iPhones will go up. [The reason-

FAP: [For every-

Randy: -that might be important is for (investors to know)

Monday, August 04, 2008, 07:00 am PT (10:00 am ET)

uations, given:

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to get us a pre r of
iPhones in a ye rn into
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Rate as
many-per-one

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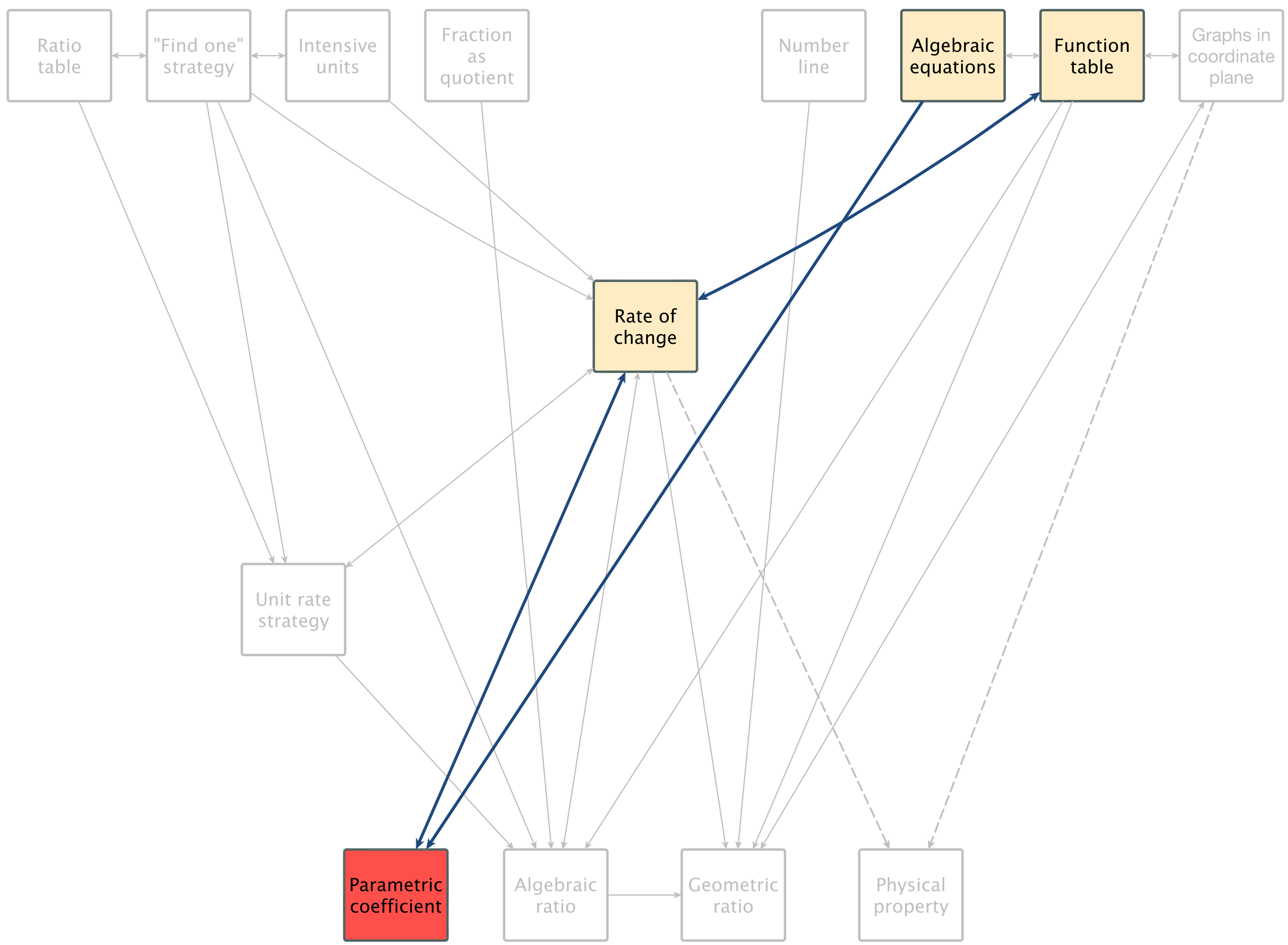
FAP: [For every-

Randy: -that m r
(investors to

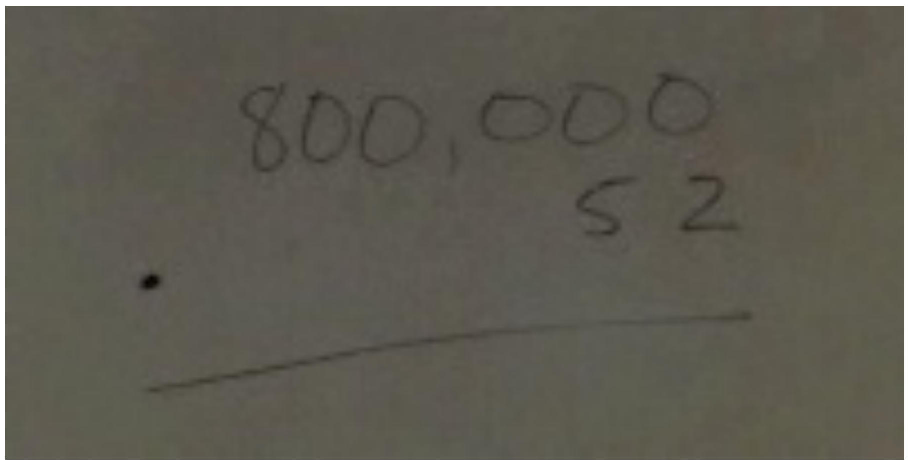
Rate as number
that can be
accumulated

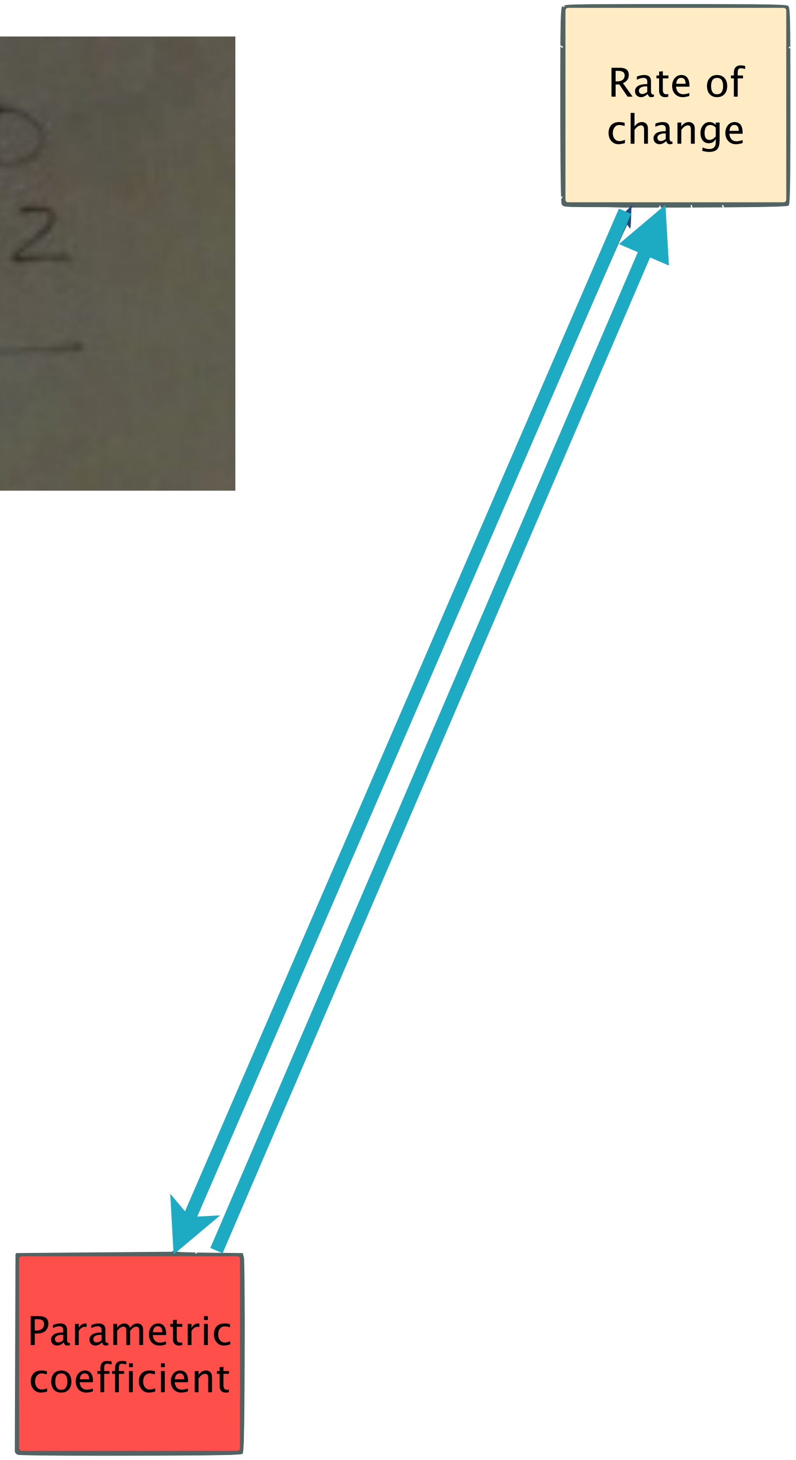
expected to be added on August 22

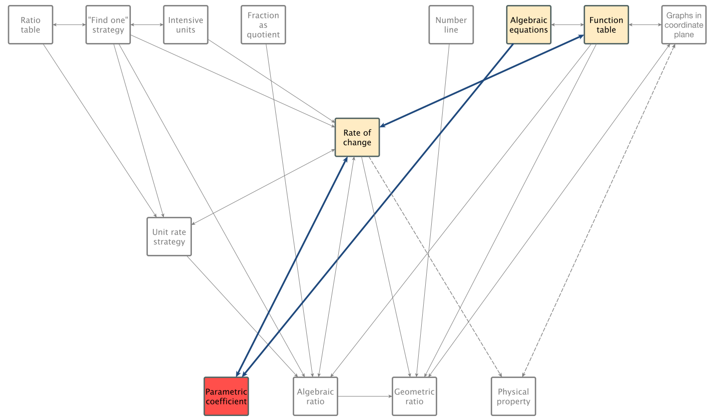
Up and down in the cascade



Up and down in the cascade





Stage	Artifacts	Characteristics of tasks
3 	Assembled and coordinated: • Algebraic equations • Rate of change Reinvented & objectified: • Parametric coefficient Objectified • Rate of change • Function tables	Making predictions in linear situations, given: • The rate of change and starting value • Multiple data points (e.g., in a table), where the independent variable increases by one.

