
# Beyond rise over run:

## Activities to invent and connect slope's five faces

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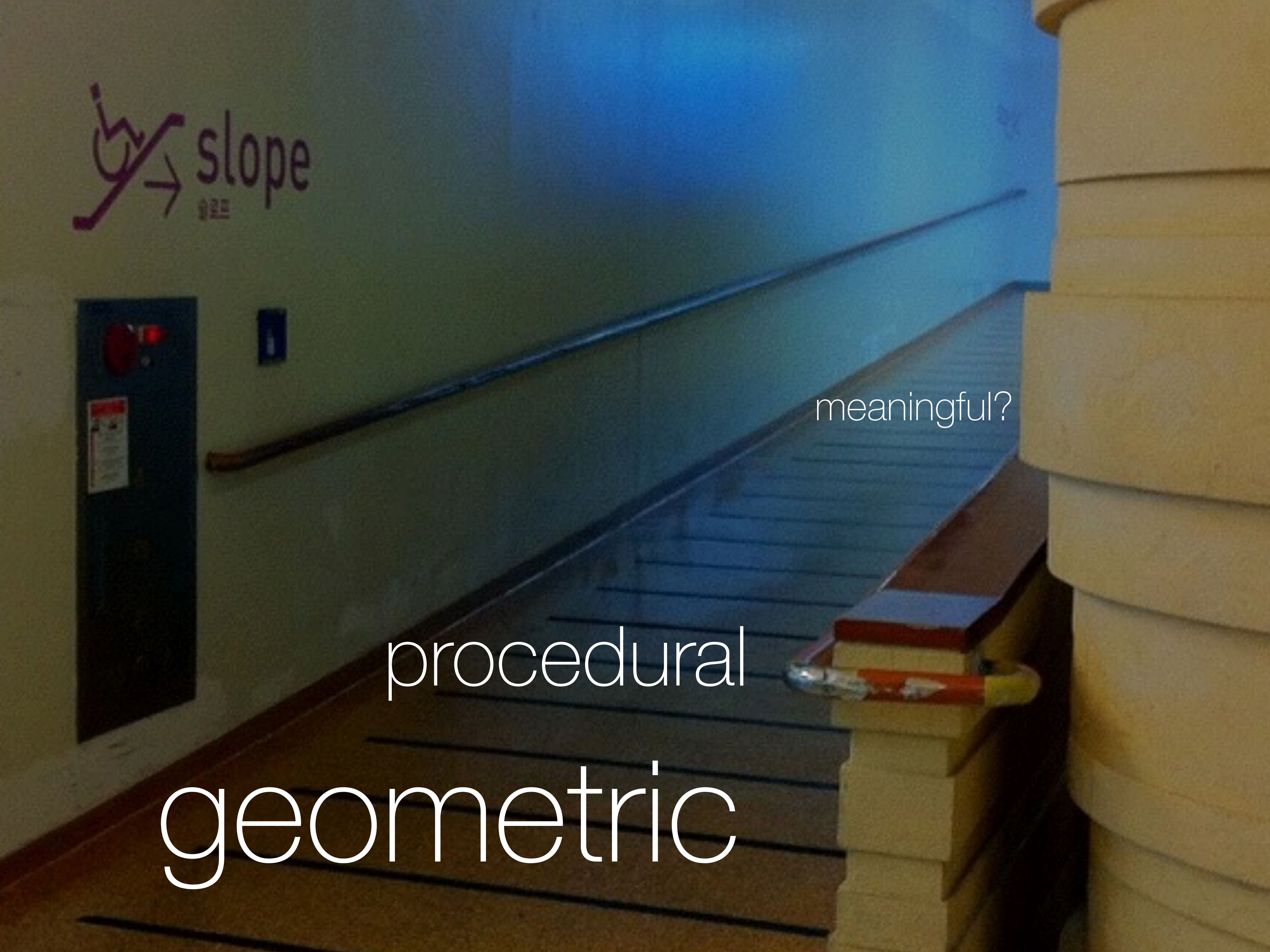
# First, let's do some math.

Feel free to write on this.

I'm going to give you a clean copy.

And all handouts are on my webpage:
www.RMEInTheClassroom.com


slope
meaningful?
procedural
geometric


meaningful?

# How do students make slope meaningful?

# slope

- Parametric coefficient
- Algebraic ratio
- Rate of change
- Geometric ratio
- Physical property

# slope

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

Physical property

$$y = a x + b$$

# slope

| Parametric coefficient | Algebraic ratio | Rate of change | Geometric ratio | Physical property |
|---|---|---|---|---|

$$\frac{y_2 - y_1}{x_2 - x_1}$$

# slope

Parametric coefficient

Algebraic ratio

**Rate of change**

Geometric ratio

Physical property

# slope

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

Physical property

$$\frac{\text{rise}}{\text{run}}$$

# slope

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

**Physical property**

“steepness”

# slope

- Parametric coefficient
- Algebraic ratio
- Rate of change
- Geometric ratio
- Physical property

Just tell them.

| Parametric coefficient | Geometric ratio | Physical property | Rate of change | Algebraic ratio |
| --- | --- | --- | --- | --- |
| $y = ax + b$ | $\frac{\text{rise}}{\text{run}}$ | "steepness" | | $\frac{y_2 - y_1}{x_2 - x_1}$ |

Algebra Lesson Page

## Slope and Rate of Change

Topic Index | Algebra Index | Regents Exam Prep Center

# Slope and Rate of Change

The word **slope** (gradient, incline, pitch) is used to describe the measurement of the steepness of a straight line. The higher the slope, the steeper the line. The slope of a line is a *rate of change*.

$$\text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{\text{Rise}}{\text{Run}}$$


Builders need to know the pitch of a roof to determine which type of shingle will be appropriate for the roof.

The building code for using asphalt shingles on roofs states that the minimum pitch must be a rise of 4" for every 12" of horizontal distance (run) covered. Asphalt shingles are not to be used on roofs that have very little steepness. Builders check to see if the pitch (slope) of the roof is $\frac{4}{12}$ or 4:12 or 4 to 12 before using asphalt shingles.

**Slope is a ratio and can be expressed as:**

change in $y$ over change in $x$. or $\frac{\text{vertical change}}{\text{horizontal change}}$ or $\frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{\text{rise}}{\text{run}}$

http://www.regentsprep.org/regents/math/algebra/AC1/Rate.htm

Parametric coefficient

$y = {\color{red}a}x + b$

# Slope and Rate of Change

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Physical property

"steepness"

# Slo[…] Rate of Change

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Geometric ratio

$$\frac{\text{rise}}{\text{run}}$$

$$\textit{Slope} = \frac{\textit{Vertical change}}{\textit{Horizontal change}} = \frac{\textit{Rise}}{\textit{Run}}$$

Rate of change

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[…]ow the pitch of a […]ich type of shingle […]te for the roof.

Algebraic ratio

$$\frac{y_2 - y_1}{x_2 - x_1}$$

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Algebraic ratio

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# meaningful?

| Parametric coefficient | Geometric ratio | Physical property | Rate of change | Algebraic ratio |
| --- | --- | --- | --- | --- |
| $y = \color{red}{a}x + b$ | $\dfrac{\text{rise}}{\text{run}}$ | “steepness” | | $\dfrac{y_2 - y_1}{x_2 - x_1}$ |

# meaningful?

Students should operate with *meaningful* quantities in situations that they can make sense of.

Parametric coefficient

$y = ax + b$

Geometric ratio

$\frac{\text{rise}}{\text{run}}$

Rate of change

Algebraic ratio

$\frac{y_2 - y_1}{x_2 - x_1}$

Physical property

"steepness"

# Why not steepness?

- Motivating?
- Robust?
  - $y = mx + b$

Parametric coefficient

$y = ax + b$

Geometric ratio

$\frac{\text{rise}}{\text{run}}$

Physical property

"steepness"

Rate of change

Algebraic ratio

$\frac{y_2 - y_1}{x_2 - x_1}$

# Why not "rise over run"?

- Meaningful?
- Focused on rule-based *position* of the numbers, and not on creating a *meaningful quantity* using meaningful operations
- e.g., $(a, b)$ vs $^{a}/_{b}$

Parametric coefficient

$y = ax + b$

Geometric ratio

$\frac{\text{rise}}{\text{run}}$

Physical property

"steepness"

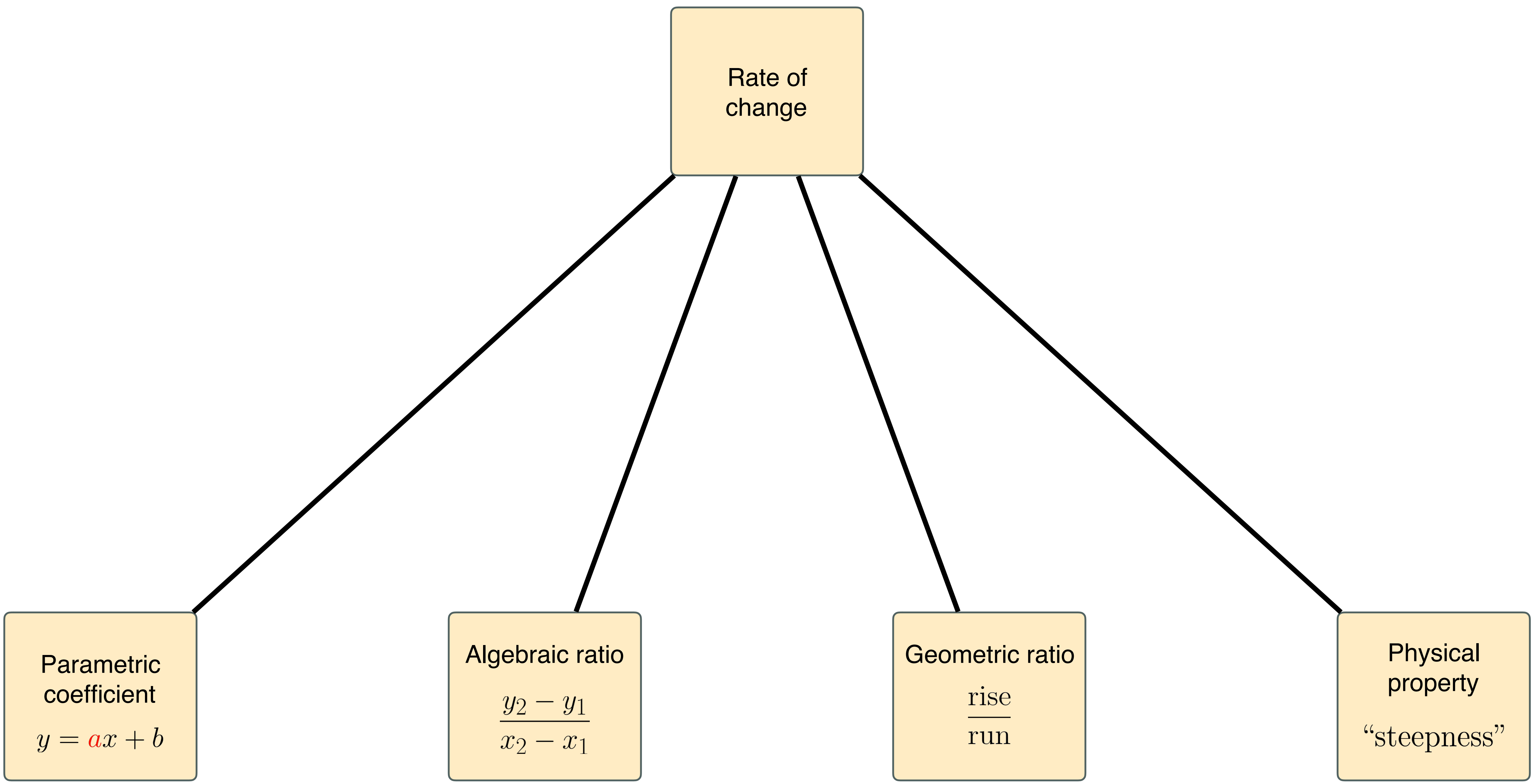
Algebraic ratio

$\frac{y_2 - y_1}{x_2 - x_1}$

Rate of change

# Why rate of change?

- Motivating :: predicting the future
- Meaningful quantity
- Robust
  - One of five NCTM "key concepts"
  - All sub-constructs of slope can be built from there


Rate of change
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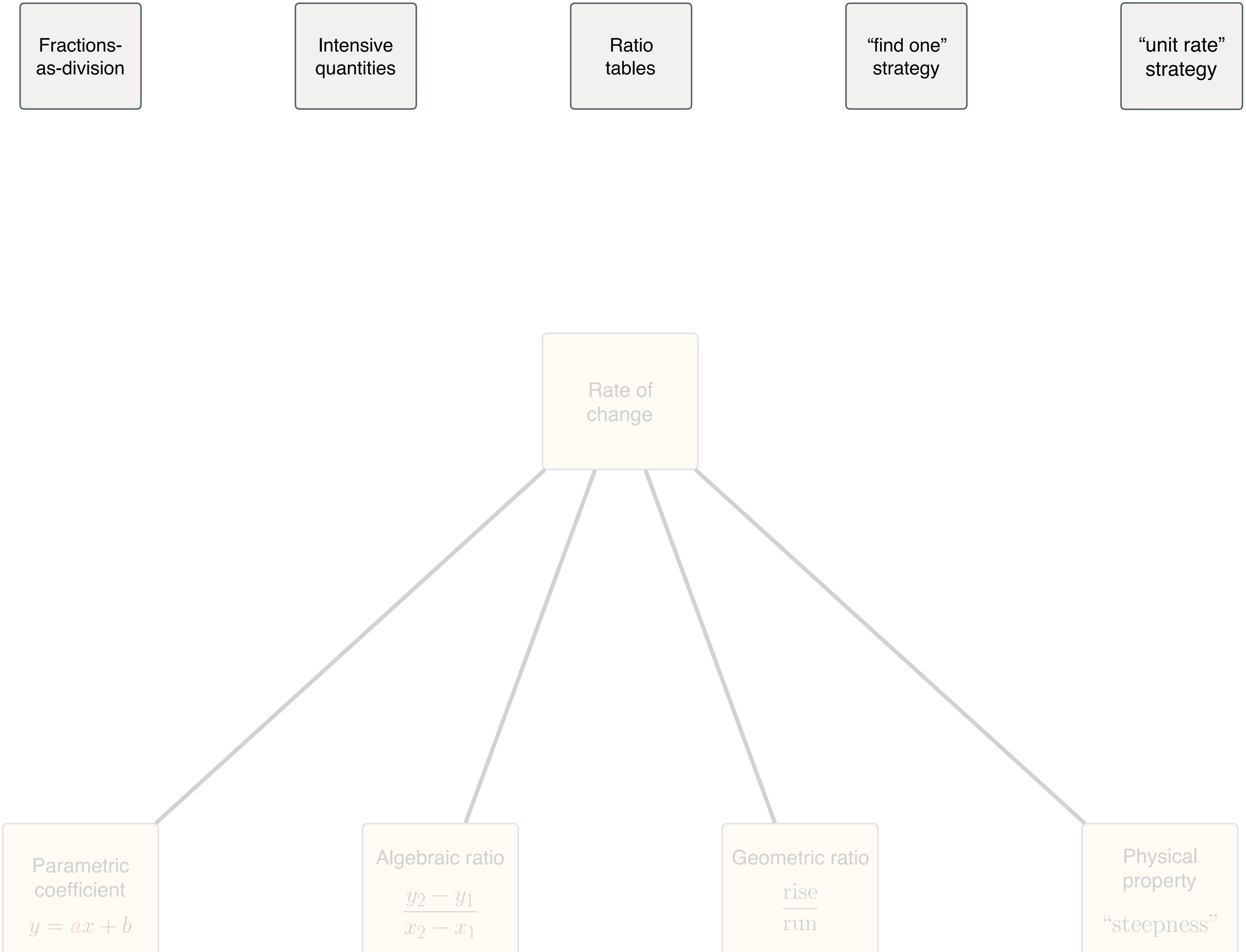

Fractions-as-division
Intensive quantities
Ratio tables
“find one” strategy
“unit rate” strategy
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# Phase 1

Fractions-as-division

Intensive quantities

Ratio tables

“find one” strategy

Fractions-as-division

Intensive quantities

Ratio tables

"find one" strategy

# Phase 1

Students **reinvent** and **learn**

- ratio table
- "find one" strategy
- intensive quantities (___per___)
- fractions-as-division

by engaging in these **activities**

"partitive division" situations

- fair sharing
- find unit values given many-to-many

Fractions-as-division

Intensive quantities

Ratio tables

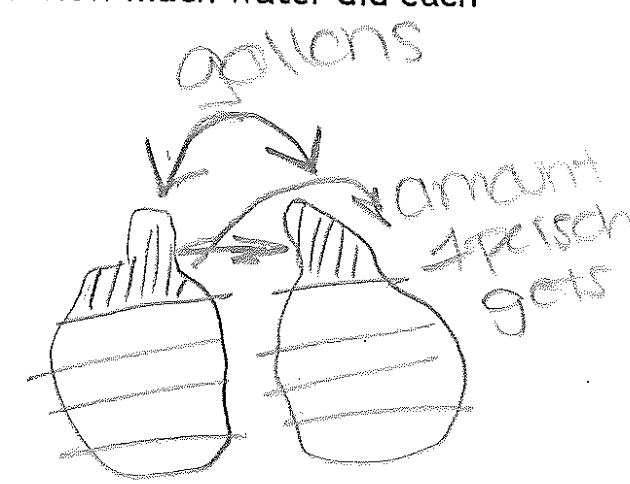
"find one" strategy

**1.** After a race, five people shared two gallons of water equally. How much water did each person receive?

*Show your work or explain your reasoning:*

÷5 5 people 2 gallons ÷5

1 2/5




State your final answer using units: 2/5 gallons *per* person.

**2.** In 7 minutes, a hot-air balloon rose 12 meters ÷7

In 1 minute, the hot-air balloon rose 12/7 meters

*Show your work or explain your reasoning:*

State your final answer using units: 12/7 meters *per* minute

Finding fair shares

Finding unit rates

1. After a race, five people shared two gallons of water equally. How much water did each person receive?

*Show your work or explain your reasoning:*

÷5 5 people 2 gallons ÷5

1 2/5

gallons

amount 1 person gets

Ratio tables

Fractions-as-division

State your final answer using units: 2/5 gallons per person.

