More than rise over run!
Activities to invent and connect the five faces of slope

Frederick Peck
Department of mathematical sciences | University of Montana

Frederick.Peck@UMontana.edu
www.RMEInTheClassroom.com

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Key points:

- Organized around rates and predictions

  Rates
  - intensive quantities ("_____per____")
  - that express covariation
  - and that can be accumulated

- Use function tables to build covariation

- Save graphs and steepness for last

Phase 1: The foundation (~5 days, beginning of the year)

Students reinvent and learn	Characteristics of tasks
• Ratio tables • "find one" strategy • Intensive quantities ("____ per ____") • Fractions-as-division	Tasks that involve the activity of **partitive division**, including: • finding fair shares • finding unit values with answers expressed as intensive quantities ("____ per ____")

Example activities

1. A class traveled on a field trip in four separate cars. The school provided a lunch of submarine sandwiches for each group. When they stopped for lunch, the subs were cut and shared as follows:

   - *The first group had 4 people and shared 3 subs equally.*
   - *The second group had 5 people and shared 4 subs equally.*
   - *The third group had 8 people and shared 7 subs equally.*
   - *The last group had 5 people and shared 3 subs equally.*

   When they returned from the field trip, the children began to argue that the distribution of sandwiches had not been fair, that some children got more to eat than the others. Were they right? Or did everyone get the same amount?

2. At a race, five people shared two gallons of water equally. How much water did each person receive?

   State your final answer using units: ________ ____________________ *per* __________________.

3. In 7 minutes, a hot-air balloon rose 12 meters

   In 1 minute, the hot-air balloon rose ___ meters

4. 3 pizzas cost 12 dollars

   1 pizza costs ____ dollars

   ______ ________________ *per* ______________.

   3 pizzas cost 12 dollars

   ____ pizzas costs 1 dollar

   ______ ________________ *per* ______________.

Phase 2: Rates and predictions (~3 days)

Students reinvent and learn	Characteristics of tasks

• **Rate of change**
  ◦ as an intensive quantity ("___per___")
  ◦ that can be accumulated
• **Parametric coefficient** ($y = \boldsymbol{a}x + b$)
	Making predictions in linear situations, given:
	• The rate of change and starting value
	• Multiple data points (e.g., in a table), where the independent variable increases by one.

Example activities

5.

Monday, August 04, 2008, 07:00 am PT (10:00 am ET)

Apple already building iPhones at rate of 40 million a year?

By Slash Lane

Apple is reportedly testing the limits of its overseas manufacturing facilities in order to keep up with demand for the new iPhone 3G, with production already cranked nearly sevenfold compared to the first-generation model.

Foxconn, the company's Taiwanese handset and iPod manufacturer, has recently ramped production of the new iPhone to 800,000 units per week, says *TechCrunch*, citing a person "close to Apple with direct knowledge of the numbers."

The build rate is said to be "above current full capacity" for the Foxconn facilities alloted to Apple's handset business, which has led to concerns that quality control may suffer. At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months.

That paces well ahead of analysts' estimates (1, 2, 3) and early reports that suggested Apple's initial iPhone 3G orders spanned only 25 million units through the expected lifespan of the product.

TechCrunch believes Apple's initial order was actually 40 million units over the course of the first twelve months, but is now hearing that "those numbers are being revised upwards sharply."

Apple said it sold 1 million iPhones in the first 72 hours the new iPhone 3G was put on sale, but has not provided an updated sales tally since. The iPhone is currently on sale in 23 countries, with 20 more expected to be added on August 22nd, and another 30 by the end of the calendar year.

(just search Google for "at the current rate" (in quotes))

Class discussion:

- "What prediction does the author make?
- "How does the author make this prediction?"
- "Why do we use multiplication to make the prediction?"
- How can you make a prediction for any number of weeks? What assumption are you making?

Goal is to help students see the role of rates and multiplication to make predictions.

6. The table below shows the cost of shipping used X-box games from CHEEP GAMZ ONLINE. Some of the data is missing.

Number of games	Total cost
0	
1	
2	20.00
3	26.00
4	32.00
5	38.00
6	44.00

a) Identify the rate of change in the table.

b) Based on the data in the table, how much would it cost to have 12 games shipped?

c) How much would it cost to have any number of games shipped?

Phase 3: The unit rate strategy (~1 day)

Students reinvent and learn	Characteristics of tasks
• The "unit rate" strategy for predictions (scale down to find a unit rate, and scale up to make a prediction)	Make predictions in proportional situations given one ratio • Within-unit values are relatively prime Problem contexts should be chosen to make clear the distinction between changes and values. • Contexts should be *dynamic*, so that it's reasonable to see both quantities changing. • Contexts should involve *negative change* in situations where negative values do not make sense

Example activity

7. Ms. Magro runs six miles every day. On average it takes her 54 minutes to run six miles. At this rate, how long will it take Ms. Magro to run an 11-mile race?