Example activities

1.

Monday, August 04, 2008, 07:00 am PT (10:00 am ET)

Apple already building iPhones at rate of 40 million a year?

By Slash Lane

Apple is reportedly testing the limits of its overseas manufacturing facilities in order to keep up with demand for the new iPhone 3G, with production already cranked nearly sevenfold compared to the first-generation model.

Foxconn, the company's Taiwanese handset and iPod manufacturer, has recently ramped production of the new iPhone to 800,000 units per week, says *TechCrunch*, citing a person "close to Apple with direct knowledge of the numbers."

The build rate is said to be "above current full capacity" for the Foxconn facilities alloted to Apple's handset business, which has led to concerns that quality control may suffer. At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months.

That paces well ahead of analysts' estimates (1, 2, 3) and early reports that suggested Apple's initial iPhone 3G orders spanned only 25 million units through the expected lifespan of the product.

TechCrunch believes Apple's initial order was actually 40 million units over the course of the first twelve months, but is now hearing that "those numbers are being revised upwards sharply."

Apple said it sold 1 million iPhones in the first 72 hours the new iPhone 3G was put on sale, but has not provided an updated sales tally since. The iPhone is currently on sale in 23 countries, with 20 more expected to be added on August 22nd, and another 30 by the end of the calendar year.

(just search Google for "at the current rate" (in quotes))

Class discussion:

- "What prediction does the author make?"
- "How does the author make this prediction?"
- "Why does multiplication make a prediction?"

Goal is to discipline perception to the role of rates and multiplication to make predictions.

2. The table below shows the cost of shipping used X-box games from CHEEP GAMZ ONLINE. Some of the data is missing. Based on the data in the table, how much would it cost to have 12 games shipped?

Number of games	Total cost
0	
1	
2	20.00
3	26.00
4	32.00
5	38.00
6	44.00

Stage	Artifacts	Characteristics of tasks
3	Assembled and coordinated: • Algebraic equations • Rate of change Reinvented & objectified: • Parametric coefficient Objectified • Rate of change • Function tables	Making predictions in linear situations, given: • The rate of change and starting value • Multiple data points (e.g., in a table), where the independent variable increases by one.

Example acti

1.

(ju

Class disc

sed X-box
e data is
much

The table shows the cost of shipping XBox games

Rate of change:
2 dollars per game

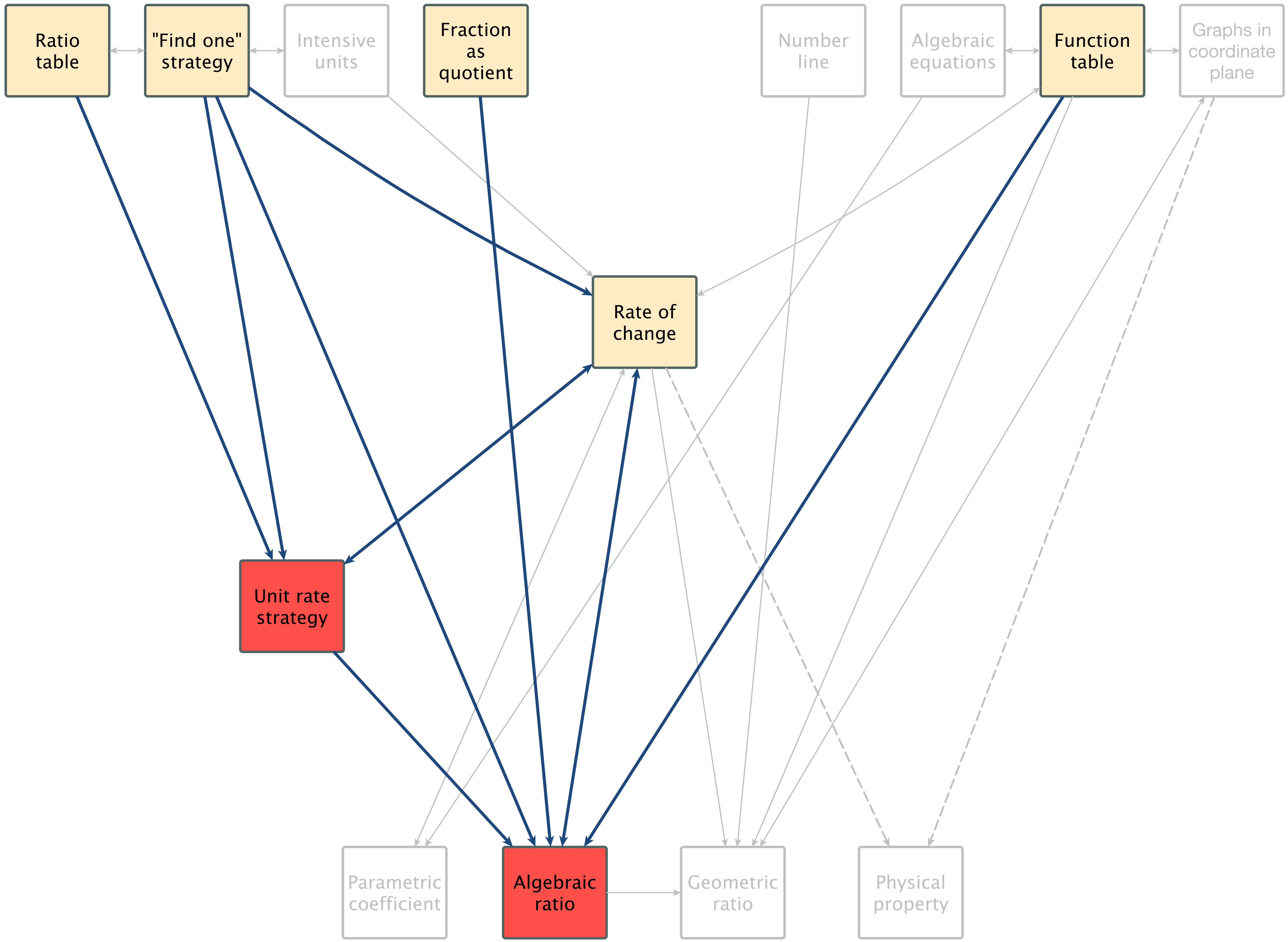
Number of games	Total cost
0	
1	
2	8.00
3	10.00
4	12.00
5	14.00
6	16.00

+1

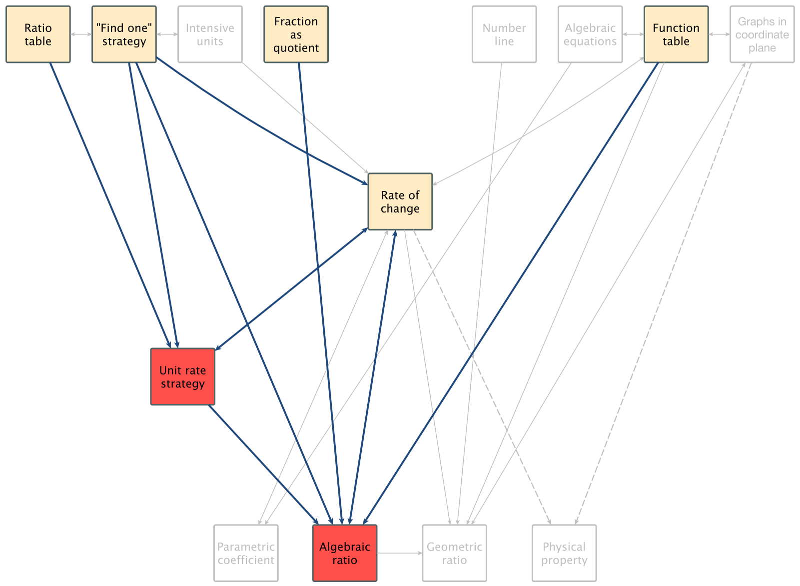
+2

Goal is to discipline perception to the role of rates and multiplication to make predictions.

stage 4



Ratio table
"Find one" strategy
Intensive units
Fraction as quotient
Number line
Algebraic equations
Function table
Graphs in coordinate plane
Rate of change
Unit rate strategy
Parametric coefficient
Algebraic ratio
Geometric ratio
Physical property



stage 4

Reinvented & made meaningful

- unit rate strategy
- algebraic ratio

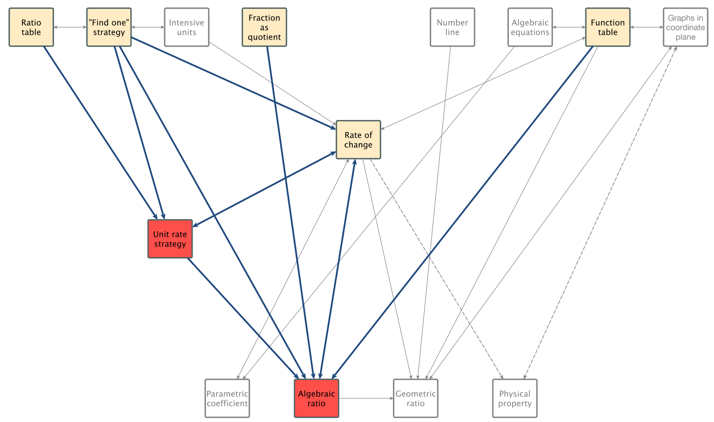
Assembled & coordinated

- ratio table
- “find one” strategy
- fraction as quotient
- rate of change
- function tables

Activities

make predictions given:

- one value in proportional situation
- two data points with $\Delta x \neq 1$

<table>
<tr><th>Stage</th><th>Artifacts</th><th>Characteristics of tasks</th></tr>
<tr>
<td>4

</td>
<td>Assembled and coordinated:
• “Find one” strategy
• Ratio table
• Fraction as quotient
• Function tables
• Rate of change
Reinvented & objectified:
• Unit rate strategy
• Algebraic ratio
Objectified
• Rate of change</td>
<td>Make predictions in linear situations given:
• A single data point, for situations where the values of the variables are proportional
• Two data points, for situations where there is a starting value.