Intensive quantities

Finding fair shares

2. In 7 minutes, a hot-air balloon rose 12 meters ÷7

÷7

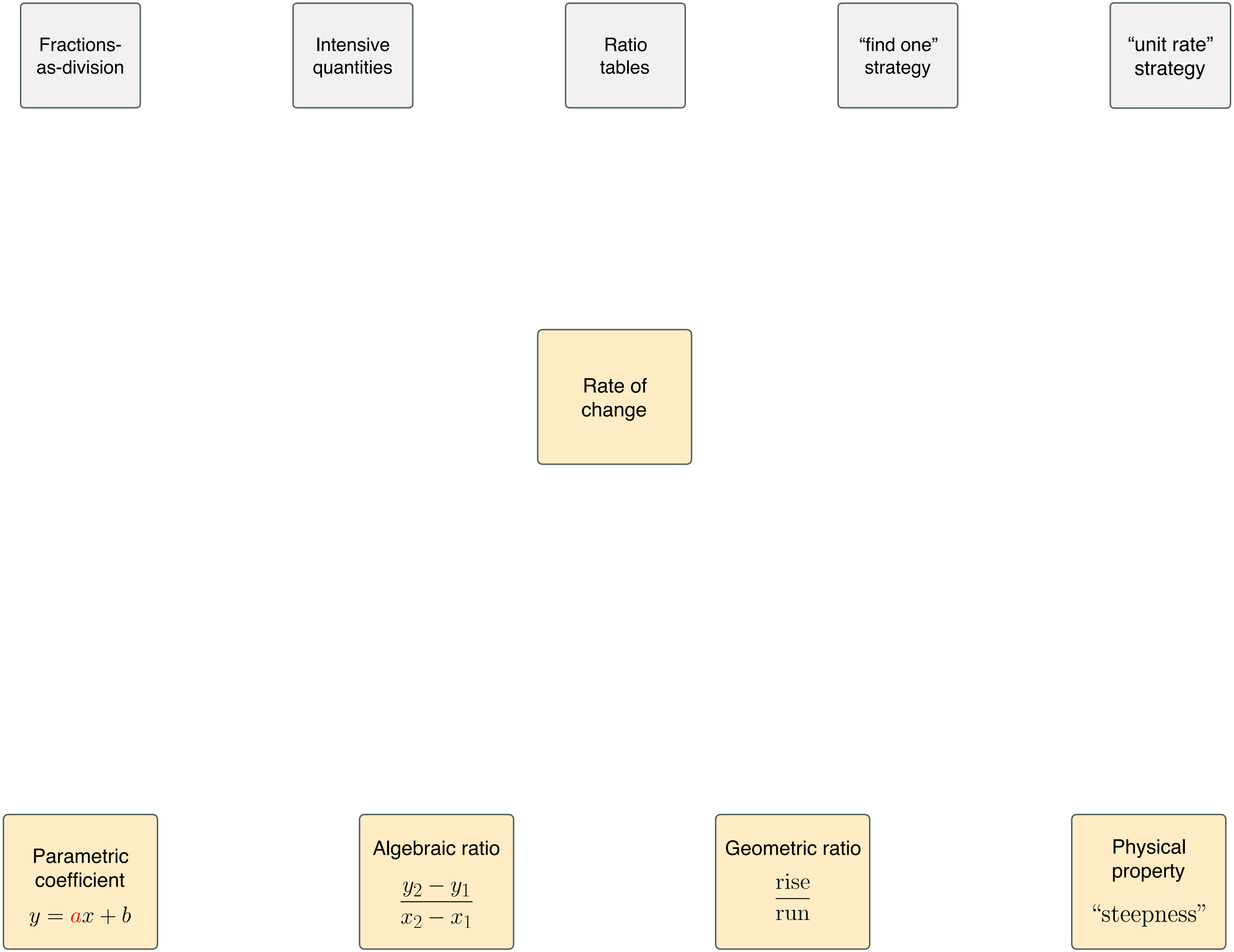
In 1 minute, the hot-air balloon rose 12/7 meters

*Show your work or explain your reasoning:*

"find one" strategy

State your final answer using units: 12/7 meters per minute

Finding unit rates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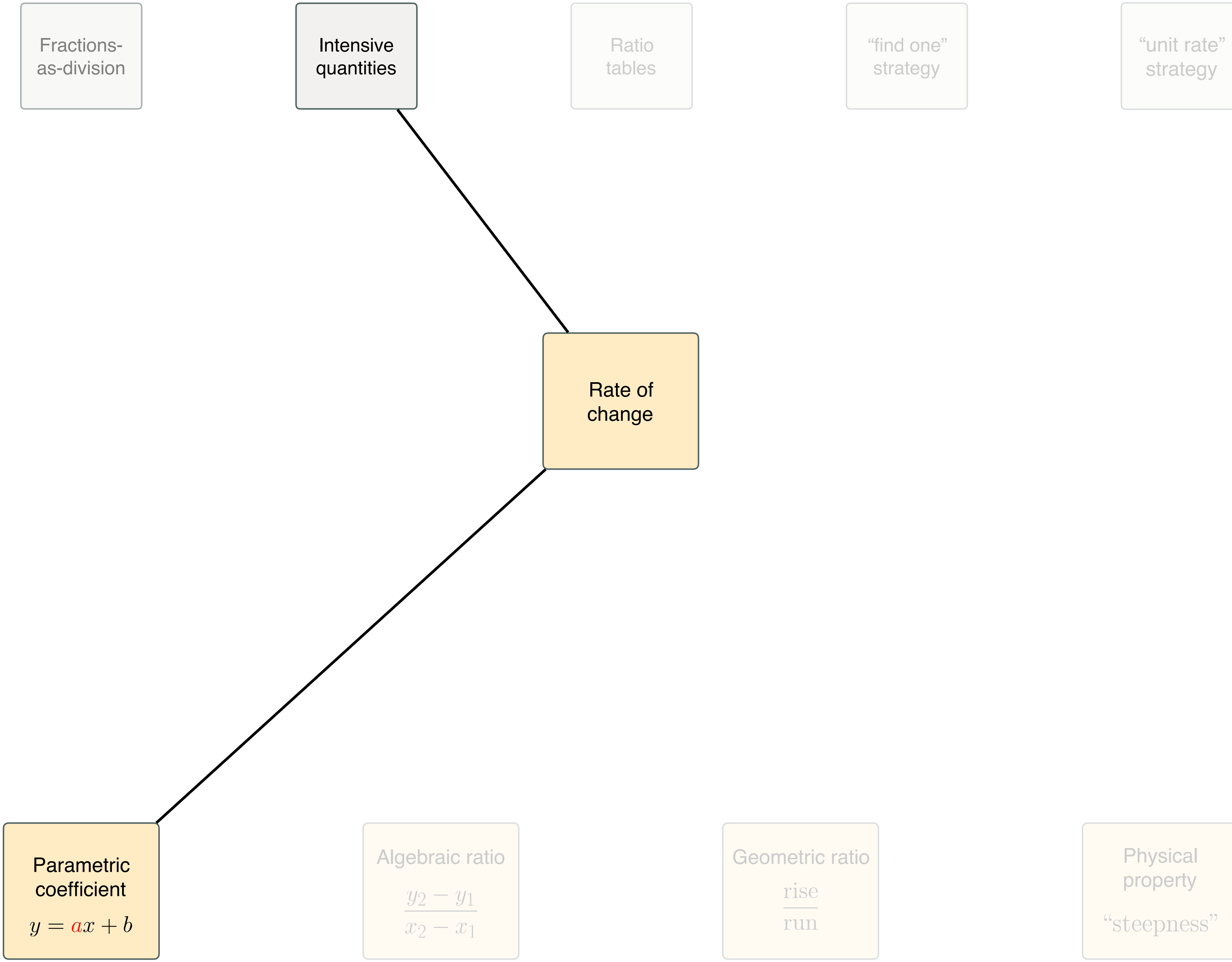
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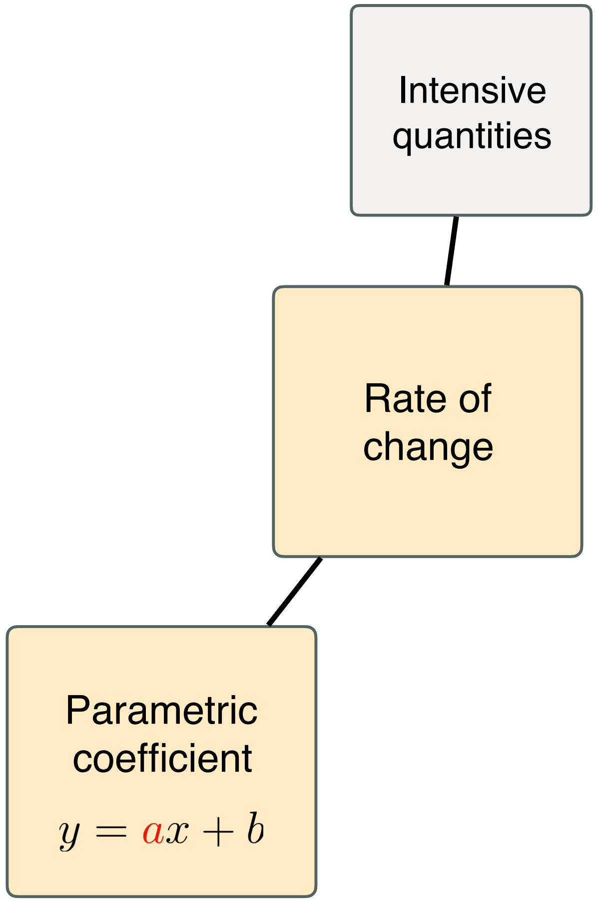
# phase 2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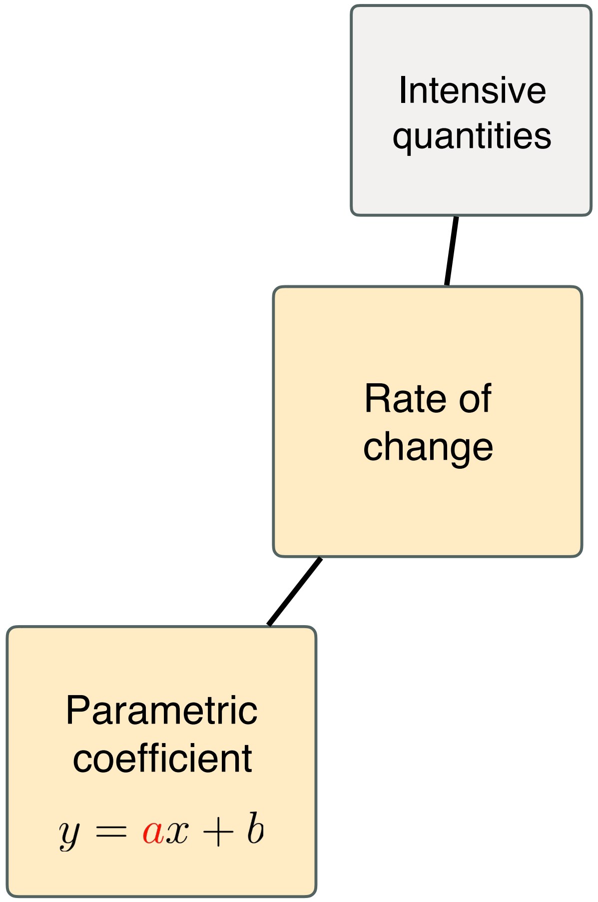



# phase 2

Students **reinvent** and **learn**

- **rate of change**
  - as an intensive quantity ("_____per_____")
  - that expresses covariation
  - and that can be accumulated
- **parametric coefficient** ($y = mx + b$)

by engaging in these **activities**

make predictions given:

- rate and start
- well-ordered function table ($\triangle x = 1$)

Monday, August 04, 2008, 07:00 am PT (10:00 am ET)

+ A -

# Apple already building iPhones at rate of 40 million a year?

By Slash Lane

**Apple is reportedly testing the limits of its overseas manufacturing facilities in order to keep up with demand for the new iPhone 3G, with production already cranked nearly sevenfold compared to the first-generation model.**

Foxconn, the company's Taiwanese handset and iPod manufacturer, has recently ramped production of the new iPhone to 800,000 units per week, says *TechCrunch*, citing a person "close to Apple with direct knowledge of the numbers."

The build rate is said to be "above current full capacity" for the Foxconn facilities alloted to Apple's handset business, which has led to concerns that quality control may suffer. At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months.

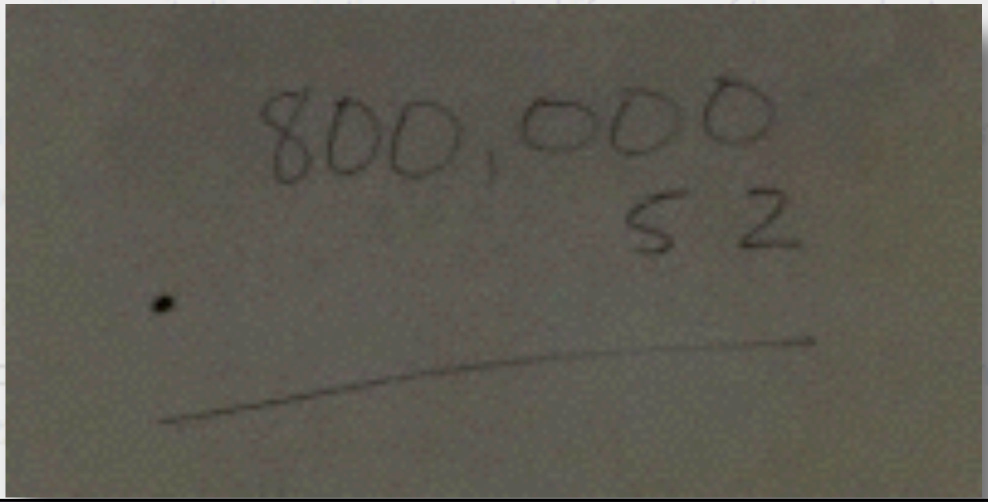
That paces well ahead of analysts' estimates (1, 2, 3) and early reports that suggested Apple's initial iPhone 3G orders spanned only 25 million units through the expected lifespan of the product.

*TechCrunch* believes Apple's initial order was actually 40 million units over the course of the first twelve months, but is now hearing that "those numbers are being revised upwards sharply."

Apple said it sold 1 million iPhones in the first 72 hours the new iPhone 3G was put on sale, but has not provided an updated sales tally since. The iPhone is currently on sale in 23 countries, with 20 more expected to be added on August 22nd, and another 30 by the end of the calendar year.

... 800,000 units per week ...

... At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months ...



Monday, August 04, 2008, 07:00 am PT (10:00 am ET)

FAP: Randy why is that [multiplication] going to get us a prediction for the number of iPhones in a year? How does weeks turn into iPhones?

Randy: Because for every week you have, you produce a certain amount of iPhones, so if you multiply it by a certain amount of weeks, the amount of iPhones will go up. [The reason-

FAP: [For every-

Randy: -that might be important is for (investors to know)

expected to be added on August 22nd, and another 30 by the end of the calendar year.

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Covariation

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Accumulation

**Apple already building iPho**

Rate as an intensive quantity

By Slash Lane

Published: 10:00 AM EST Monday

http://www.appleinsider.com/article ... rate_of_40_million_a_year.html

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RATE OF CHANGE

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The build rate is said to be "above current full capacity" for the Foxconn facilities allotted to Apple's handset business, which has led to concerns that quality control may suffer. At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months.

Prediction

| | Number of games | Total cost | |
|---|---|---|---|
| | 0 | | |
| | 1 | | |
| +1 | 2 | 8.00 | +2 |
| | 3 | 10.00 | |
| | 4 | 12.00 | |
| | 5 | 14.00 | |
| | 6 | 16.00 | |

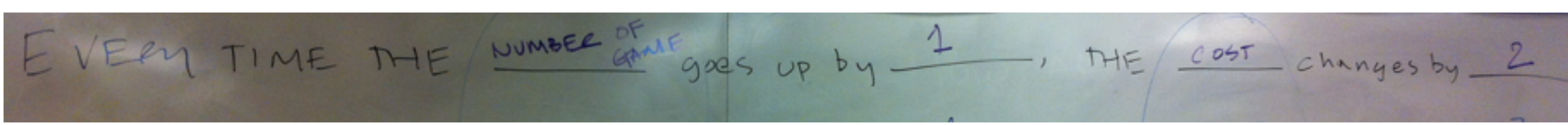
Rate of change: 2 dollars per game.

FAP: Stacy, where do you see that rate of change in the table?

Stacy: Um, for every number of games, the total cost goes up by two

FAP: Ever::y what about the number of games?

Stacy: The, each time the number of games increases by one, the total cost increases by two.



Rate as an intensive quantity

Rate of change: 2 dollars per game.

Covariation

| N... of... | ...st |
|---|---|
| 1 | |
| 2 | 8.00 |
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| 6 | |

+1

+2

FAP: Stacy, where do you see that rate of change in the table?

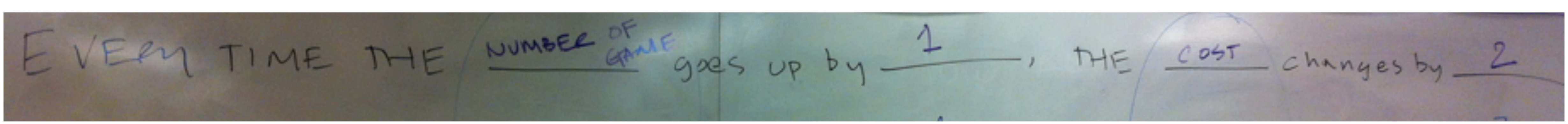
Stac... ...f game... ...by two

that expresses covariation

FA... number...

...cy: The, each time the number ...games increases by one, the ...l cost increases by two.

Covariation



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# Accumulation

# Covariation

GAME goes up by 1

Prediction: How much would it cost to ship 12 Xbox games?

*Show your work or explain your reasoning in the space below*

$28.00 because 2·12=24 +4=28

Prediction: How much would it cost to ship 12 Xbox games?

*Show your work or explain your reasoning in the space below*

$$\begin{array}{r} 12 \\ \times\ 2 \\ \hline 24 \end{array} +4 = \$28$$

Prediction: How much would it cost to ship 12 Xbox games?

*Show your work or explain your reasoning in the space below*

7=18 10=24
8=20 11=26
9=22 12=28

$28 for 12 games

| | |
|---|---|
| 5 | 14.00 |
| 6 | 16.00 |
| 7 | 1800 |
| | 2000 |
| 9 | 2200 |
| 10 | 2400 |
| | 2600 |
| 12 | [illegible] |

| Number of games | Total cost |
|---|---|
| 0 | |
| 1 | |
| 2 | 8.00 |
| 3 | 10.00 |
| 4 | 12.00 |
| 5 | 14.00 |
| 6 | 16.00 |

+1 +2

Equation: $y = 2(x) + (4)$

Melissa: Okay, um Y is like the final, cost, and, two is the one time fee times how many games you have- or not the one time fee- like, how much dollars it is per game, and four is the one time fee.