Phase 4: The algebraic ratio $\frac{y_2 - y_1}{x_2 - x_1}$ (~5 days)

Students reinvent and learn	Characteristics of tasks

• The algebraic ratio: $\frac{y_2 - y_1}{x_2 - x_1}$	• Make predictions in **non-proportional**, linear situations given:
	  • Two or more points
	  • Well-ordered table where $\Delta x \neq 1$
	  • Non-well ordered table
	• Activities to *generalize*:
	  • Explain your strategy
	  • Does that always work?
	  • Write a formula in Excel
	
	Problem contexts should be chosen to make clear the distinction between changes and values.
	  • Contexts should be *dynamic*, so that it's reasonable to see both quantities changing.
	  • Contexts should involve *negative change* in situations where negative values do not make sense

Example activities

8. At the end of the summer, the YMCA drains their swimming pool. Raif and Julie are in charge of measuring the height of the pool as it drains.

> **Raif says:** I checked the pool two hours after we started draining it. When I checked, the height of the water was 517 mm.
>
> **Julie says:** I checked the pool seven hours after we started draining it. When I checked, the height of the water was 402 mm.

Imagine you checked the height of the pool five hours after the YMCA started draining it. What would the height have been? What assumptions did you make?

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9. Malik is a window installer. He charges a cost per window, as well as a fixed cost. The table below shows information for Malik's last five window installations

Number of windows	Total cost (dollars)
3	121
9	199
6	160
2	108
7	173

a. Choose any two rows in the table. Using these two data points, calculate the rate of change. *Show your work or explain your reasoning in the space below:*

State your final answer using units: ________________________________

b. What does the rate of change mean, in terms of Malik's windows?

c. Choose two **<u>different</u>** rows in the table. Using these **<u>new</u>** data points, calculate the rate of change. *Show your work or explain your reasoning in the space below:*

State your final answer using units: ________________________________

d. Does it matter which data points you choose when you calculate the rate of change? Why or why not? *You should write at least two sentences to explain.*

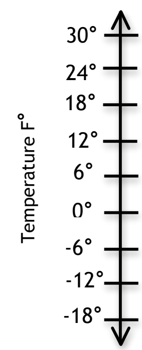
Phase 5: The geometric ratio $\frac{\text{rise}}{\text{run}}$ (~4 days)

Students reinvent and learn	Characteristics of tasks
• The geometric ratio: $\frac{\text{rise}}{\text{run}}$	• Show change on number-line diagrams. • Make predictions in linear situations, given a graph of a function in a coordinate plane. Problem contexts should be chosen to make clear the distinction between changes and values. • Contexts should be *dynamic*, so that it's reasonable to see both quantities changing. • Contexts should involve *negative change* in situations where negative values do not make sense

Example activities

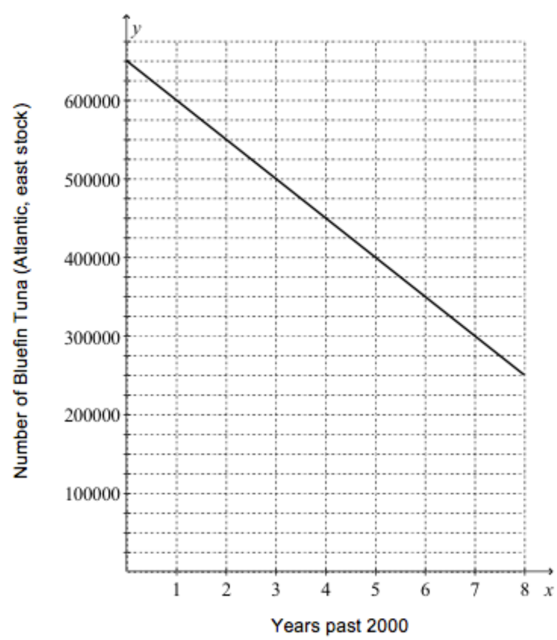
10. The temperature in Helena, MT rose from -12 degrees to 24 degrees.

    Draw an arrow on the number line to show this change



11. Use the graph on the right:

    - What are two variables that are changing in the graph?
    - Write a sentence that describes how these two variables are changing
    - Find the rate of change in the graph
    - At this rate, when will the stock of Bluefin tuna be depleted?



Phase 6: The physical property (steepness) (~3 days)

Students reinvent and learn	Characteristics of tasks
• The geometric ratio: $\frac{\text{rise}}{\text{run}}$	• Compare rates given two intersecting linear functions graphed in a coordinate plane. • Measure and compare the steepness of objects

Example activity

12. Charlie and Linus are running along a straight track. A position vs. time graph for both runners is shown on the right.

    - At the instant, $t = 2$ sec, who is running faster, Charlie or Linus?
    - Do Linus and Charlie ever have the same speed? If so, at what time?