Problem contexts should be chosen to make clear the distinction between changes and values.</td>
</tr>
</table>

Example activities:

1. Unit rate strategy

Ms. Magro runs six miles every day. On average it takes her 54 minutes to run six miles. At this rate, how long will it take Ms. Magro to run an 11-mile race?

2. Algebraic ratio

At the end of the summer, the YMCA drains their swimming pool. Raif and Julie are in charge of measuring the height of the pool as it drains.

> **Raif says:** I checked the pool two hours after we started draining it. When I checked, the height of the water was 517 mm.
>
> **Julie says:** I checked the pool seven hours after we started draining it. When I checked, the height of the water was 420 mm.

Imagine you checked the height of the pool five hours after the YMCA started draining it. What would the height have been?

Ratio table

"Find one" strategy

stage 4

Rate of change

Unit rate strategy

make predictions given one value in proportional situation

Ms. Magro runs 6 miles every day. On average, she can run six miles in 54 minutes. At this rate, how long would it take Ms. Magro to run an 11-mile race?

6 mile > 54 min

6÷(÷6

1 mile 9 min

9×11 = 99.

Takes 99 minutes

Ratio table

"Find one" strategy

stage 4

Rate of change

Unit rate strategy

make predictions given one value in proportional situation

3 pizzas cost 12 dollars

1 pizza costs ____ dollars

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$6\div$ 1 mile $\div 6$ 9 min

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Takes 99 minutes

stage 4

Ratio table

"Find one" strategy

Rate of change

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"Find one" strategy

Unit rate strategy

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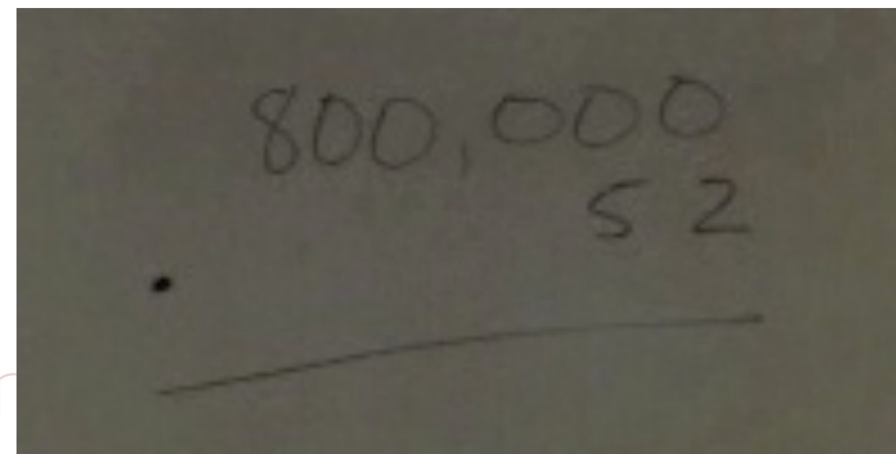

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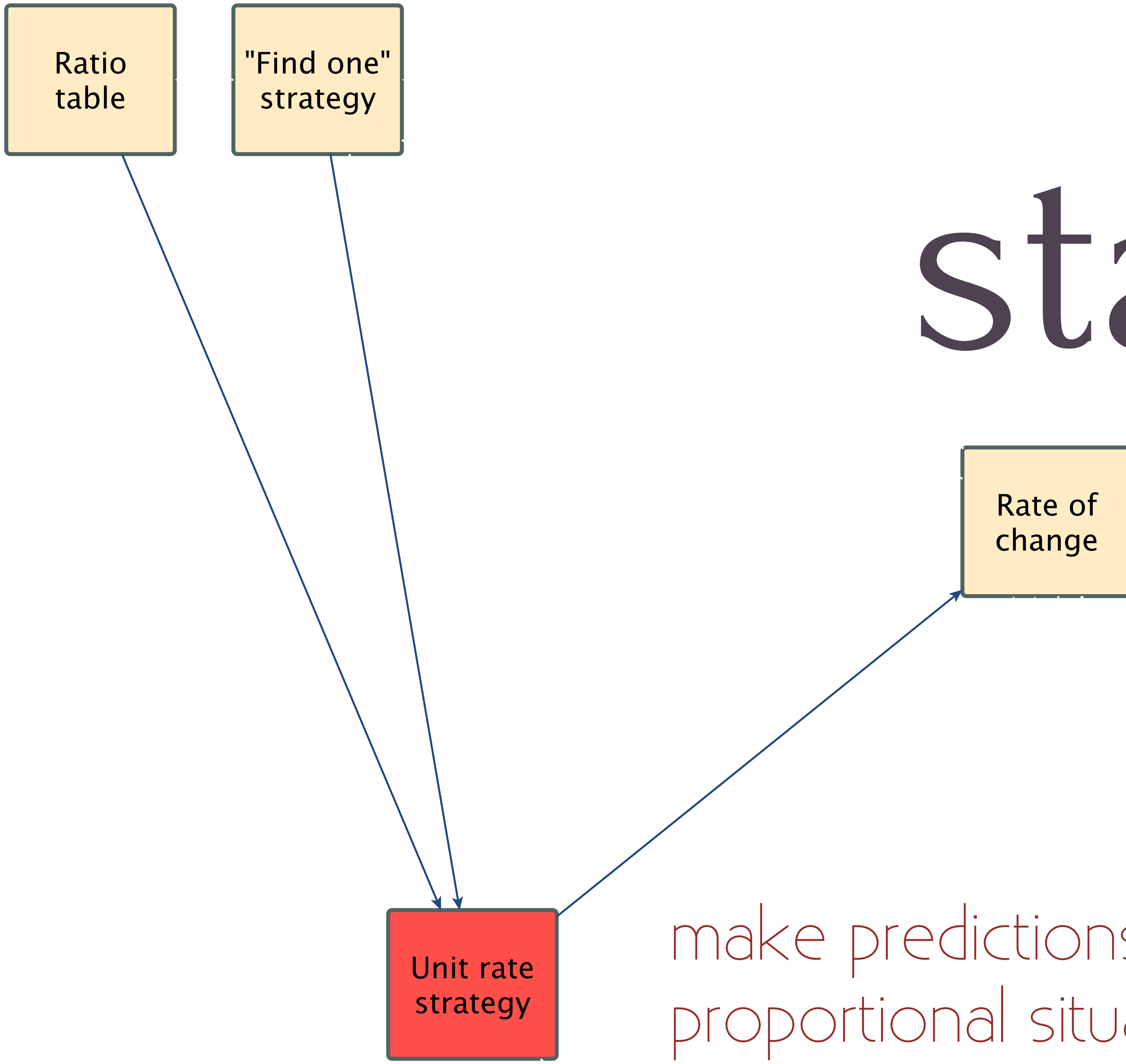