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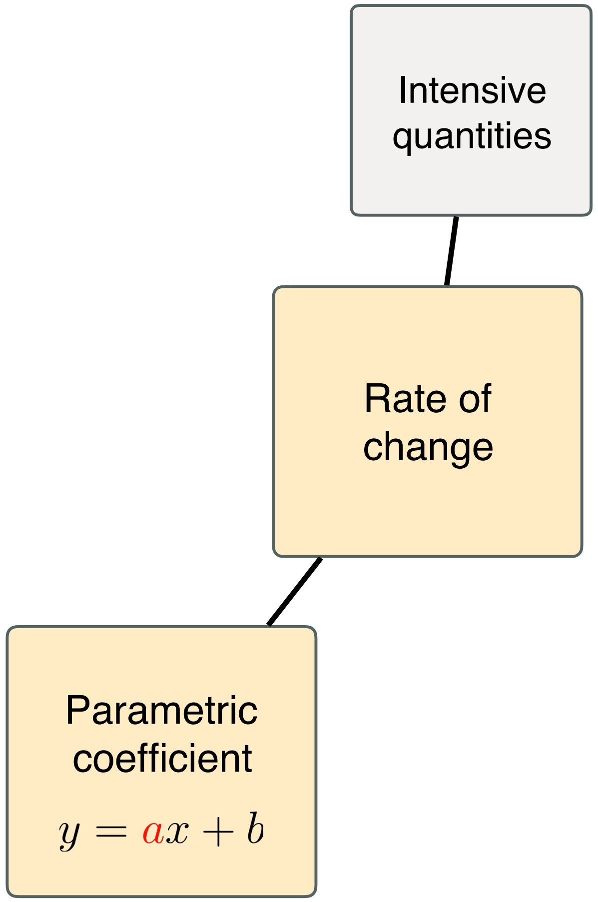
Rate as an intensive quantity that can be accumulated




Rate as an intensive quantity

e 2

Students **reinvent** and **learn**

- **rate of change**
  - as an intensive quantity ("_____ per _____")
  - that expresses covariation
  - and that can be accumul ed
- **parametric coefficie** $b$)

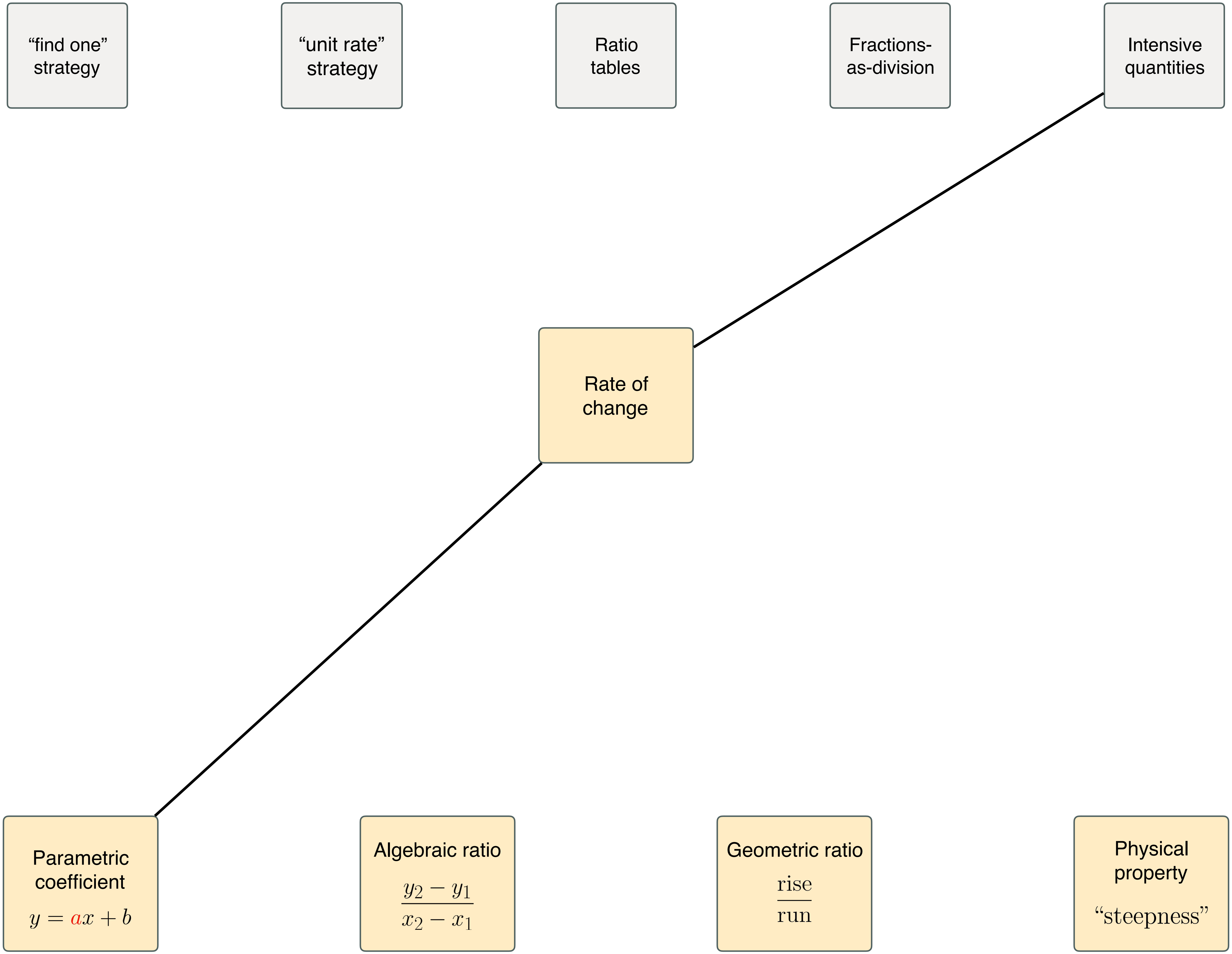
that expresses covariation

by engaging in these **activities**

make predictions given:

- well-ordered function table ($\triangle x = 1$)
- rate and start

and can be accumulated


"find one" strategy
"unit rate" strategy
Ratio tables
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Intensive quantities
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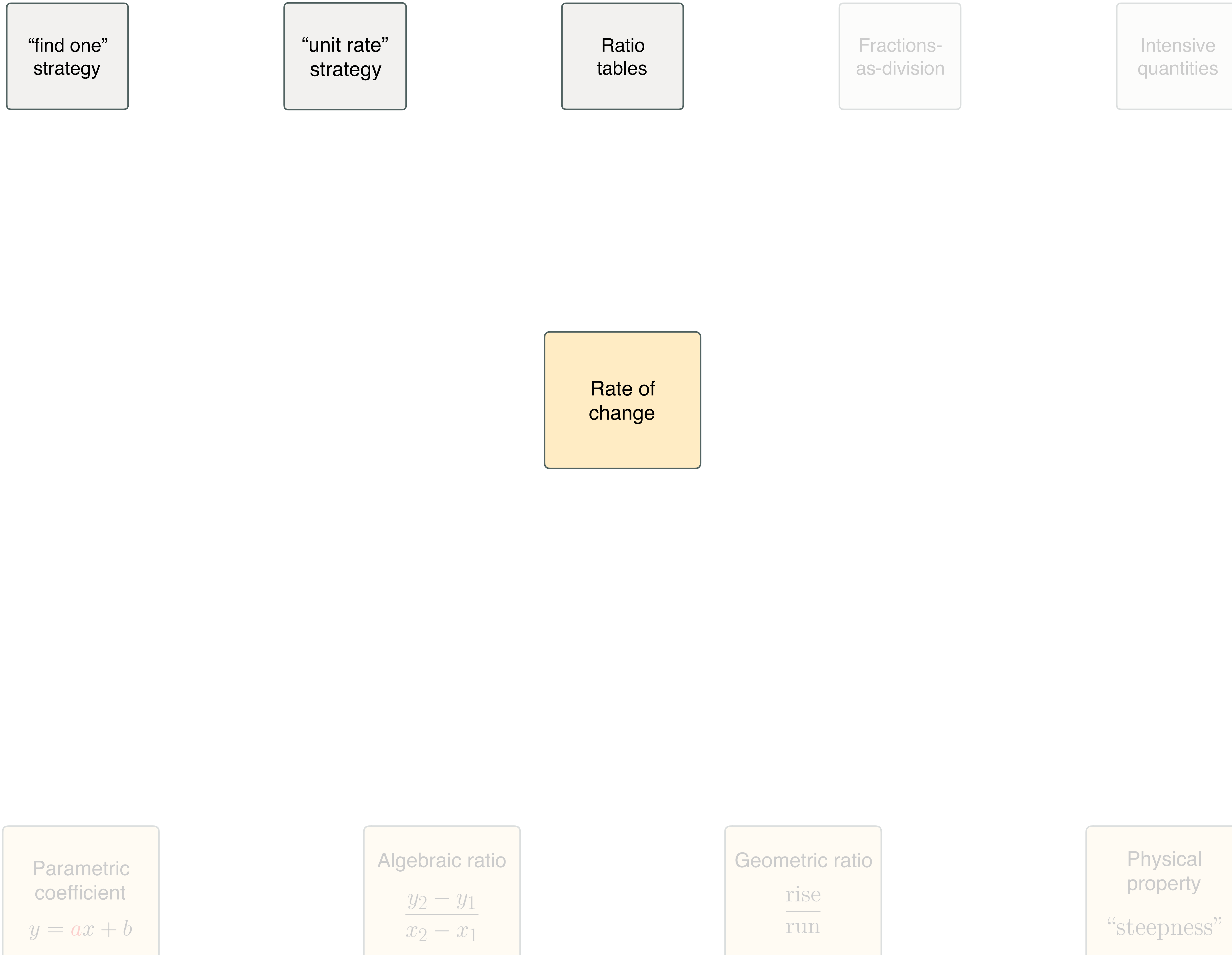

“find one” strategy
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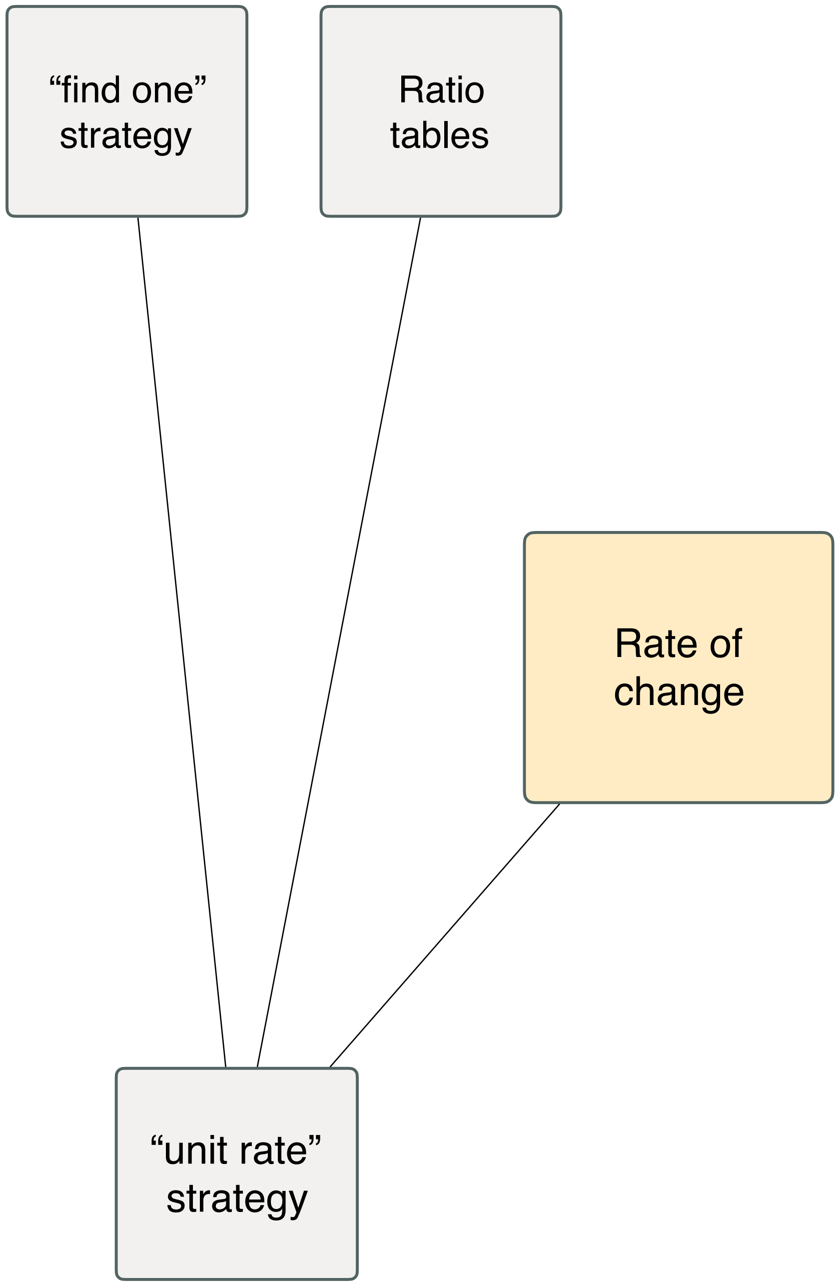
# phase 3



“find one” strategy
Ratio tables
Rate of change
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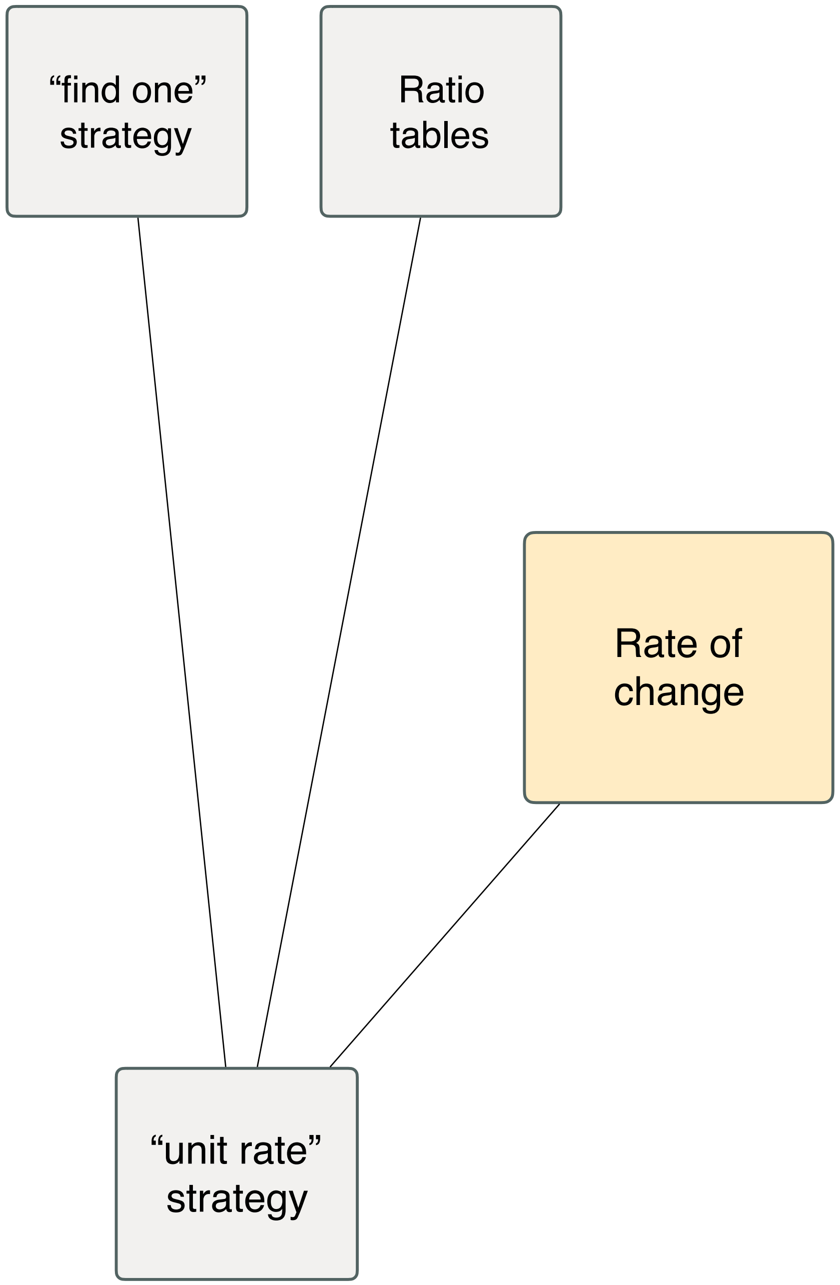



# phase 3

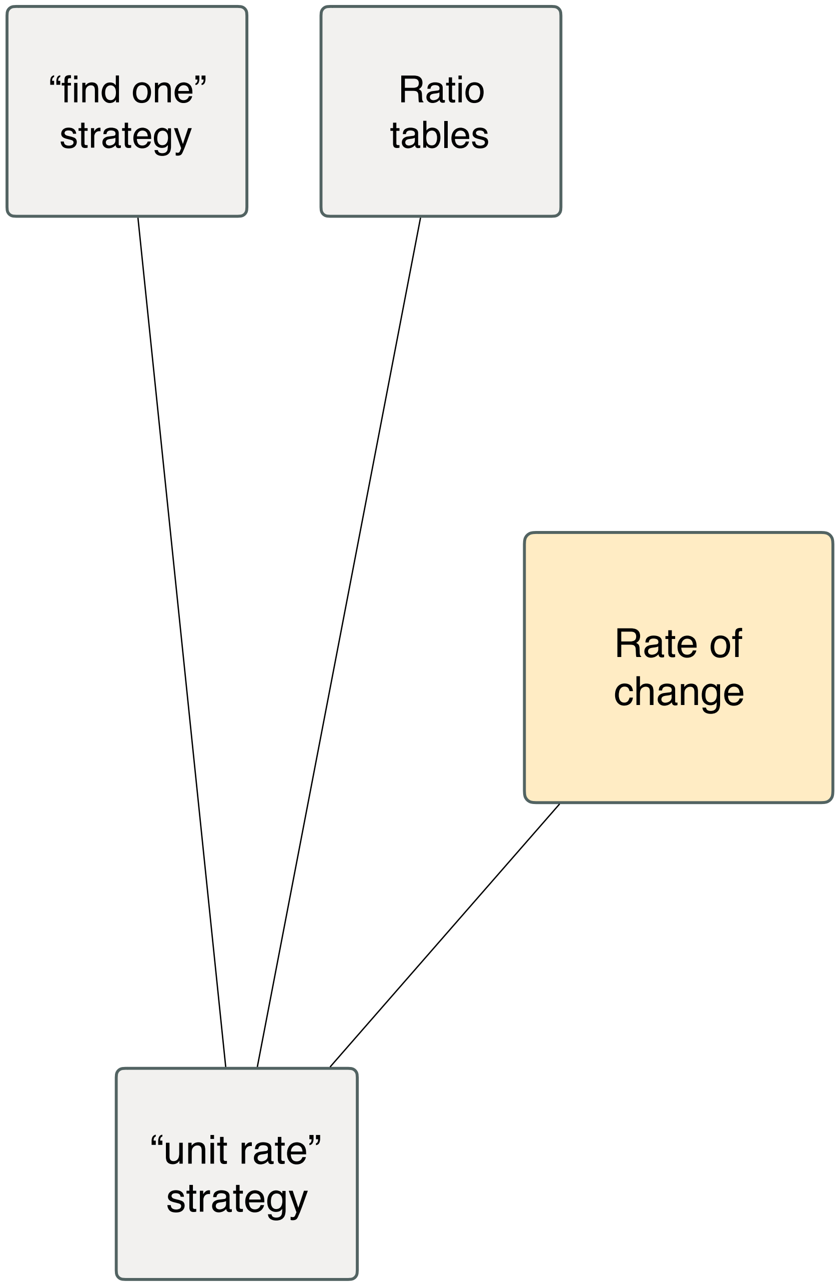
Students **reinvent** and **learn**

unit rate strategy
(scale down to find a unit rate, and scale up to make a prediction)

by engaging in these **activities**

make predictions

- proportional situations
- within-unit values are relatively prime




Ms. Magro runs 6 miles every day. On average, she can run six miles in 54 minutes. At this rate, how long would it take Ms. Magro to run an 11-mile race?

6 mile > 54 min .

6÷( 1 mile 9 min )÷6

9x11 = 99 .

Takes 99 minutes

"find one" strategy

Ratio tables

Rate of change

"unit rate" strategy

**2.** In 7 minutes, a hot-air balloon rose 12 meters

In 1 minute, the hot-air balloon rose $12/7$ meters

$\div 7$ $\div 7$

*Show your work or explain your reasoning:*

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Takes 99 minutes

"find one" strategy

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"unit rate" strategy

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6÷( 6 mile > 54 min )÷6

1 mile 9 min

$\boxed{9 \times 11 = 99}$

Takes 99 minutes

“find one” strategy

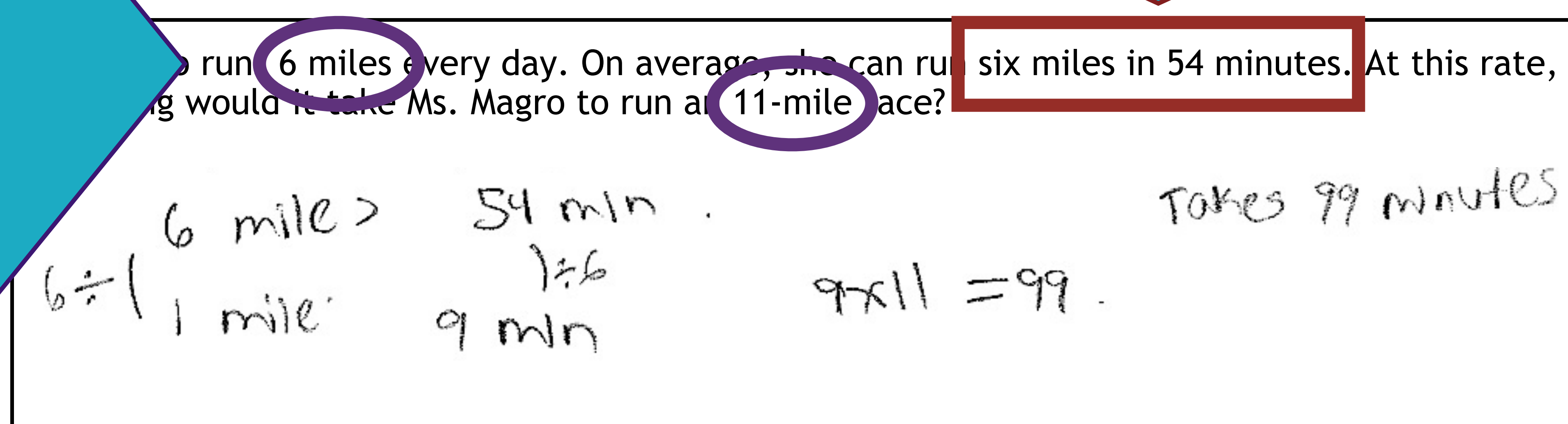
Ratio tables

Rate of change

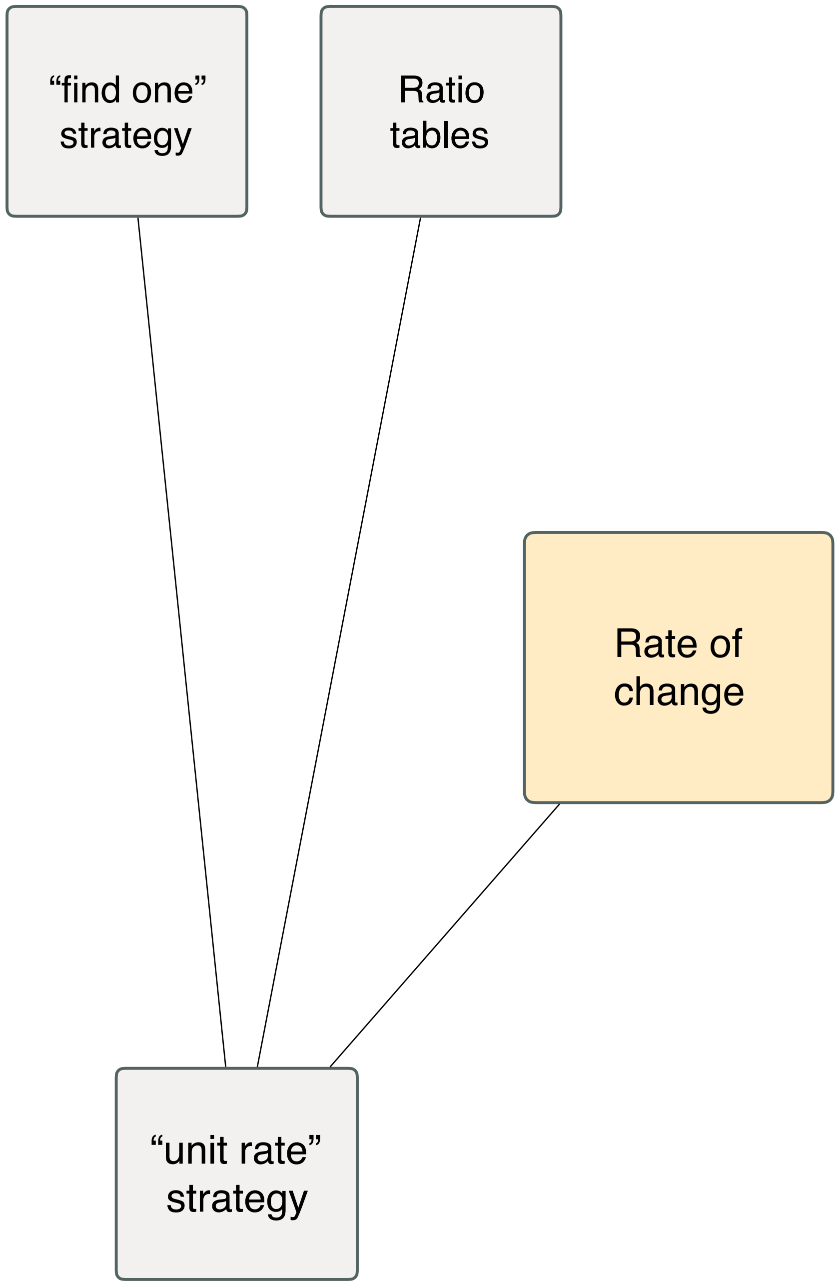
“unit rate”

Miles are **proportional** to minutes

Within unit values are **relatively prime**



# phase 3



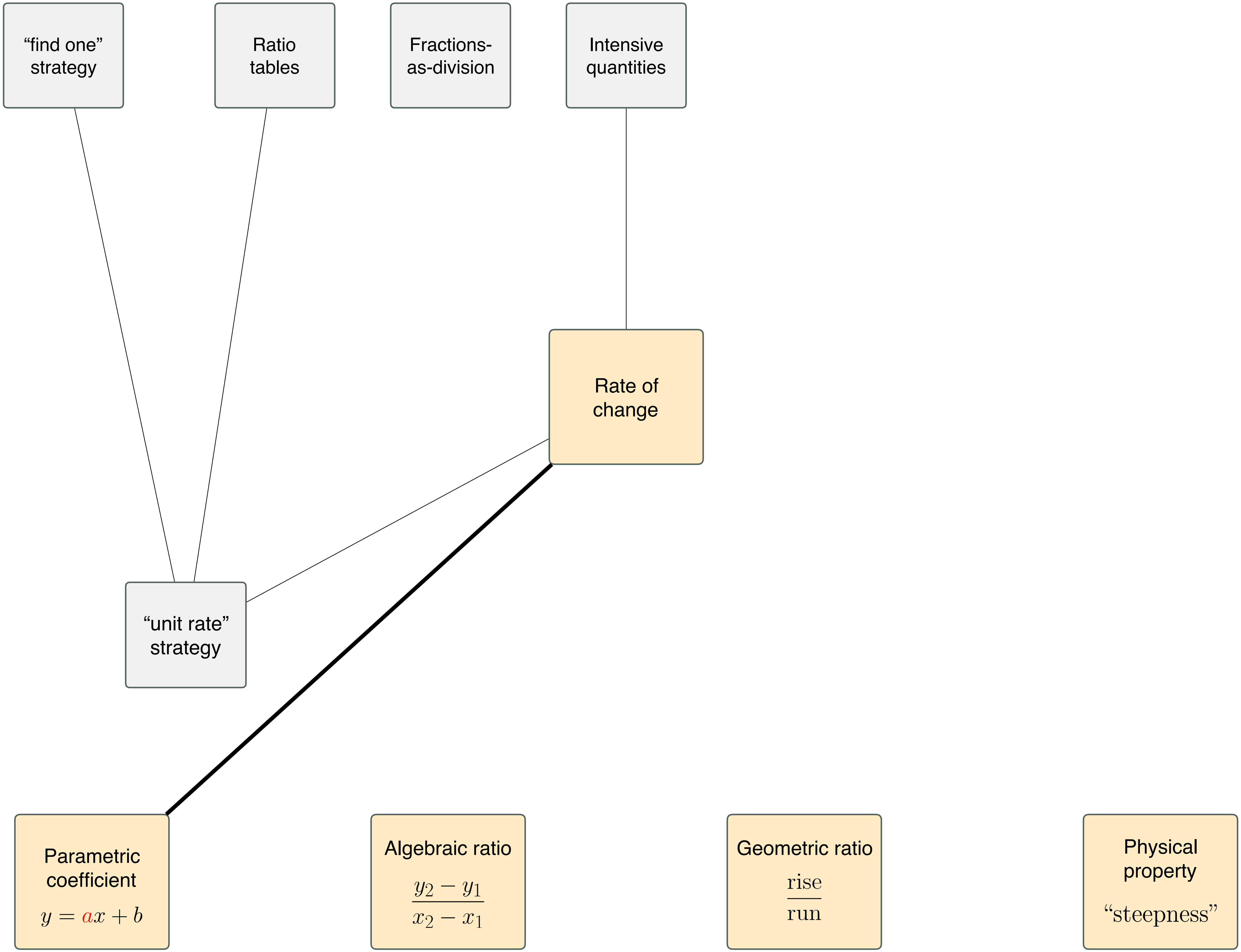

Students **reinvent** and **learn**

unit rate strategy
(scale down to find a unit rate, and scale up to make a prediction)

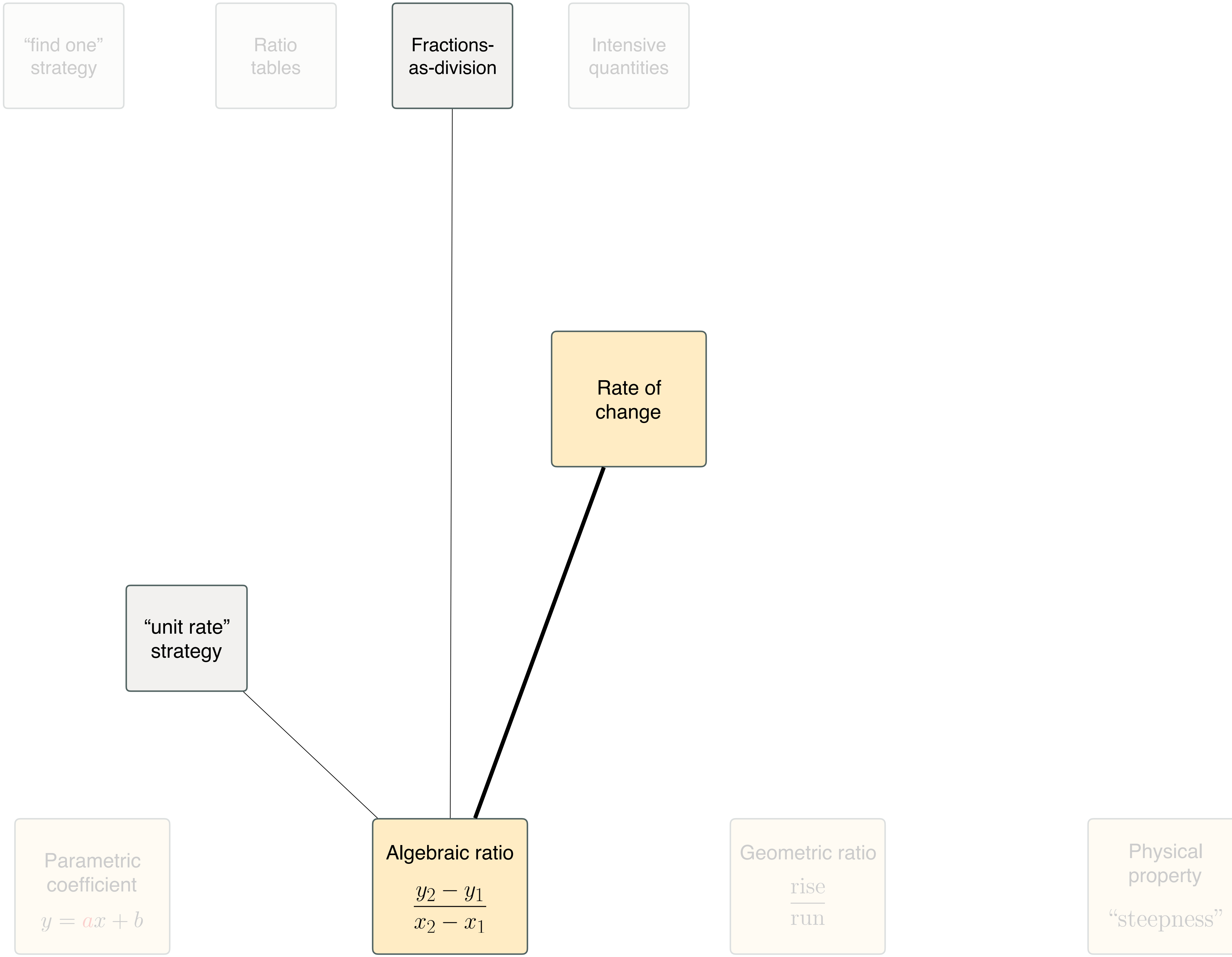
by engaging in these **activities**

make predictions

- proportional situations
- within-unit values are relatively prime



“find one” strategy
Ratio tables
Fractions-as-division
Intensive quantities
Rate of change
“unit rate” strategy
Parametric coefficient
$y = ax + b$
Algebraic ratio
$\frac{y_2 - y_1}{x_2 - x_1}$
Geometric ratio
$\frac{\text{rise}}{\text{run}}$
Physical property
“steepness”