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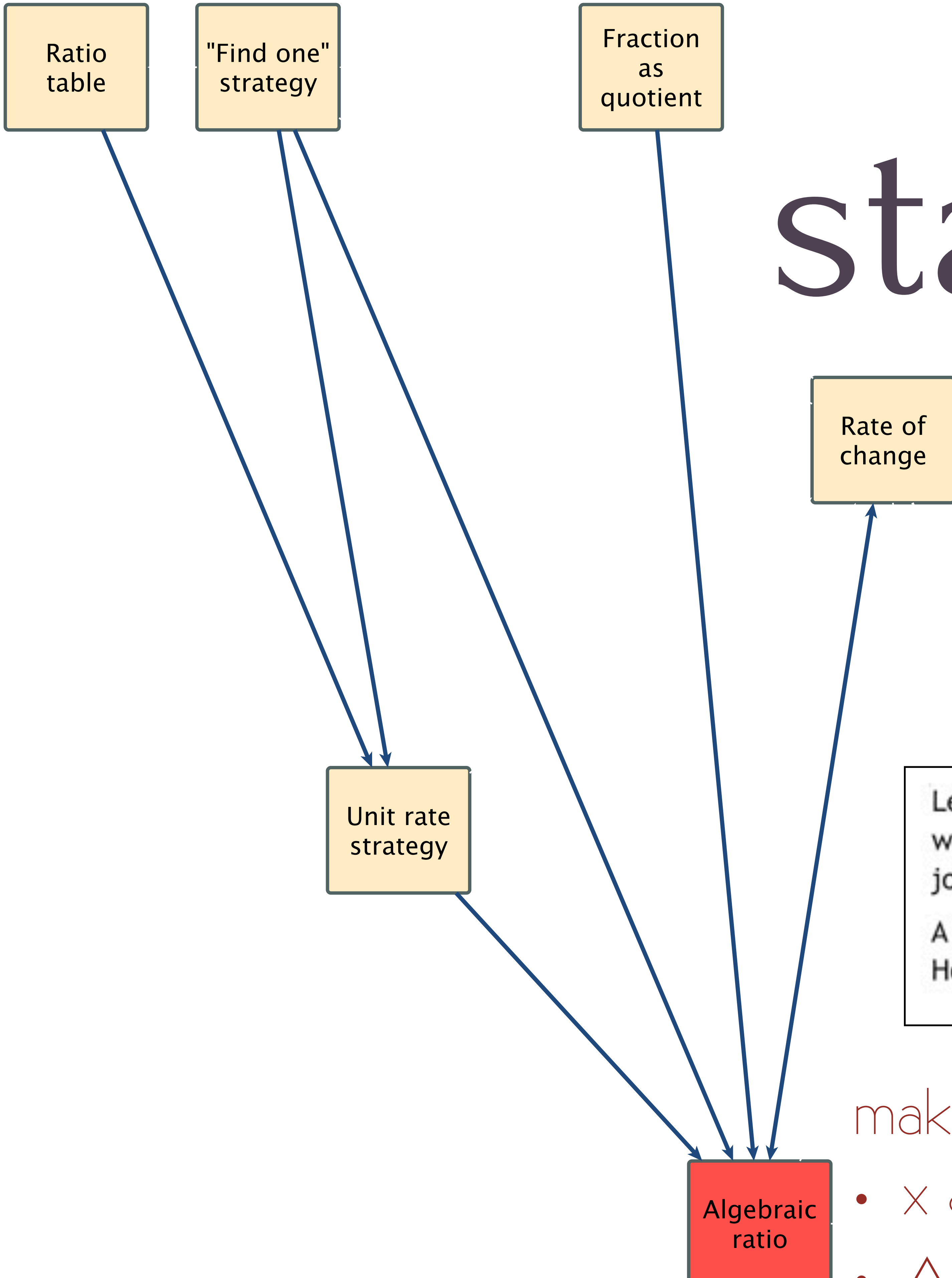
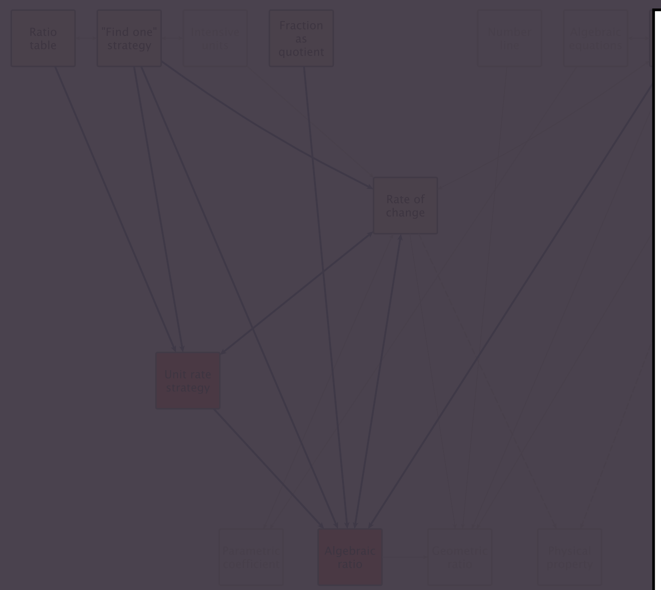


Leslie is a window installer. On Friday, she installed two windows, and charged 402 dollars. Last week, on another job, she charged 517 dollars to install seven windows.

A new customer has asked Leslie to install five windows. How much will this cost?

make predictions given two data points:

- x & y not proportional
- $\Delta x \neq 1$



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2 windows = 402 dollers

7 windows = 517 dollars

$\frac{517}{7}$ = 1 window

\$364 = 5 windows

2 = 402

$\frac{\$402}{2}$ = 1 window

\$1,005 = 5 windows

7 = 517

$\frac{5}{1} \cdot \frac{517}{7} = \frac{2585}{7} \rightarrow 364$

$\frac{402}{2} \cdot \frac{5}{1} = \frac{2010}{2} = \frac{1005}{1}$

23

0	\$356
1	\$379
2	\$402
3	\$425
4	\$448
5	\$471
6	\$494
7	\$517

−23, +23, +1

$517 - 402 = 115$

$\frac{115}{5} = 23$

$402 - 23 = 379$

$379 - 23 = 356$

Rate of change = \$23 per window

starting cost = \$356

5 windows = \$475

4

Reinvent... & object...

Assem... & coordina...

Activ... ...tional situation

... $\Delta x \neq 1$

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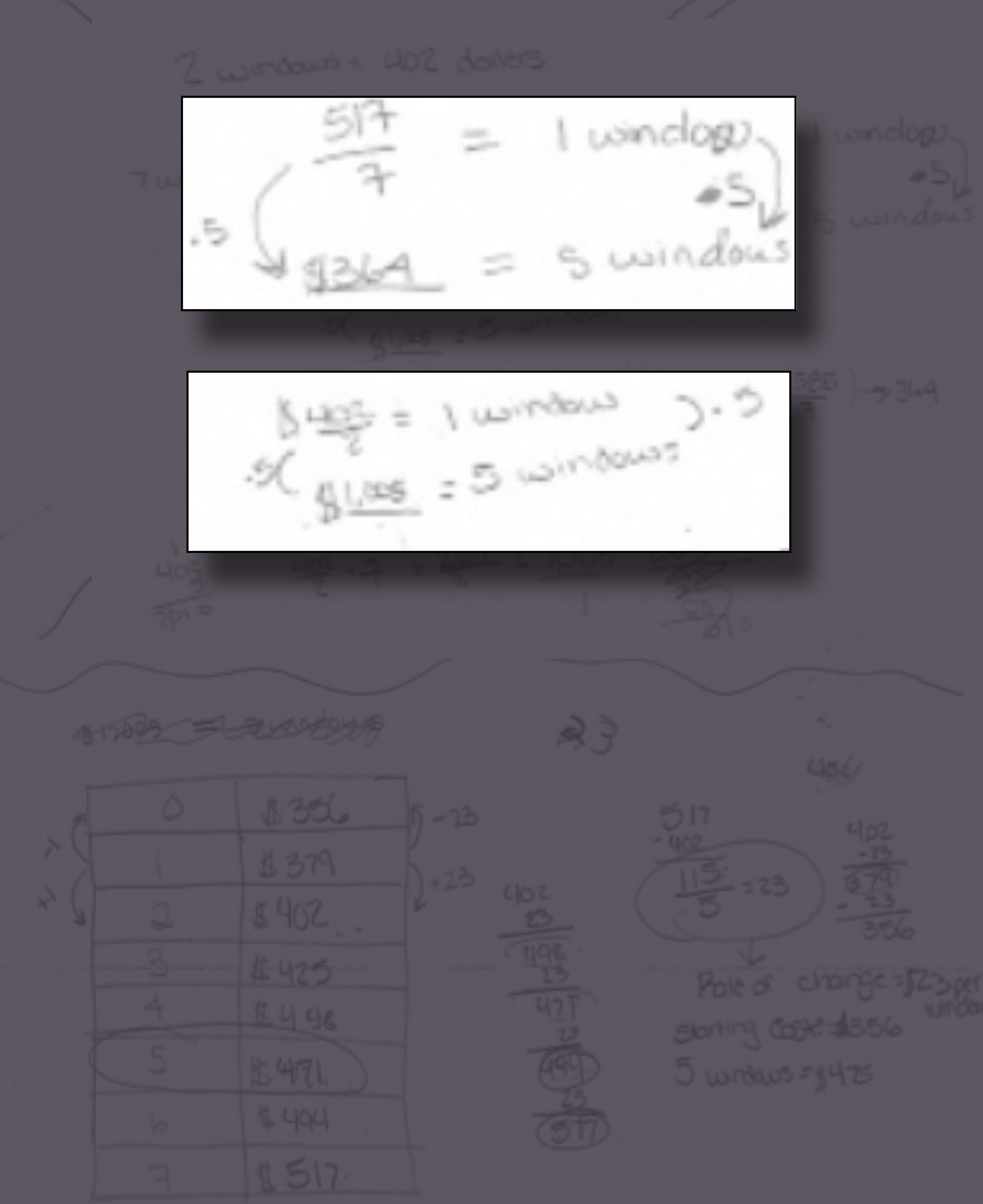

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+23

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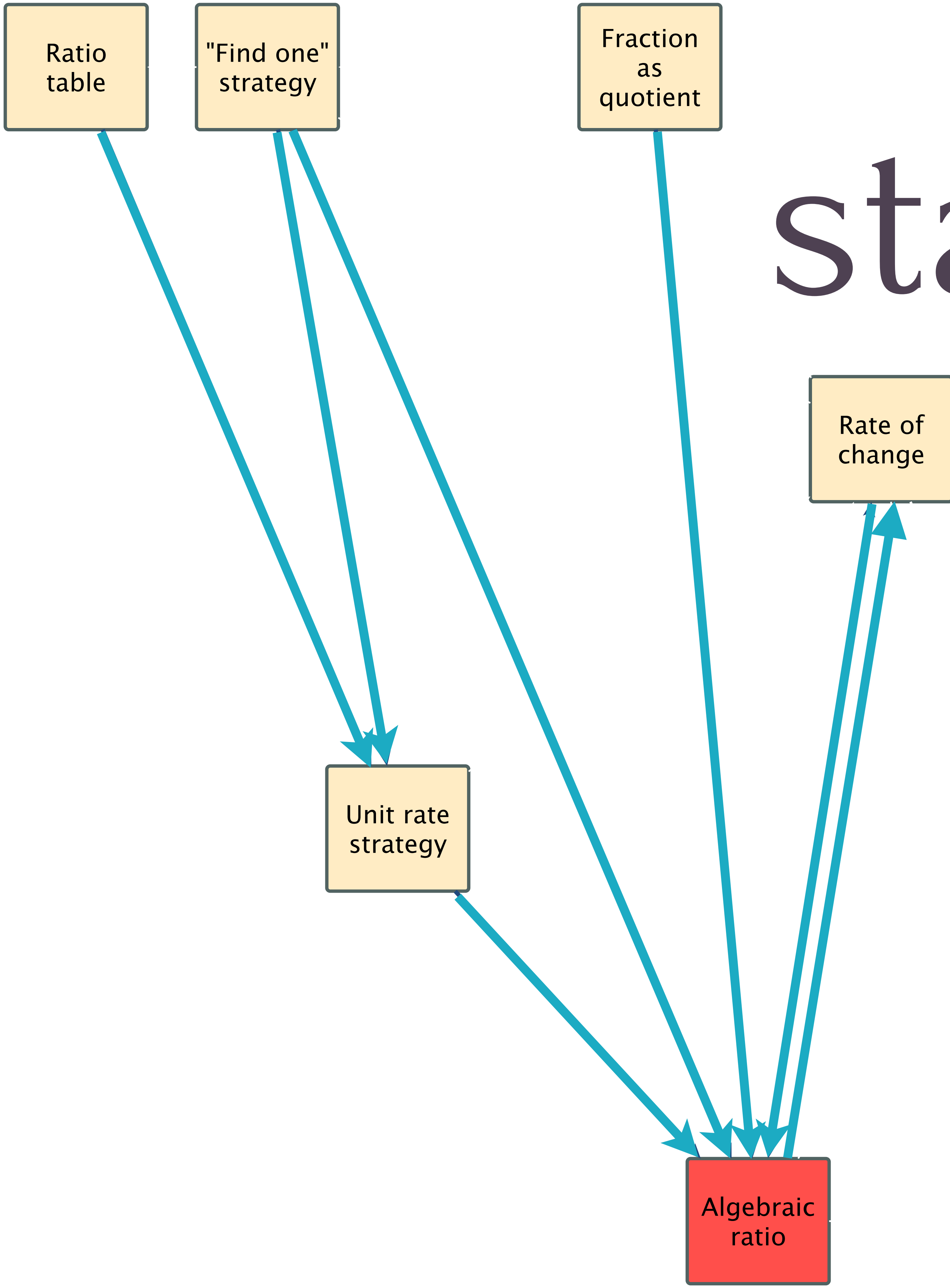
stage 4