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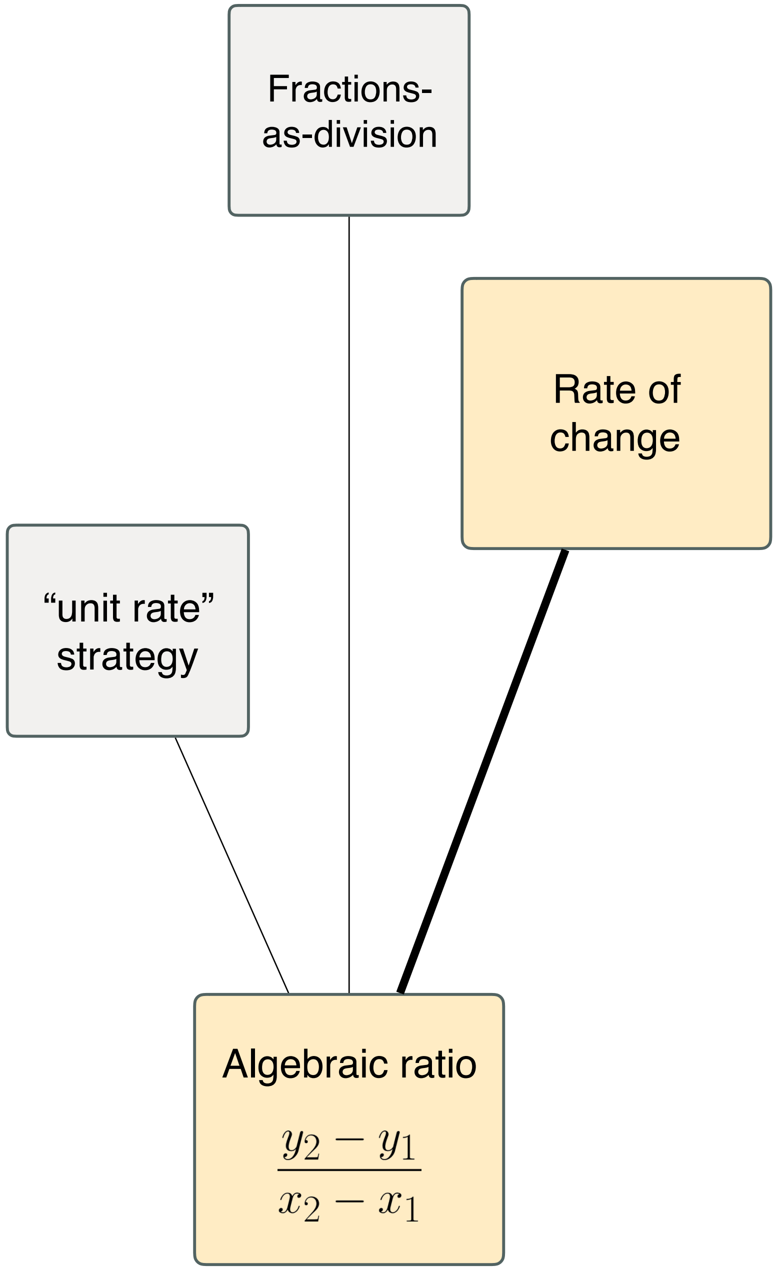
# phase 4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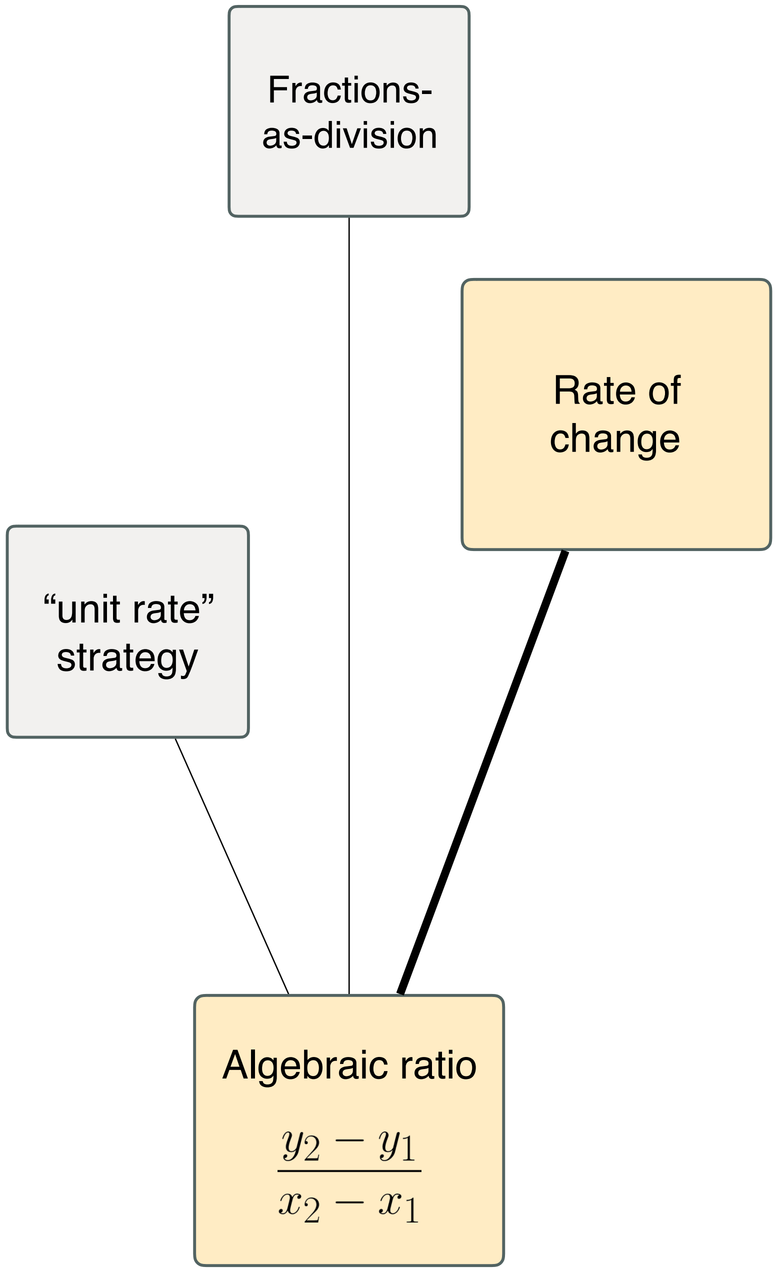



# phase 4

Students **reinvent** and **learn**

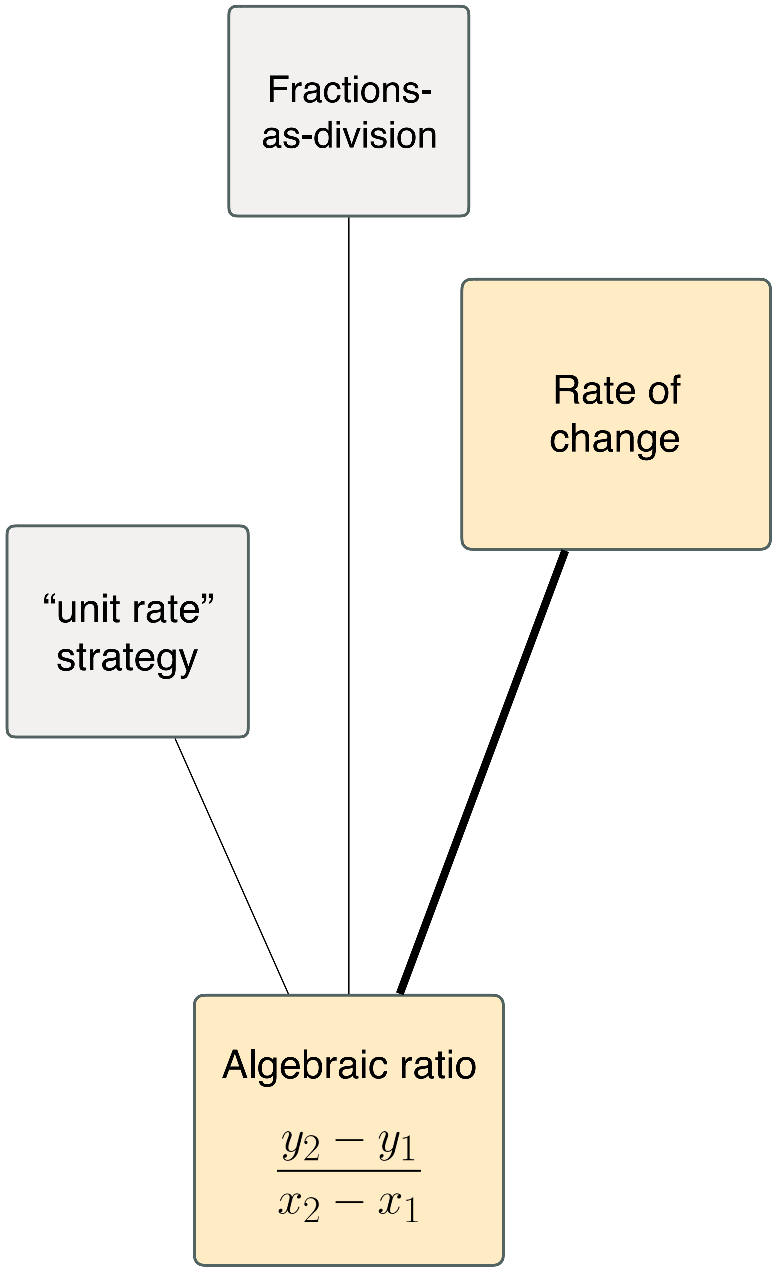
algebraic ratio

$$\frac{y_2 - y_1}{x_2 - x_1}$$

by engaging in these **activities**

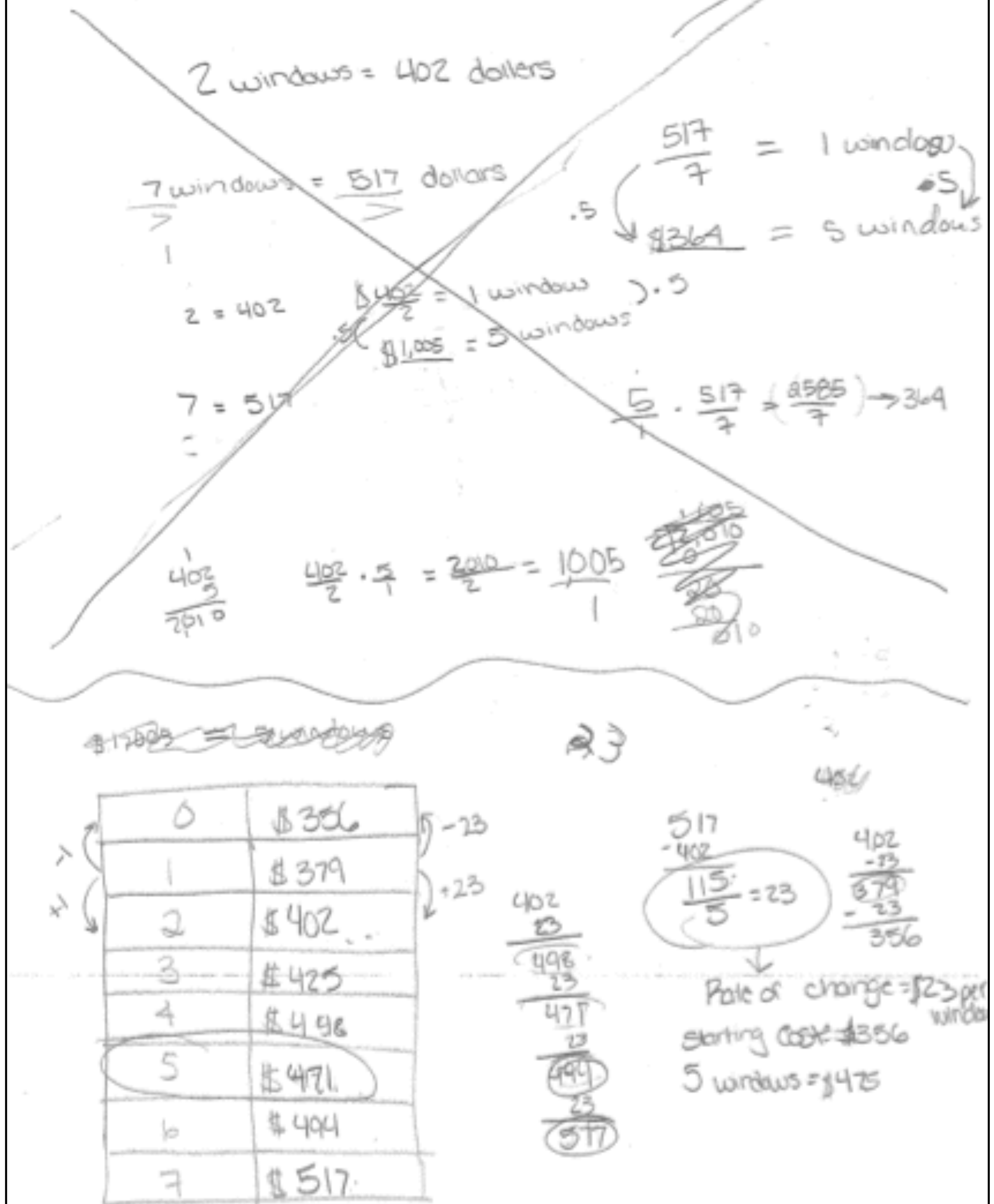
make predictions

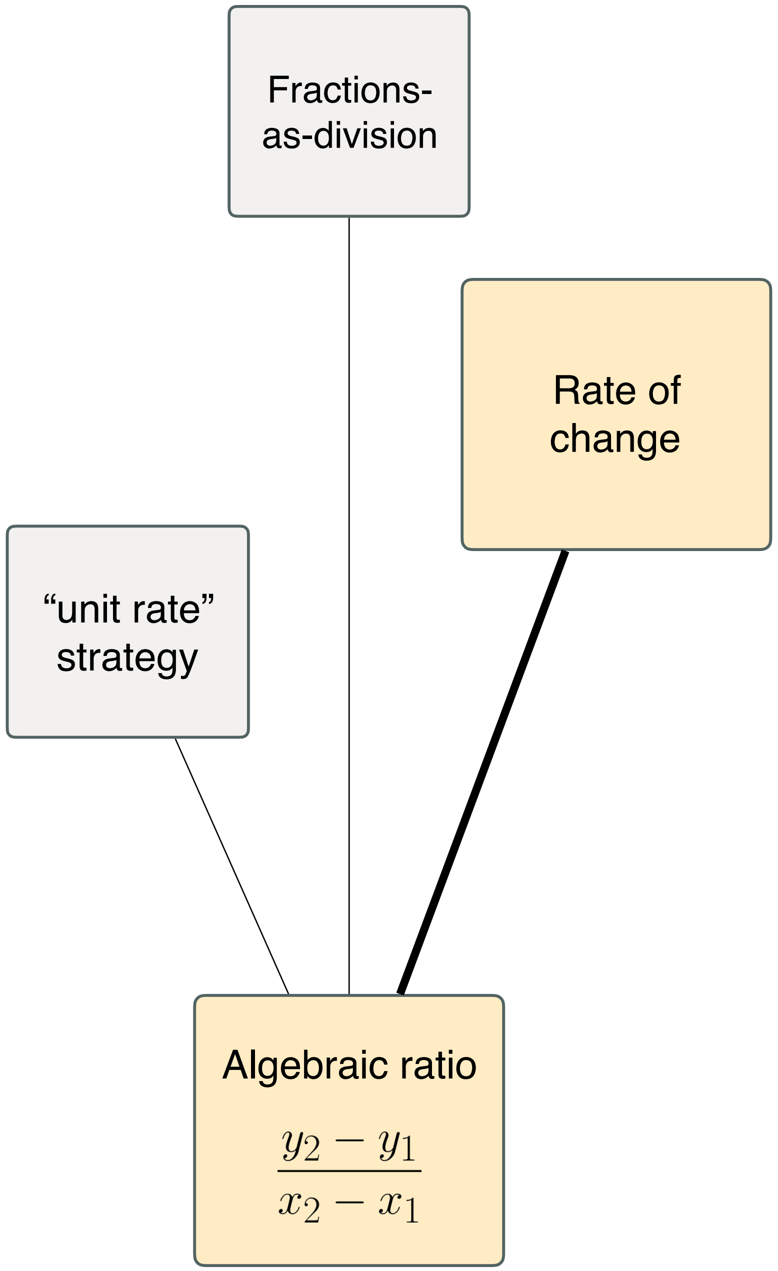
- linear, non-proportional situations
- two data points with $\triangle x \neq 1$

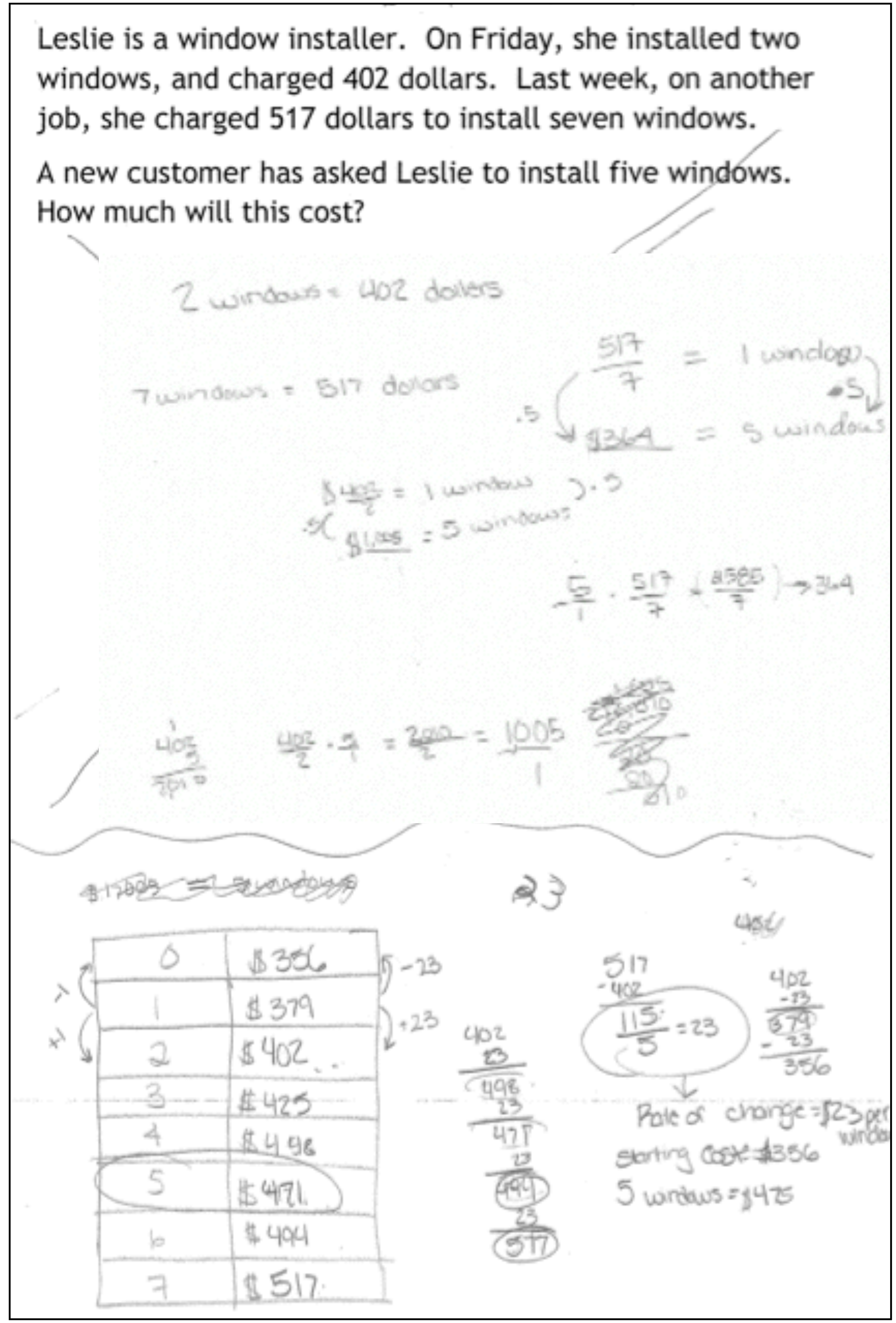



Leslie is a window installer. On Friday, she installed two windows, and charged 402 dollars. Last week, on another job, she charged 517 dollars to install seven windows.

A new customer has asked Leslie to install five windows. How much will this cost?



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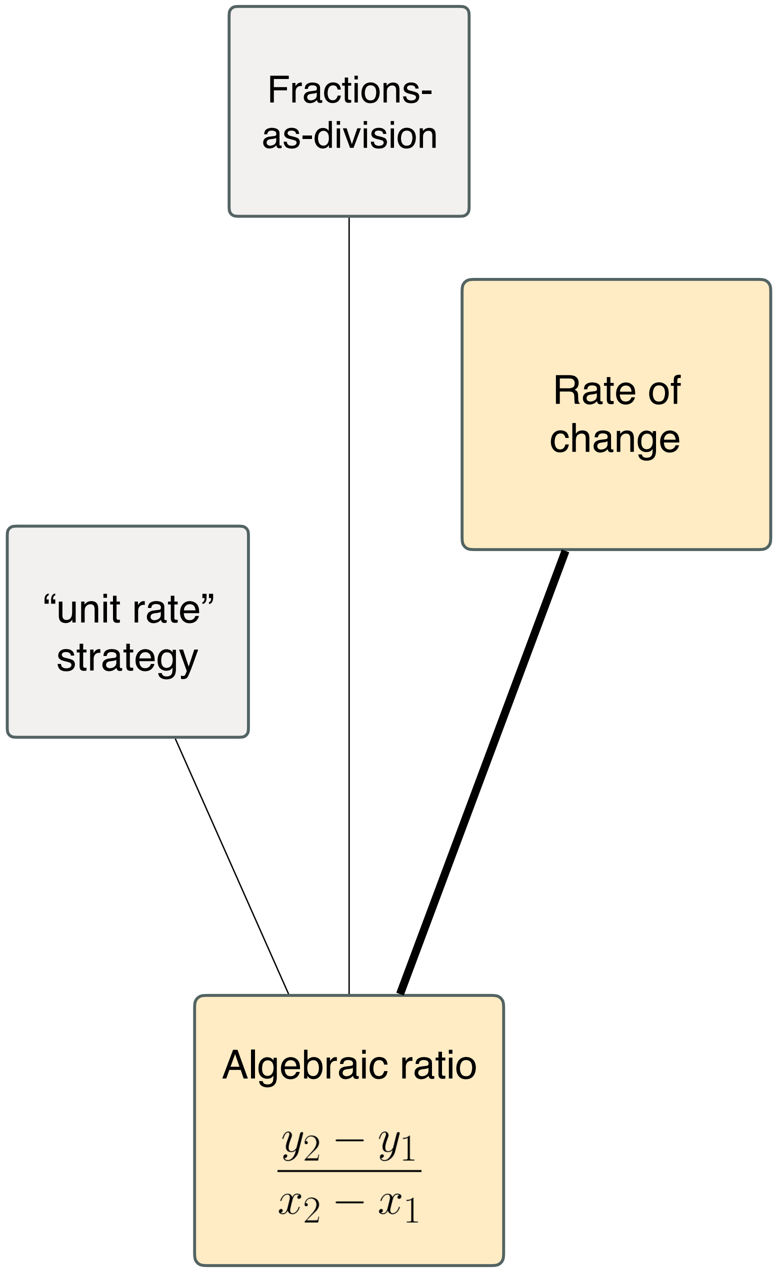
| | |
|---|---|
| 0 | \$356 |
| 1 | \$379 |
| 2 | \$402 |
| 3 | \$425 |
| 4 | \$498 |
| 5 | \$471 |
| 6 | \$494 |
| 7 | \$517 |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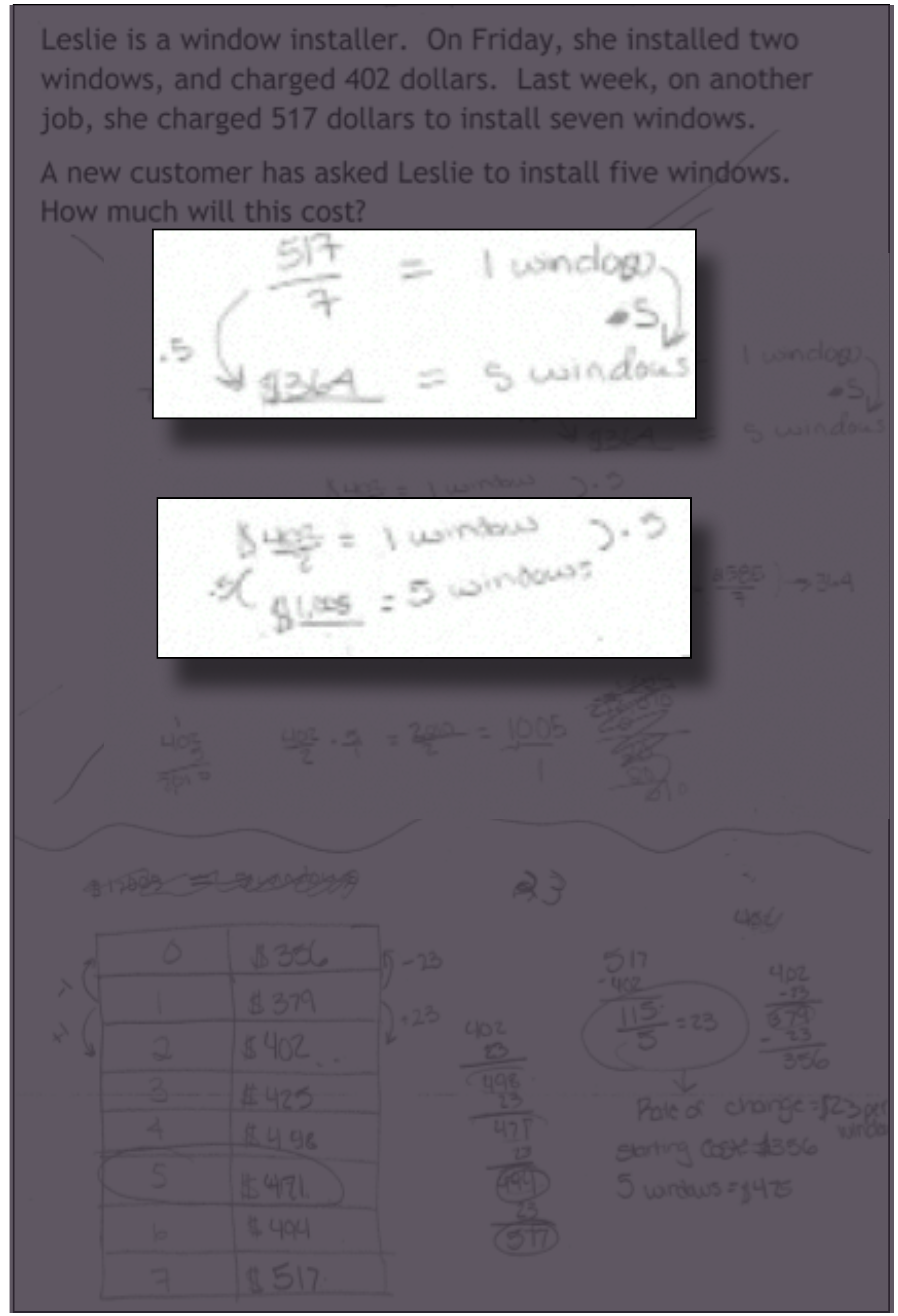
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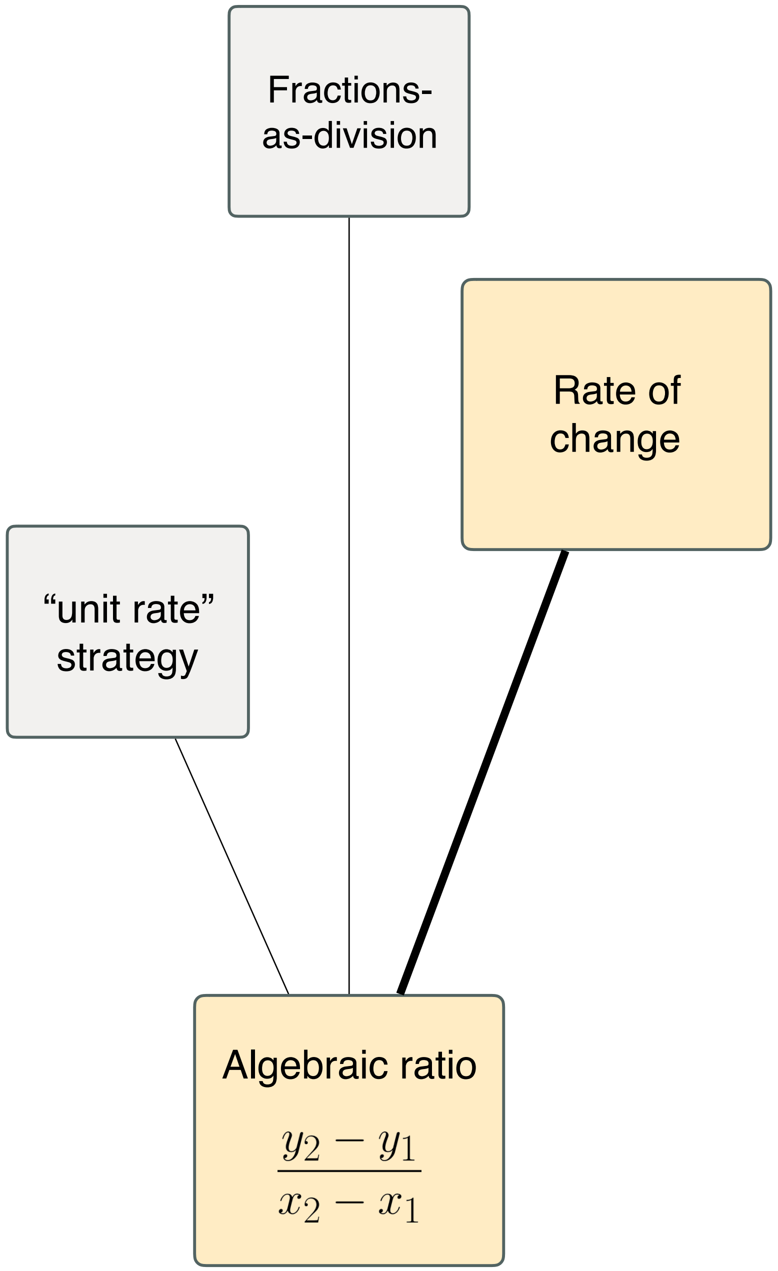
A new customer has asked Leslie to install five windows. How much will this cost?
517/7 = 1 window
•5
.5
$364 = 5 windows
$402/2 = 1 window
•5
$1,005 = 5 windows
Rate of change = $23 per window
Starting Cost = $356
5 windows = $475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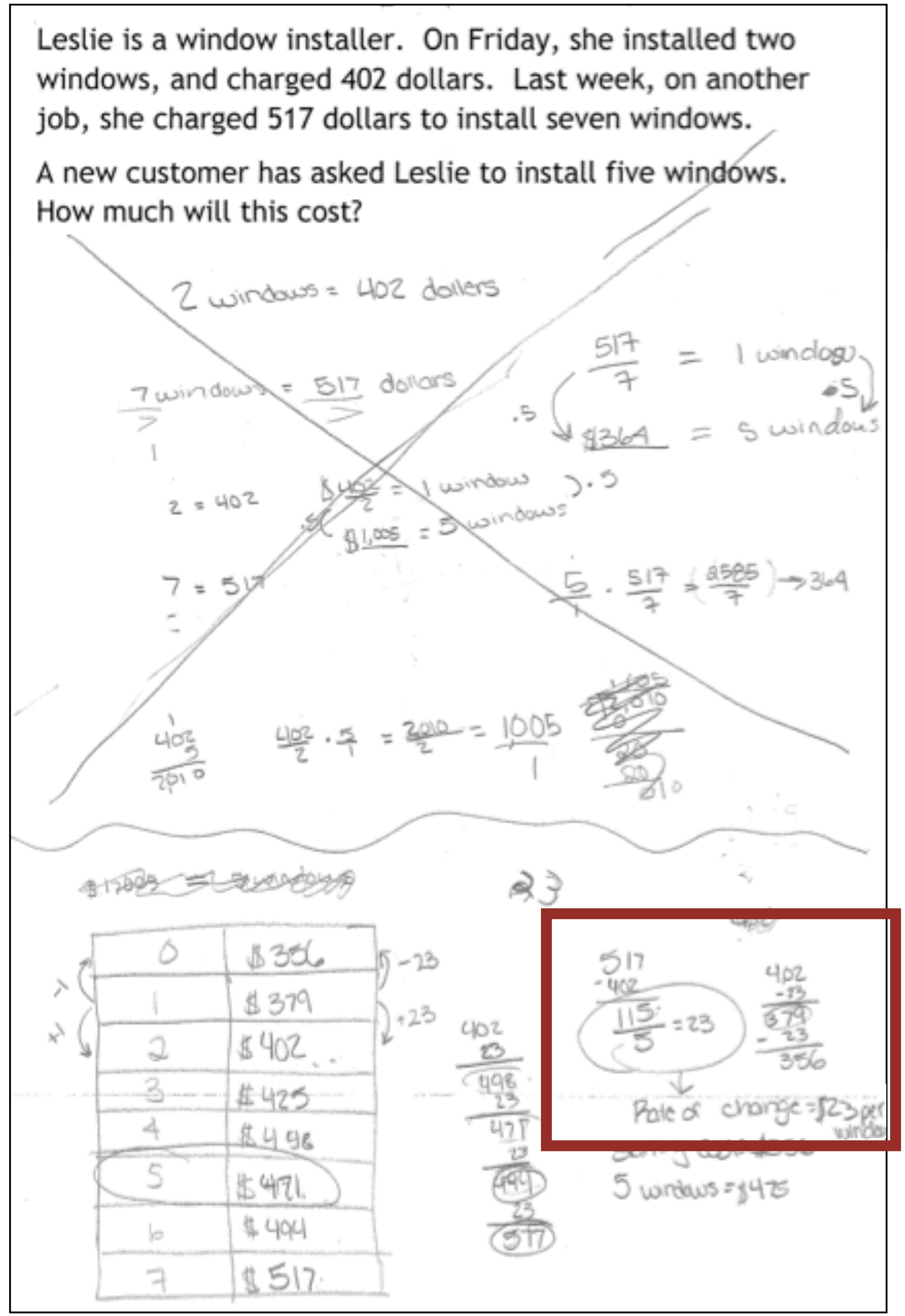
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A new customer has asked Leslie to install five windows. How much will this cost?
2 windows = 402 dollers
7 windows = 517 dollars
517/7 = 1 window
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$1,005 = 5 windows
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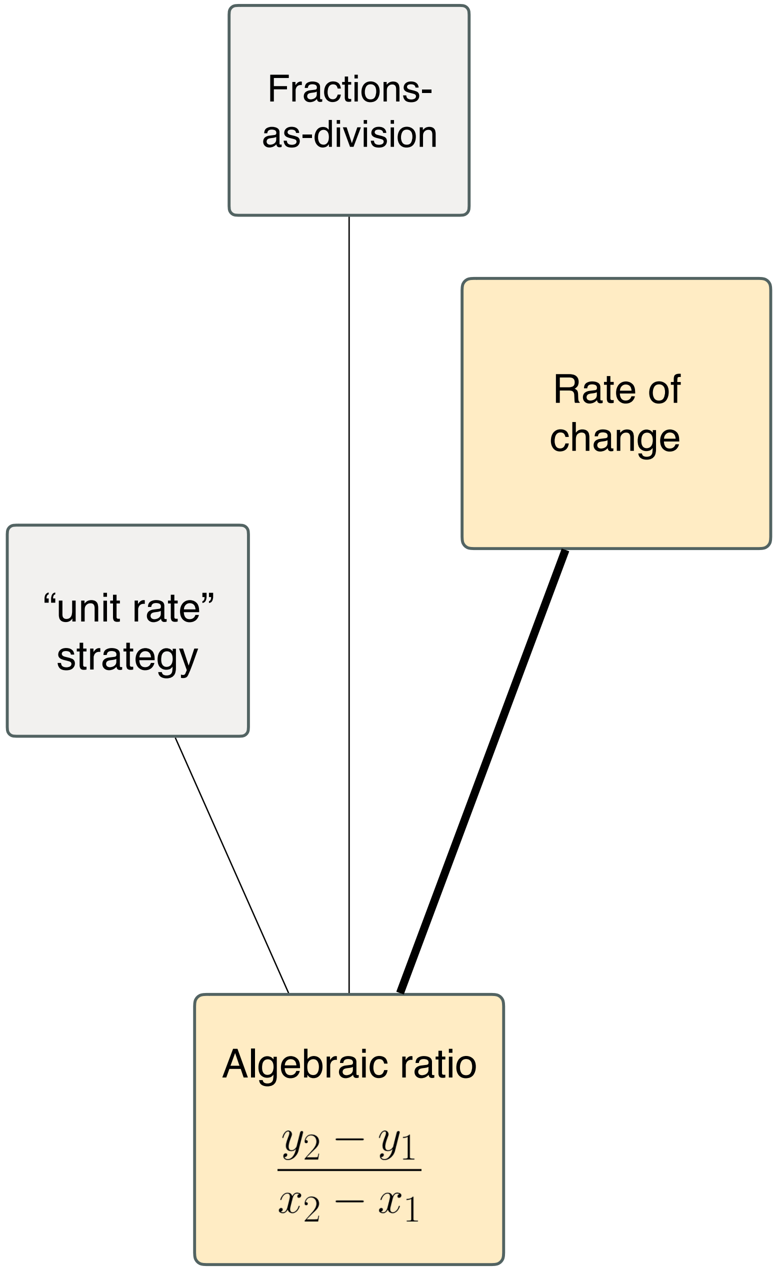
5 windows = $475