2. Algebraic ratio

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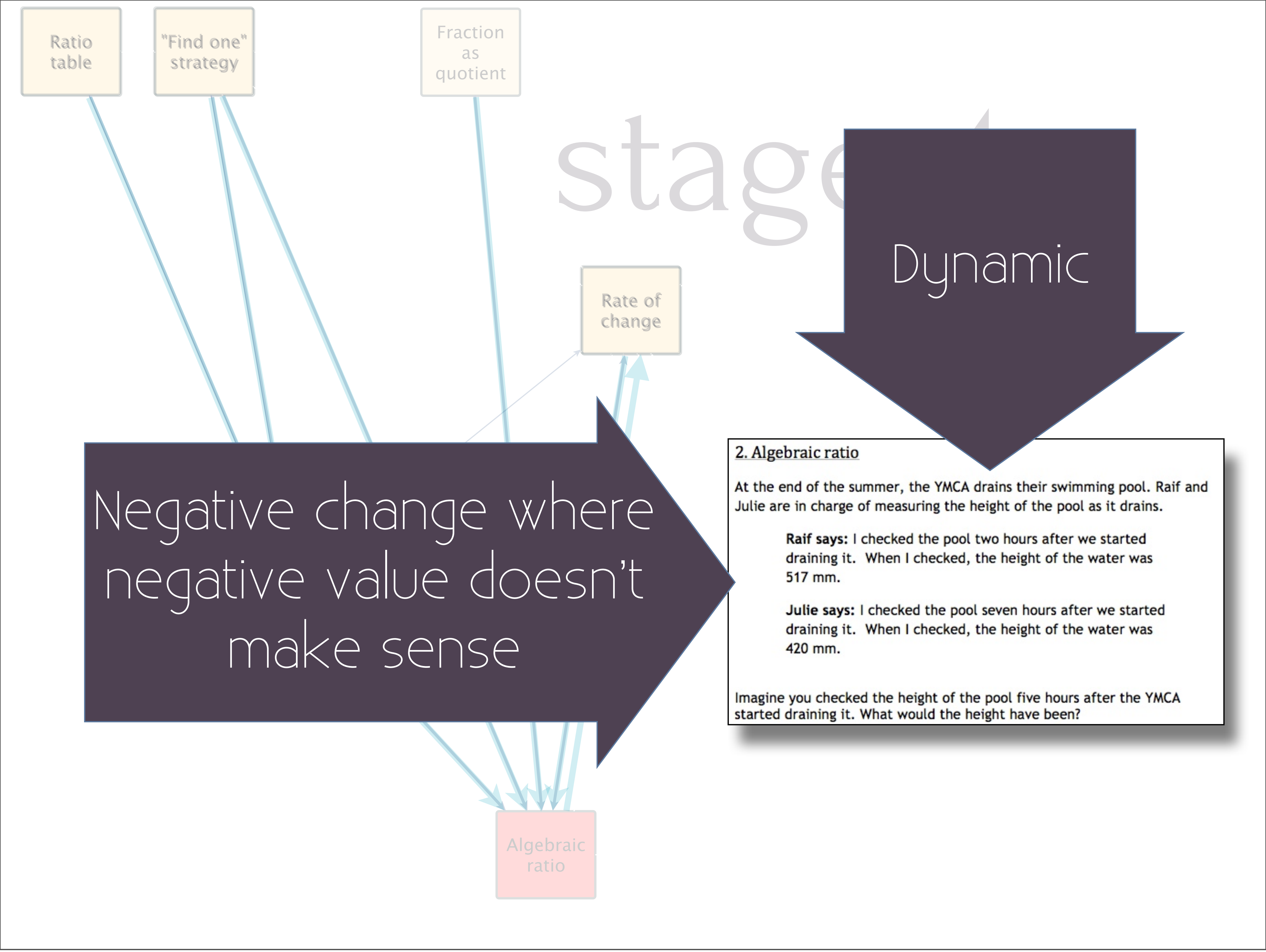

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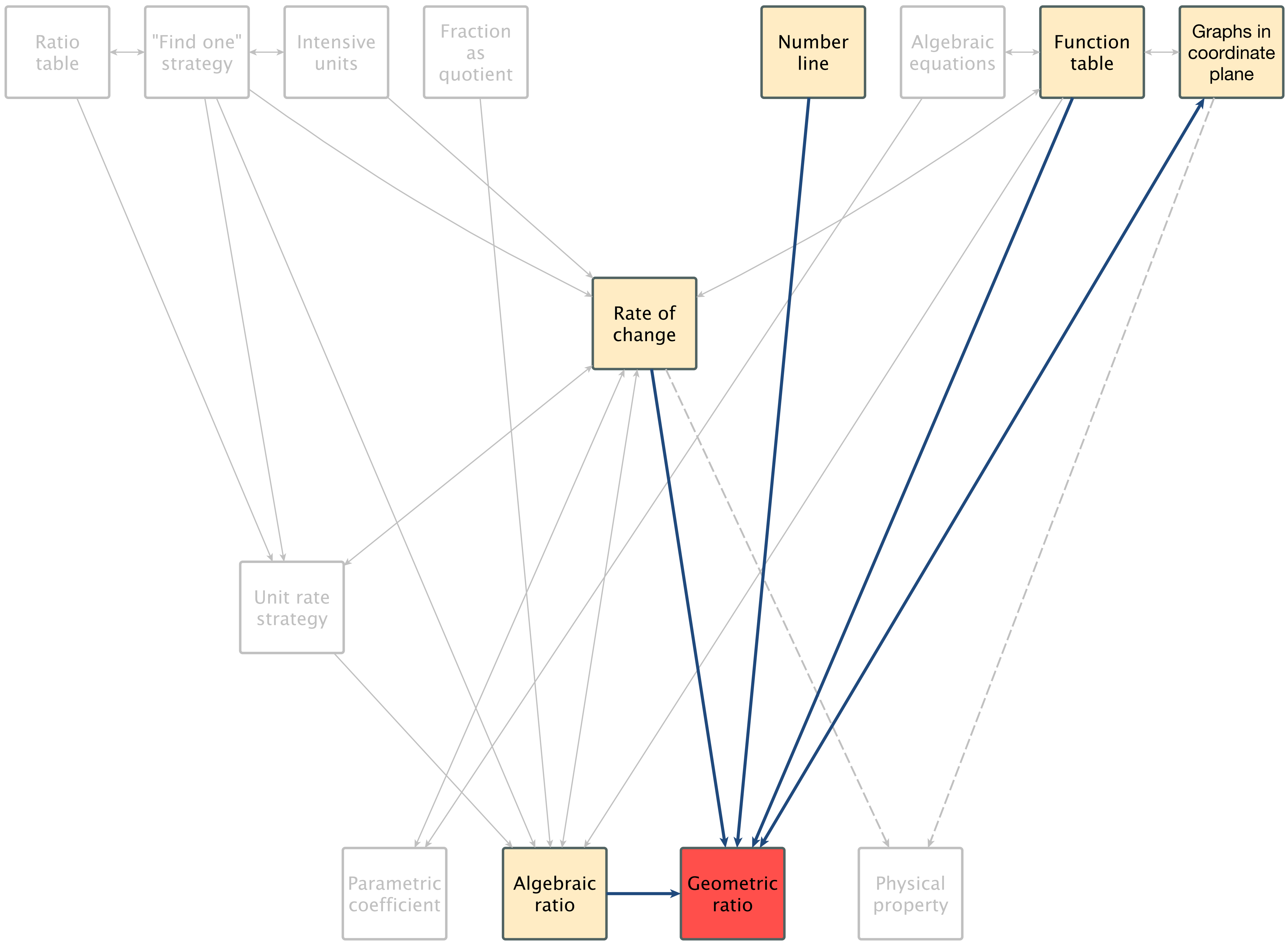
stage 5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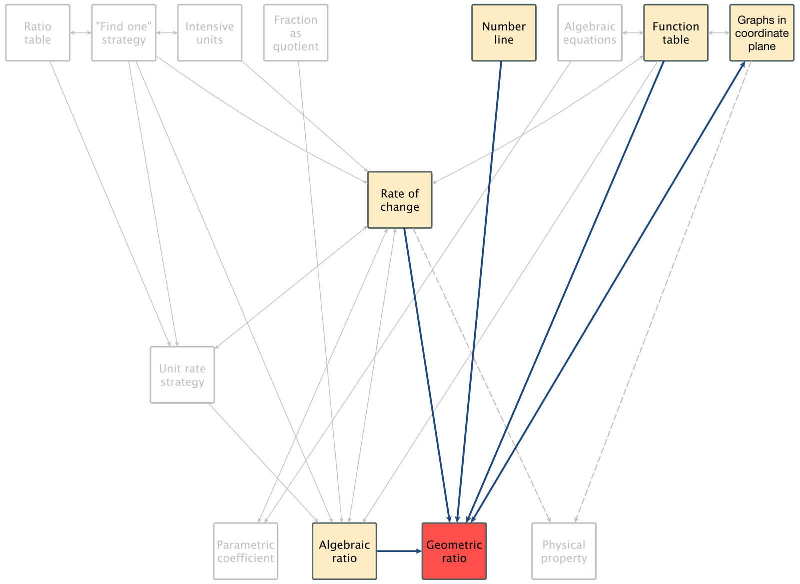