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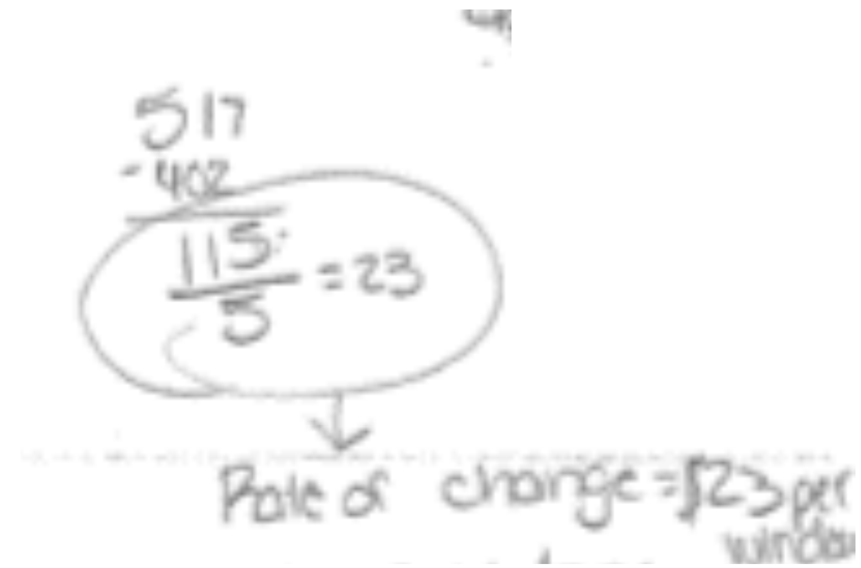
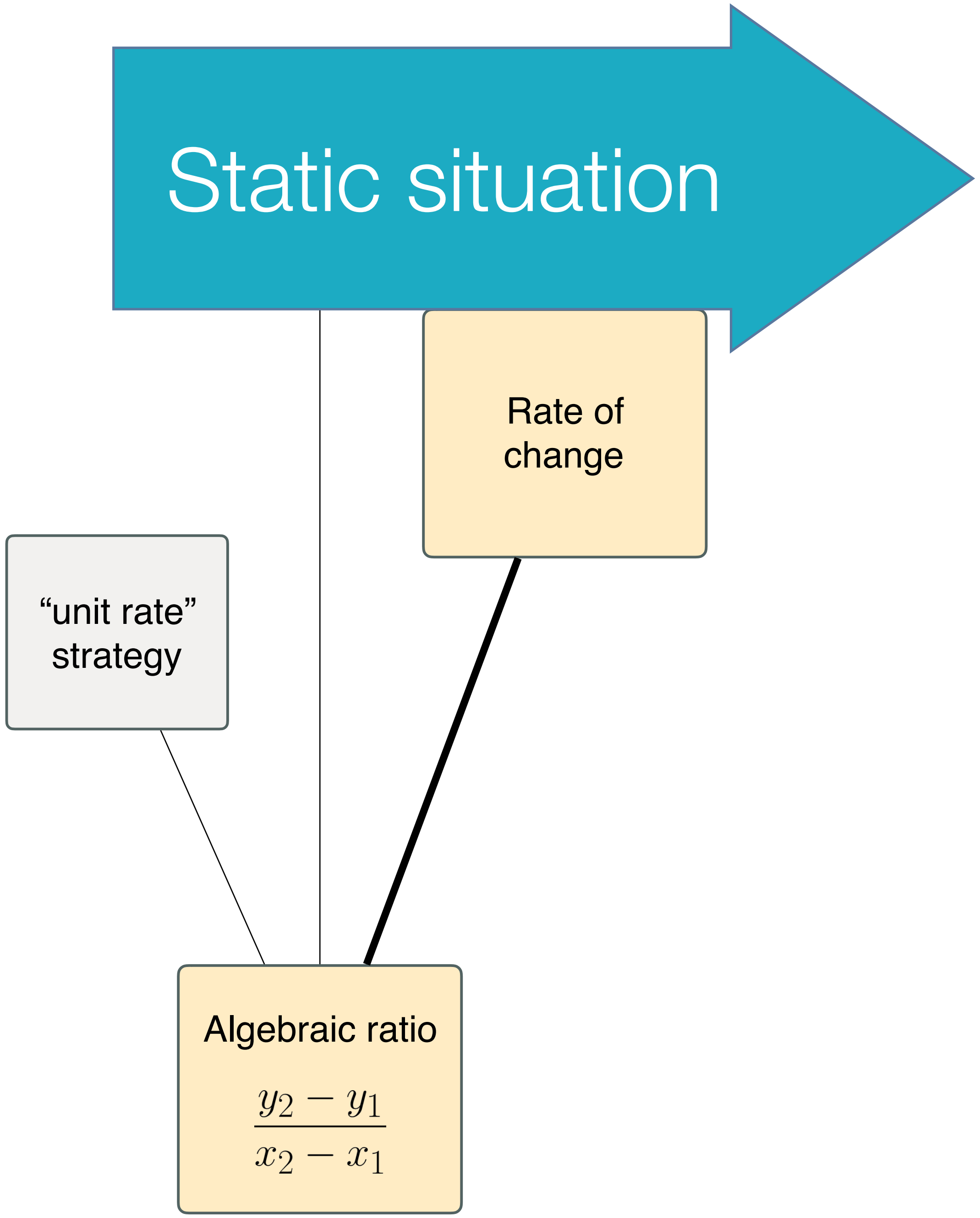



"The cost of one window without the delivery cost"

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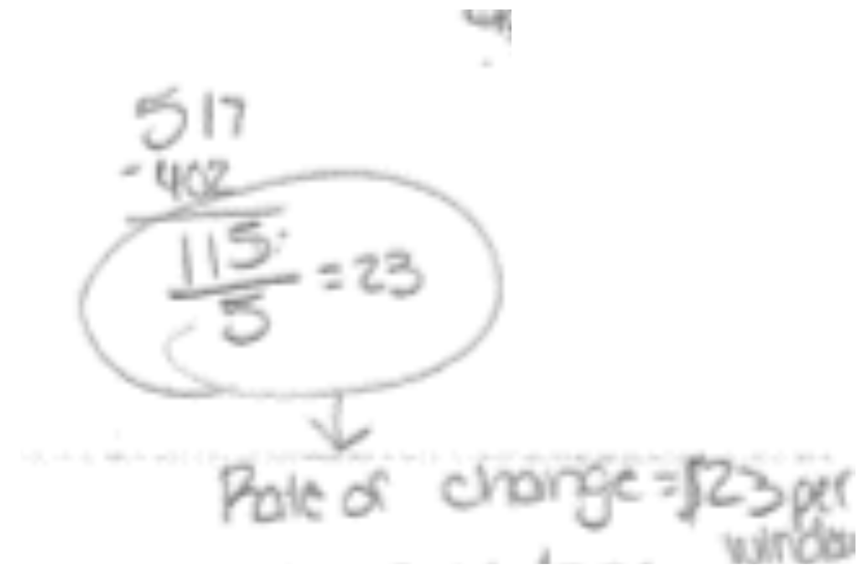
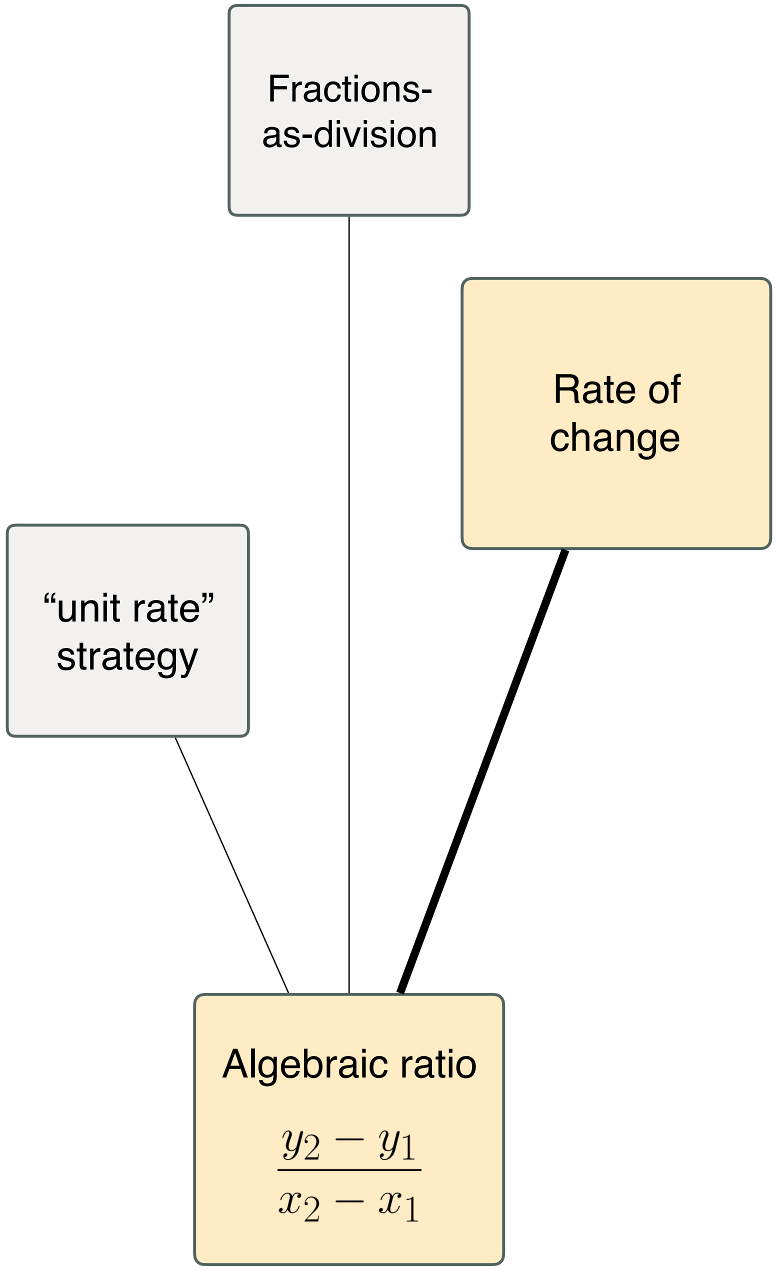



“The cost of one window without the delivery cost”

Rate as a different kind of *value*, not in terms of *change or covariation*

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8. At the end of the summer, the YMCA drains their swimming pool. Raif and Julie are in charge of measuring the height of the pool as it drains.

> **Raif says:** I checked the pool two hours after we started draining it. When I checked, the height of the water was 517 mm.
>
> **Julie says:** I checked the pool seven hours after we started draining it. When I checked, the height of the water was 402 mm.

Imagine you checked the height of the pool five hours after the YMCA started draining it. What would the height have been? What assumptions did you make?

Fractions-as-division

Rate of change

"unit rate" strategy

Negative change where negative value doesn't make sense

Leslie is a window installer. On Friday, she installed two windows, and charged 402 dollars. Last week, on another job, she charged 517 dollars to install seven windows.

A new cu ... ll five windows.
How muc ...

Dynamic

At the end of the summer, the YMCA drains their swimming pool. Raif and Julie are in charge of measuring the height of the pool as it drains.

**Raif says:** I checked the pool two hours after we started draining it. When I checked, the height of the water was 517 mm.

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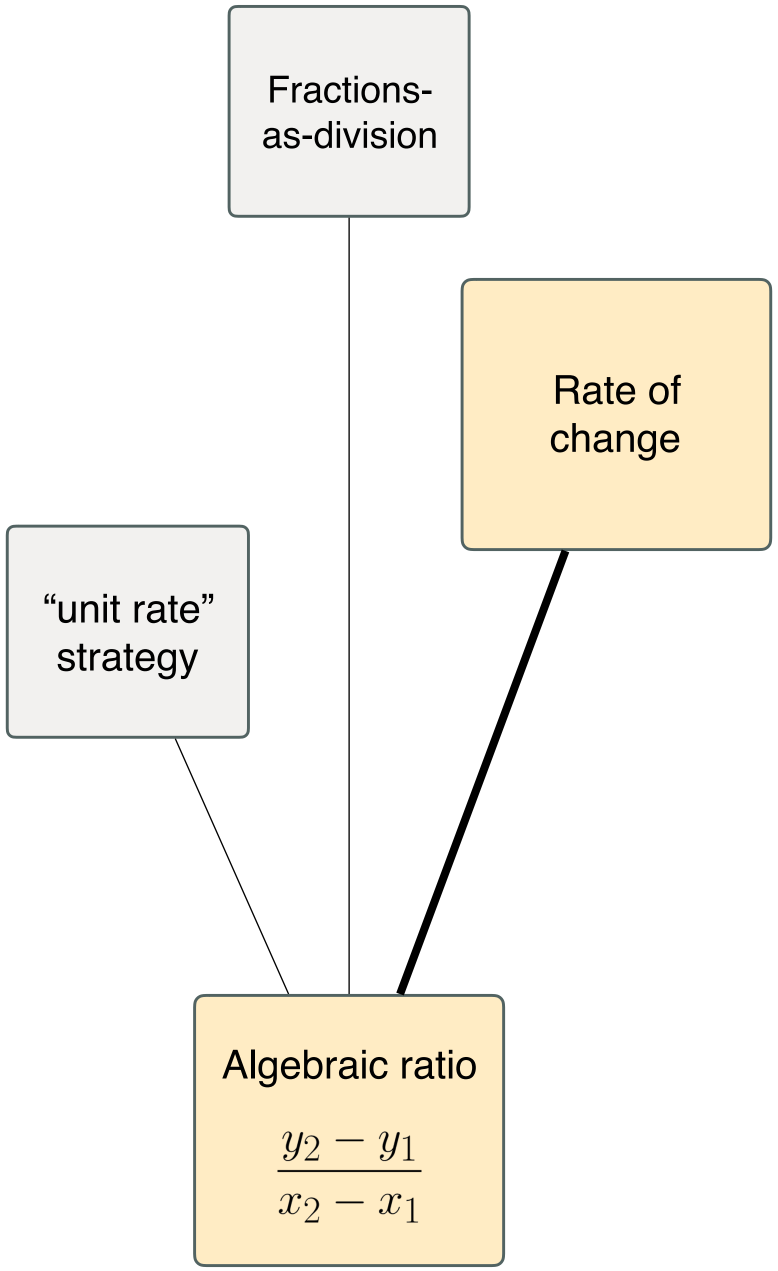
Imagine you checked the height of the pool five hours after the YMCA started draining it. What would the height have been? What assumptions did you make?




# phase 4

Students **reinvent** and **learn**

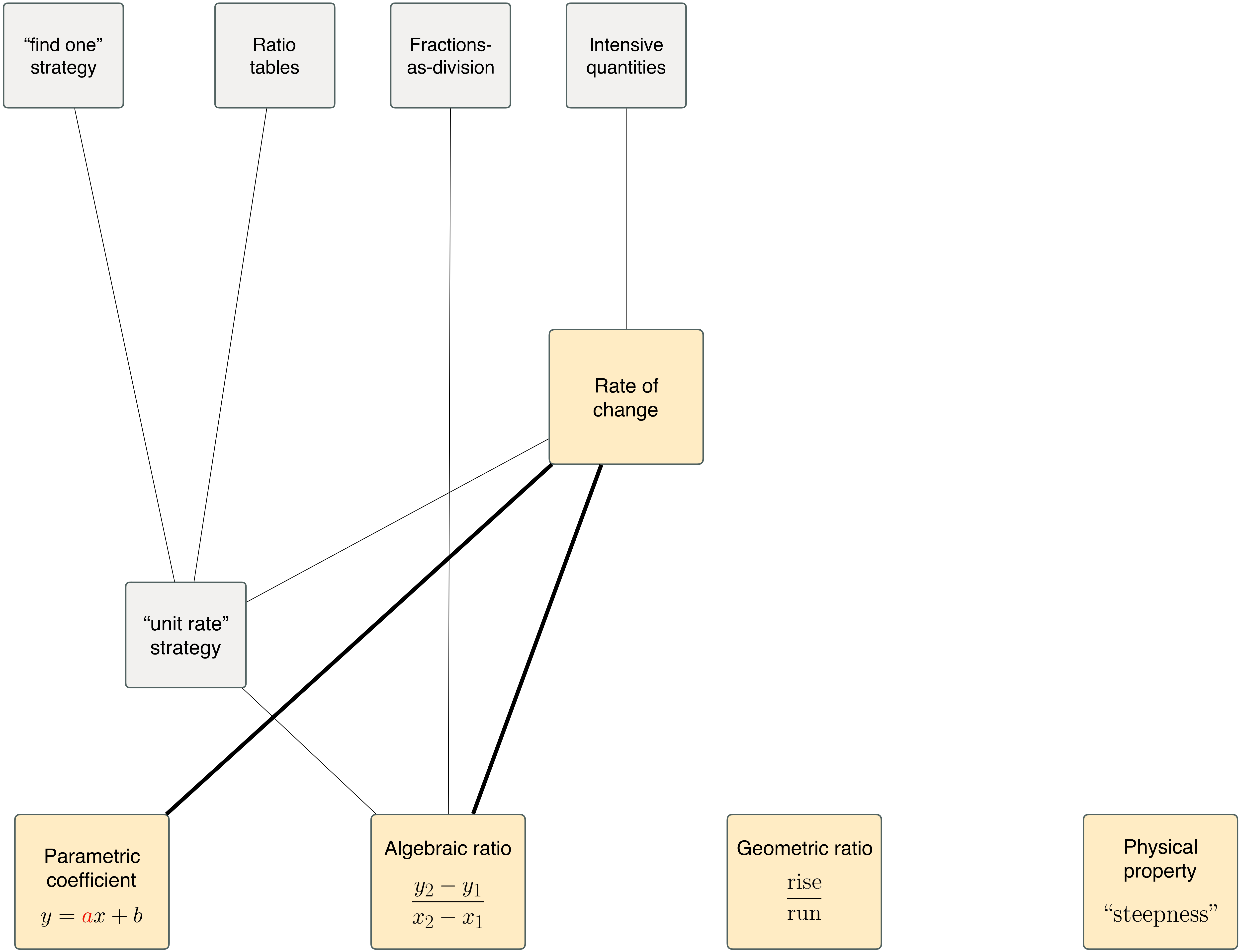
algebraic ratio

$$\frac{y_2 - y_1}{x_2 - x_1}$$

by engaging in these **activities**

make predictions

- linear, non-proportional situations
- two data points with $\triangle x \neq 1$