stage 5

Reinvented & made meaningful

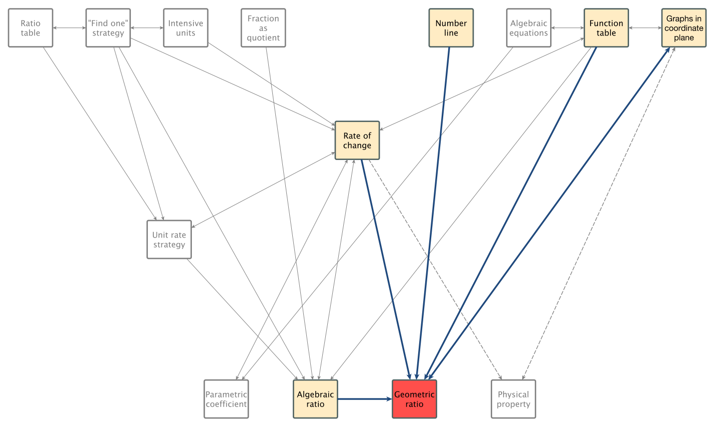
- geometric ratio

Assembled & coordinated

- algebraic ratio
- rate of change
- number line
- function tables
- graphs in coordinate plane

Activities

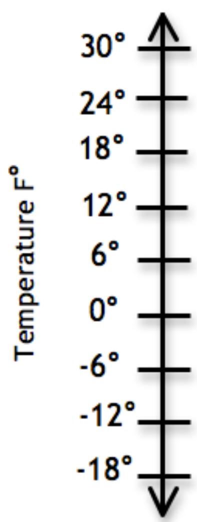
- show change on number line diagrams
- make predictions given graph

Stage	Artifacts	Characteristics of tasks
5 	Assembled and coordinated: • Number line • Graphs in coord. plane • Rate of change • Algebraic ratio Reinvented & objectified: • Geometric ratio Objectified: • Graphs in coord. plane	• Show change on number-line diagrams. • Make predictions in linear situations where there is a starting value, given a graph of a function in a coordinate plane.

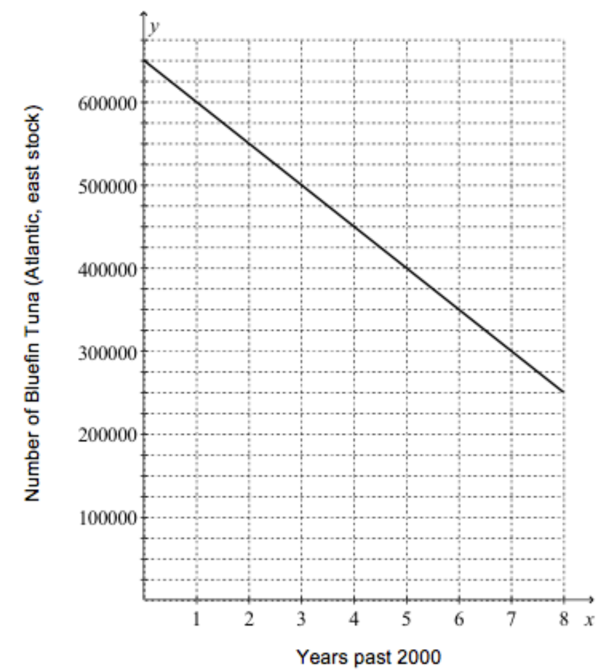
Example activities:

1. The temperature in Alamosa, Colorado rose from -12˚ to 24˚.

   Draw an arrow on the number line below to show this change.



2.



- What are two variables that are changing in the graph?
- Write a sentence that describes how these two variables are changing
- Find the rate of change in the graph
- At this rate, when will the stock of Bluefin tuna be depleted?

Frederick Peck
Frederick.Peck@Colorado.edu
www.RMEInTheClassroom.com

Stage	Artifacts	Characteristics of tasks
5	Assembled and coordinated:	• Show change on number-line diagrams. • Make predictions in linear situations where

Explain or show how you found the rate of change in the graph.

X	Y
2	550,000
4	450,000

$$\begin{array}{r} 450{,}000 \\ -\ 550{,}000 \\ \hline -100{,}000 \\ \div\quad 2 \\ \hline -50{,}000 \end{array}$$

We picked 2 points of the graph, subtracted output_1 from output_2, and then ÷ that by 2 which was $\text{input}_2 - \text{input}_1$.

1. The temperature in Alamosa, Colorado rose from -12° to 24°.

   Draw an arrow on the number line below to show this chang

2.

1	600000
3	5,00000

2( ) 100000

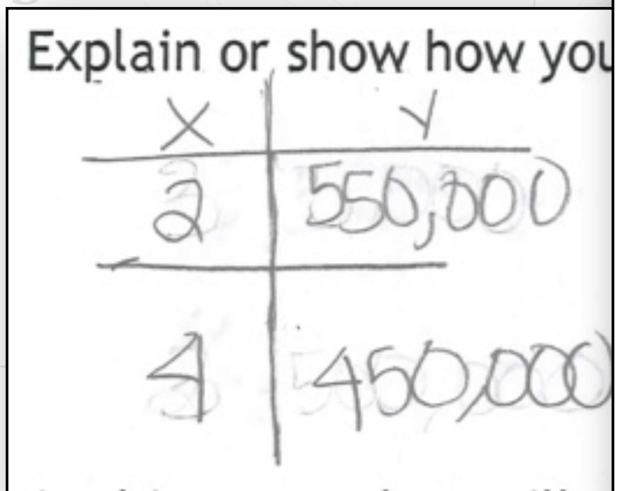
Friday, September 26, 14

Stage | Artifacts | Characteristics of tasks

5

...w change on number-line diagrams.

...ke predictions in linear situations where



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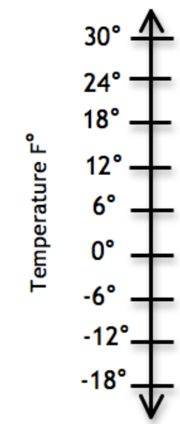

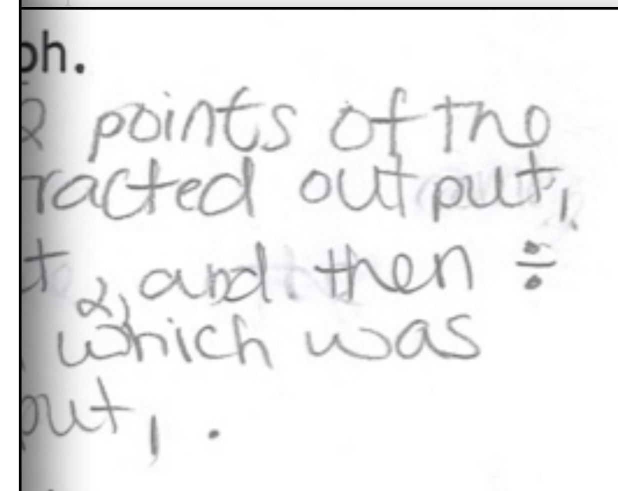

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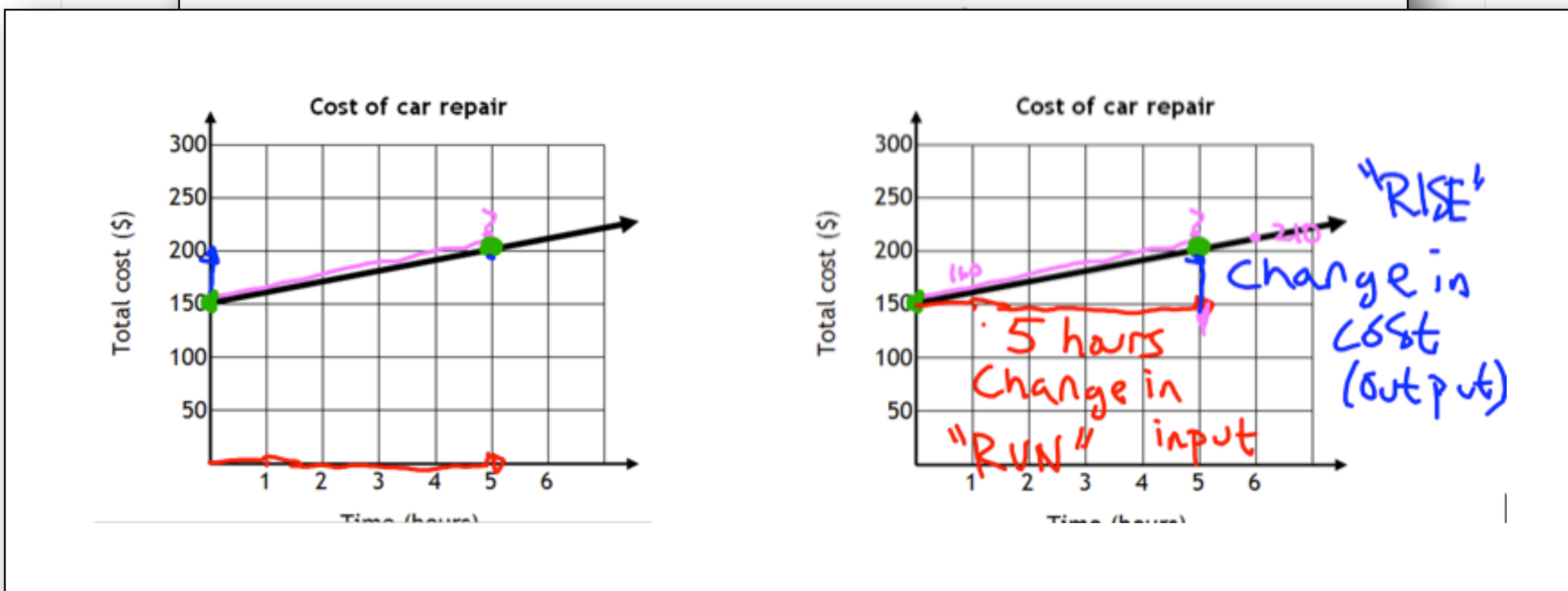
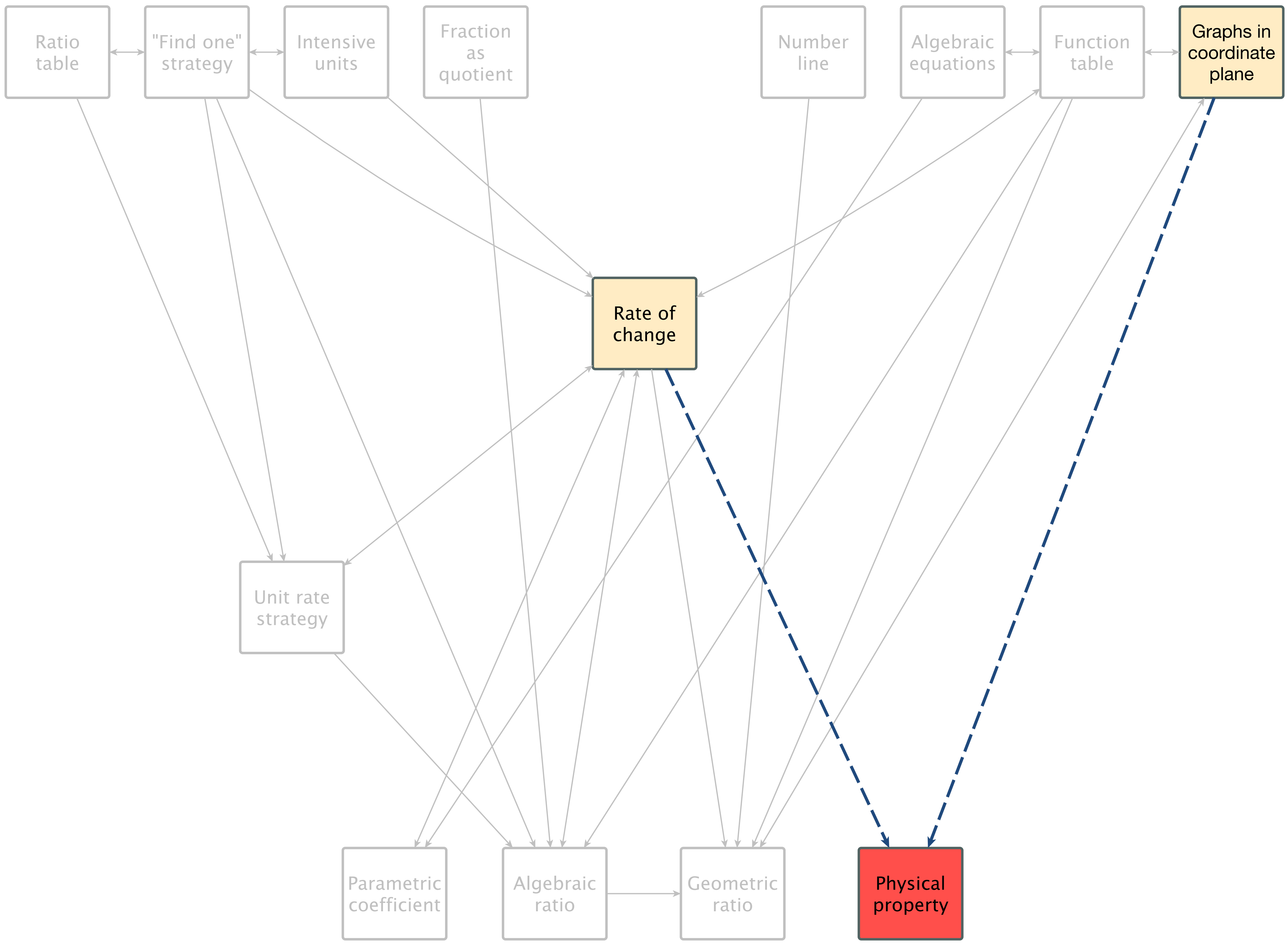


stage 6



Ratio table
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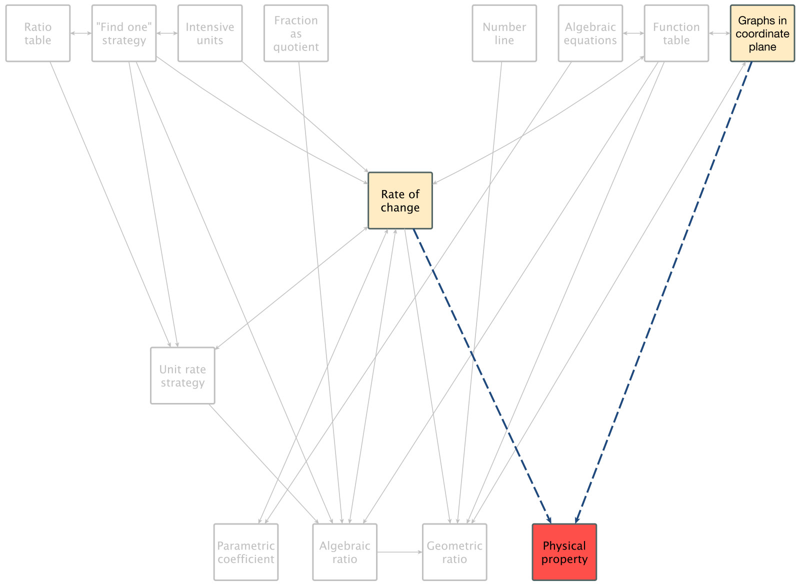


stage 6

Reinvented & made meaningful

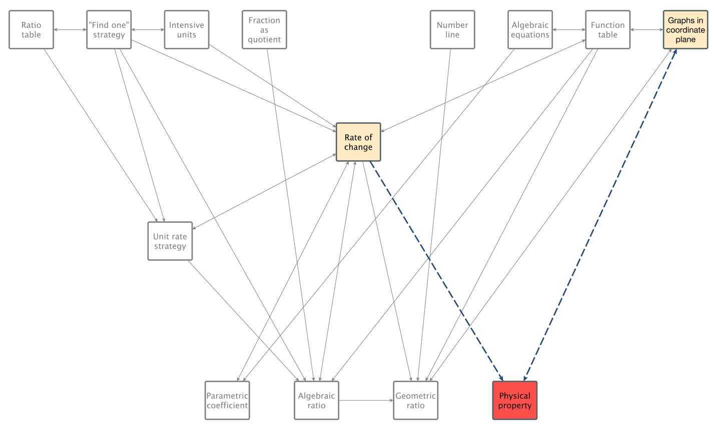
- physical property

Assembled & coordinated

- rate of change
- graphs in coordinate plane

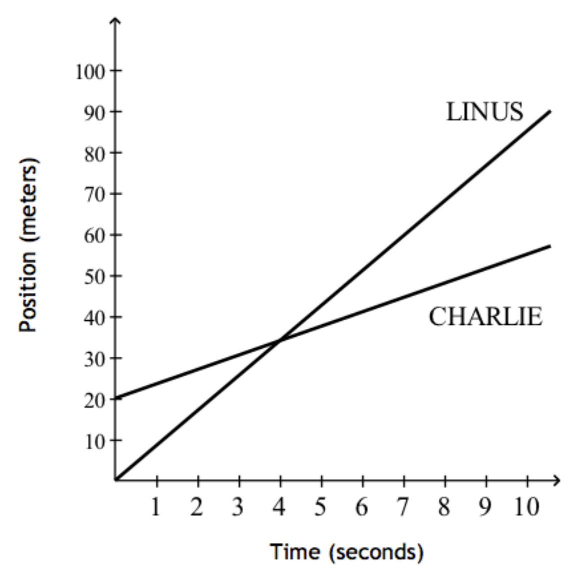
Activities

- compare rates given graph of two intersecting linear functions
- measure steepness of objects

Stage		Artifacts	Characteristics of tasks
6		Assembled and coordinated: • Rate of change • Graphs in coord. plane Reinvented & objectified • Physical property	• Compare rates given two intersecting linear functions graphed in a coordinate plane. • Measure and compare the steepness of objects

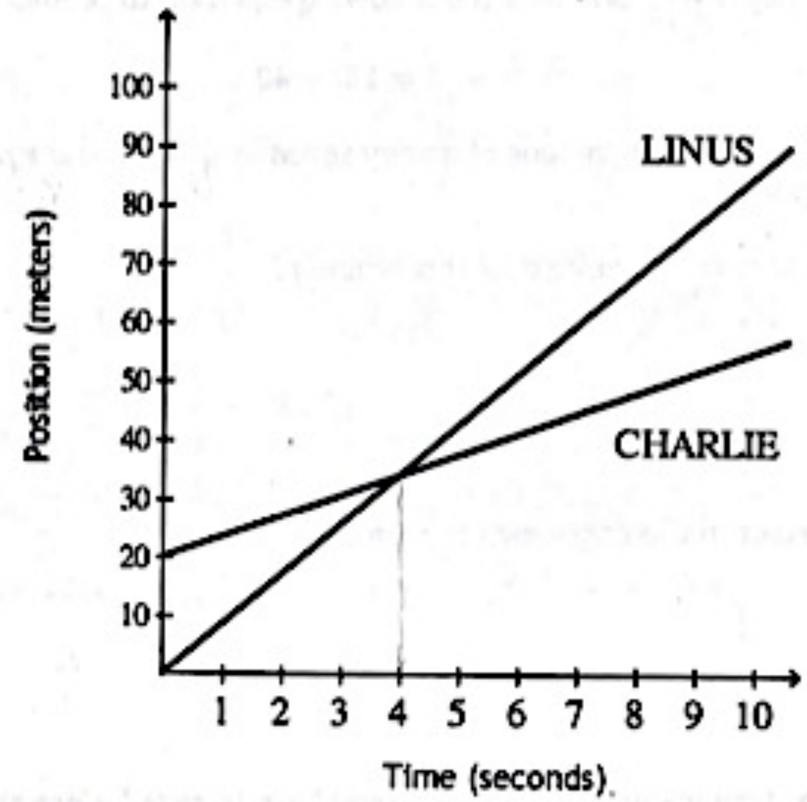
Example activities

Charlie and Linus are running along a straight track. A position vs. time graph for both runners is shown below.



- At the instant, $t = 2$ sec, who is running faster, Charlie or Linus?
- Do Linus and Charlie ever have the same speed? If so, at what time?

8. Charlie and Linus are running along a straight track. A position vs. time graph for both runners is shown below.



a. At the instant, $t = 2$ sec, who is running faster, Charlie or Linus? linus

Explain your reasoning

linuses line is steeper so he is running faster

b. Do Linus and Charlie ever have the same speed? If so, at what time?

Explain your reasoning.

yes at 4 seconds they are going at the same speed

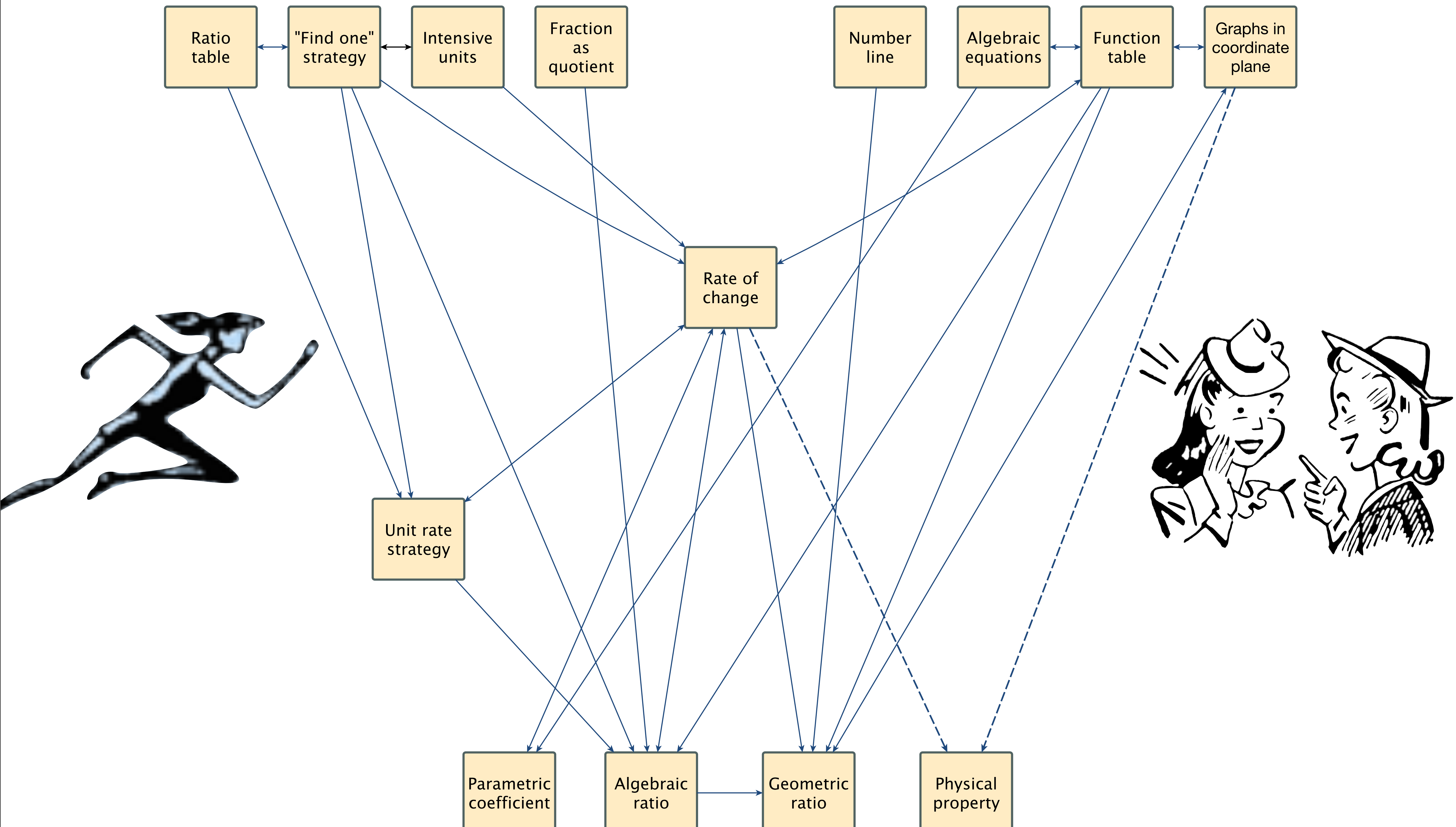
summary

How do students make slope meaningful?

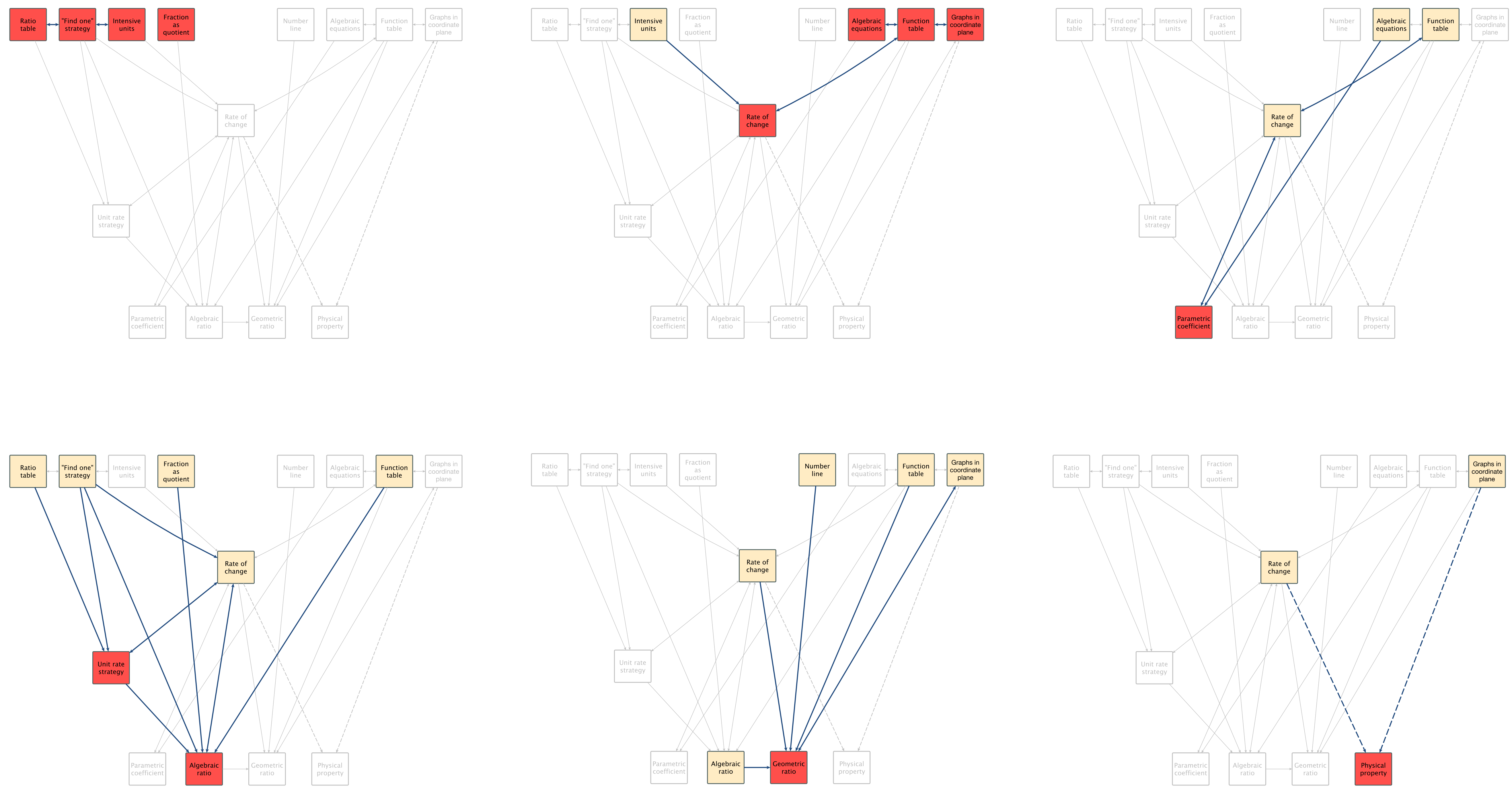
slope

- Parametric coefficient
- Algebraic ratio
- Rate of change
- Geometric ratio
- Physical property

cascade of artifacts



progression of learning



Questions and discussion

Questions and discussion

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University of Colorado, USA

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