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Geometric ratio
$\frac{\text{rise}}{\text{run}}$
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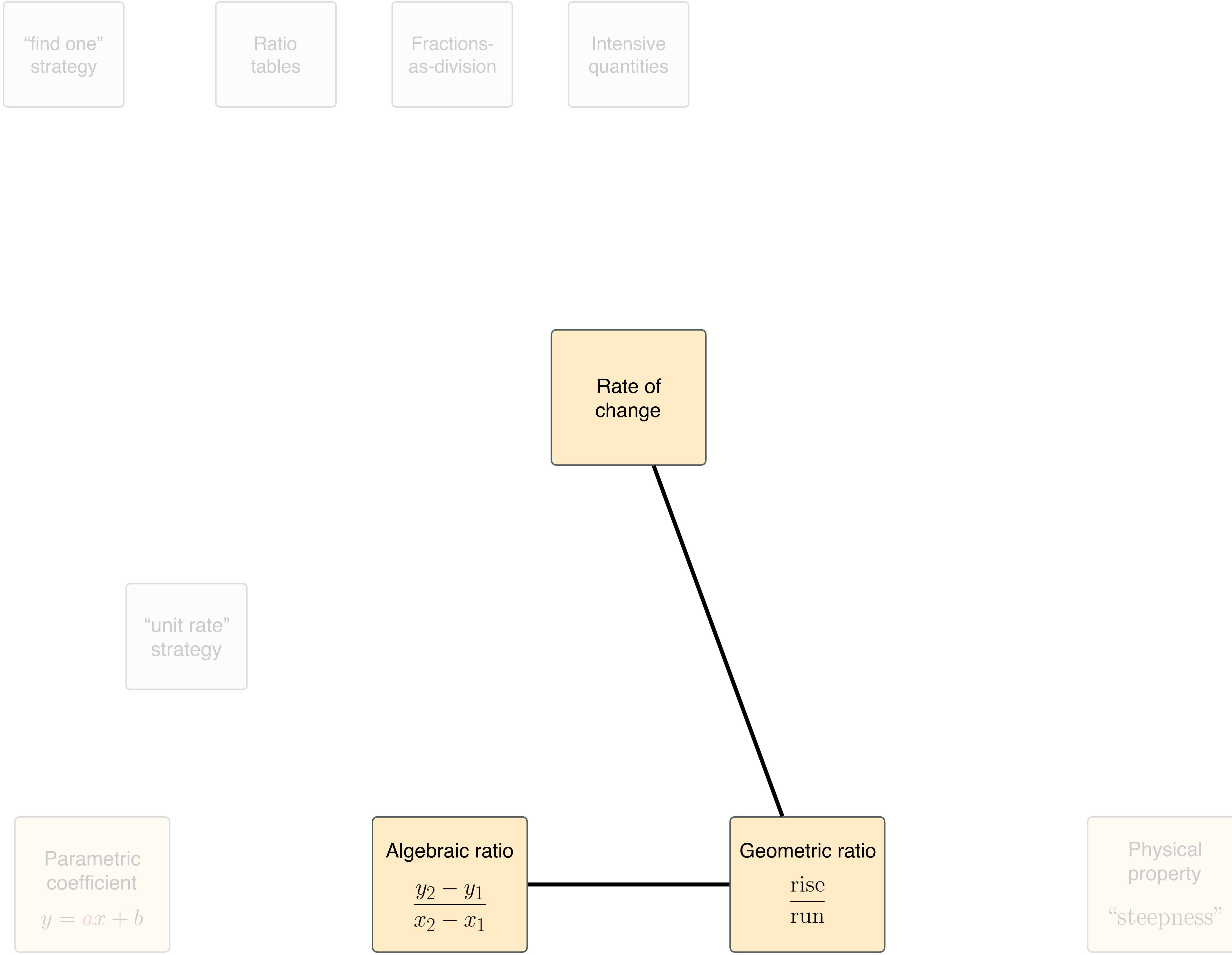
“steepness”



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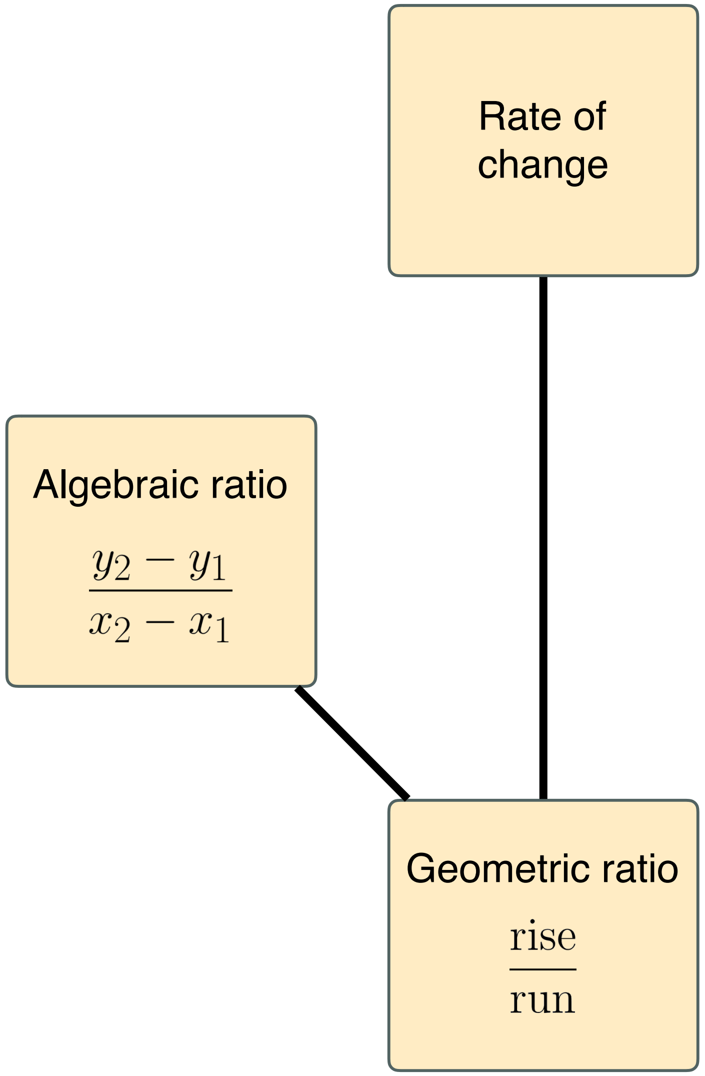
# phase 5


Rate of change
Algebraic ratio
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Geometric ratio
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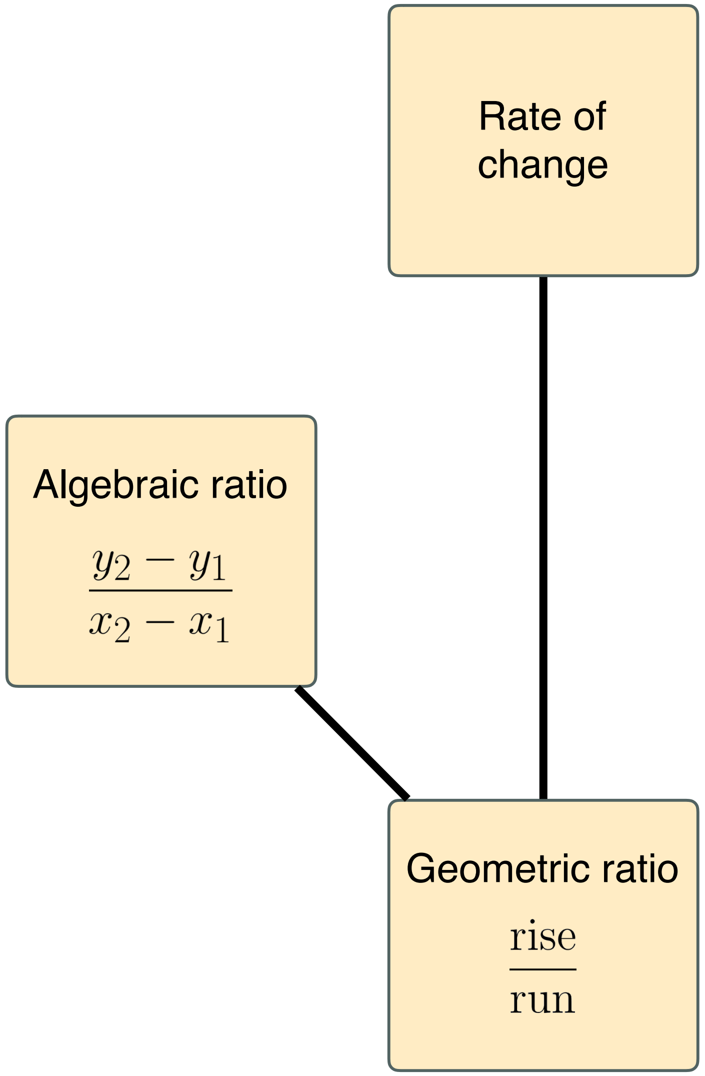



# phase 5

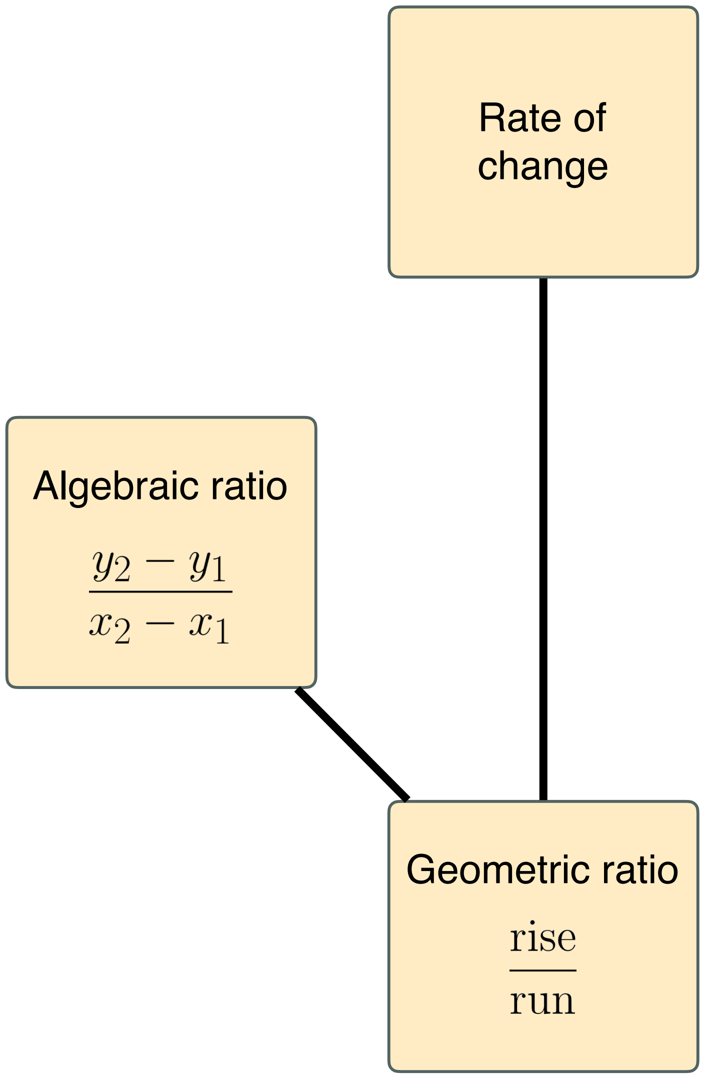
Students **reinvent** and **learn**

geometric ratio

$$\frac{\text{rise}}{\text{run}}$$

by engaging in these **activities**

- Show change on number-line diagrams.
- Make predictions in linear situations, given a graph of a function in a coordinate plane.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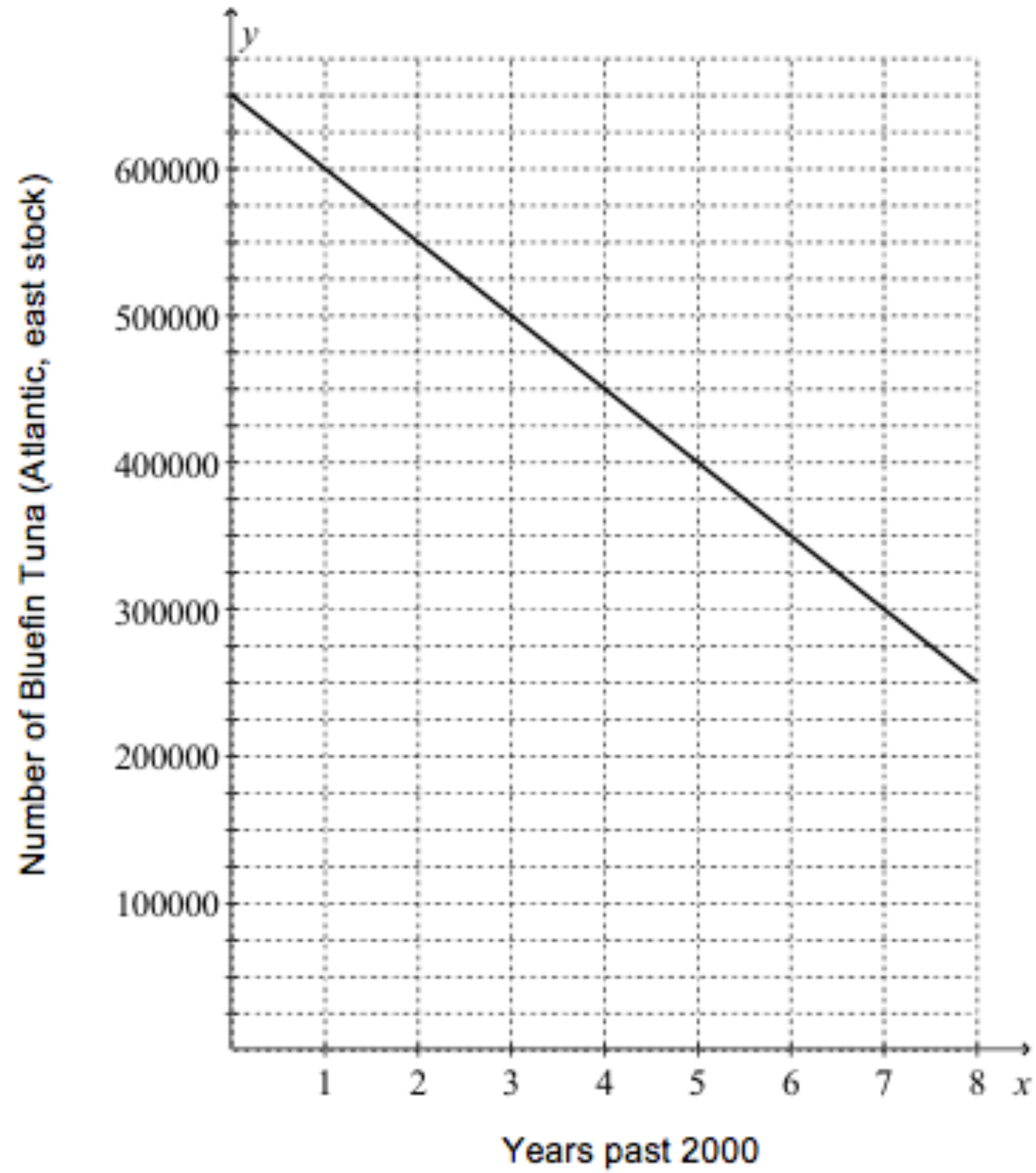

y
600000
500000
400000
300000
200000
100000
1
2
3
4
5
6
7
8
x
Number of Bluefin Tuna (Atlantic, east stock)
Years past 2000

Rate of change

Algebraic ra

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Explain or show how you found the rate of change in the graph.

| X | Y |
| --- | --- |
| 2 | 550,000 |
| 4 | 450,000 |

$$\begin{array}{r} 450,000 \\ -\ 550,000 \\ \hline -100,000 \\ \div\ 2 \\ \hline -50,000 \end{array}$$

We picked 2 points of the graph, subtracted $\text{output}_1$ from $\text{output}_2$, and then ÷ that by 2 which was $\text{Input}_2 - \text{Input}_1$.

400000

| | |
| --- | --- |
| 1 | 600000 |
| 3 | 5,00000 |

2( )100000

Rate of change

Algebraic ra

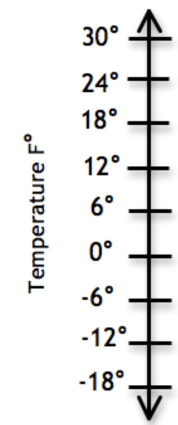
$\frac{y_2 - y_1}{x_2 - x_1}$

Explain or show how you

| X | Y |
|---|---|
| 2 | 550,000 |
| 4 | 450,000 |

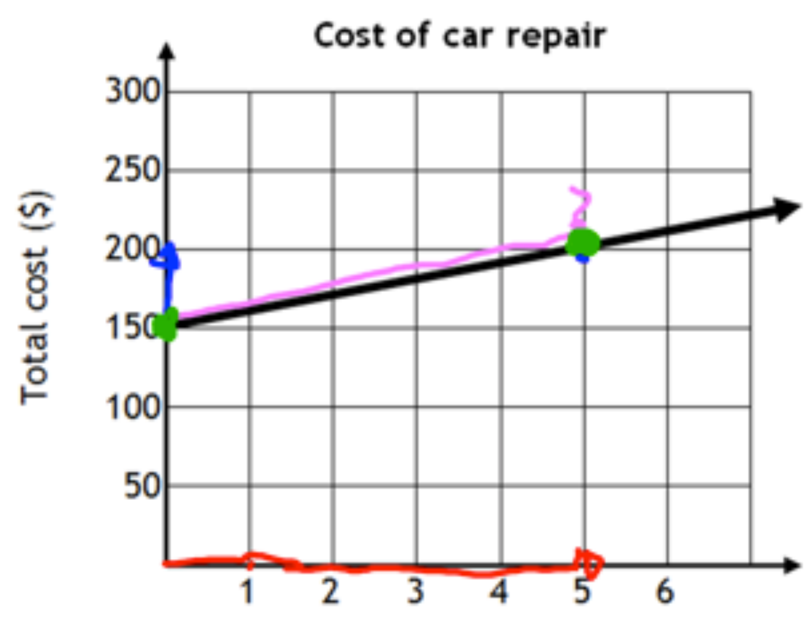
1. The temperature in Alamosa, Colorado rose from -12° to 24°.

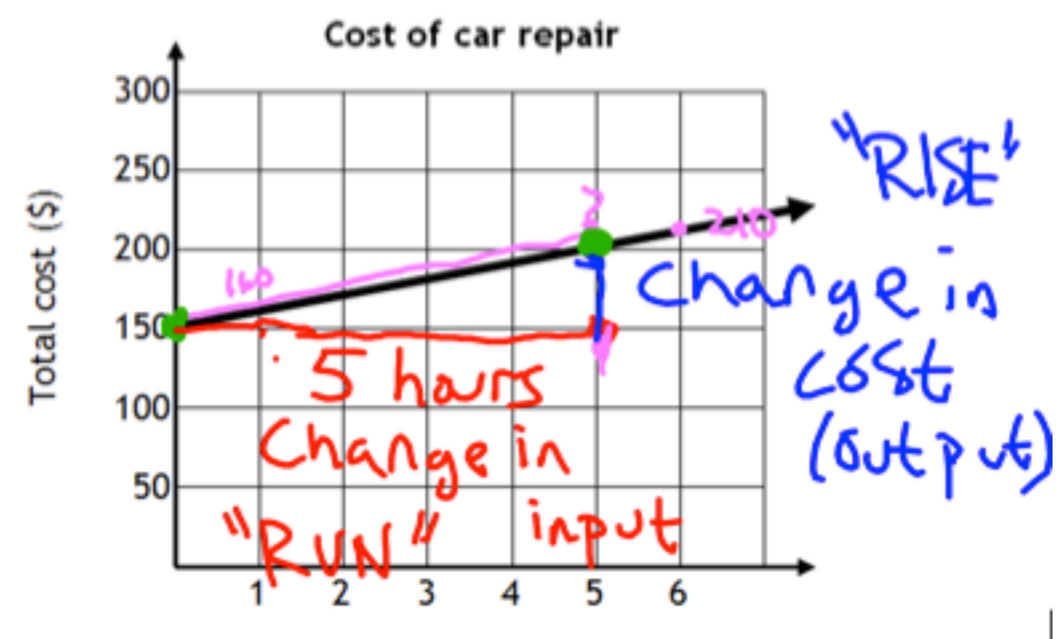
   Draw an arrow on the number line below to show this change.

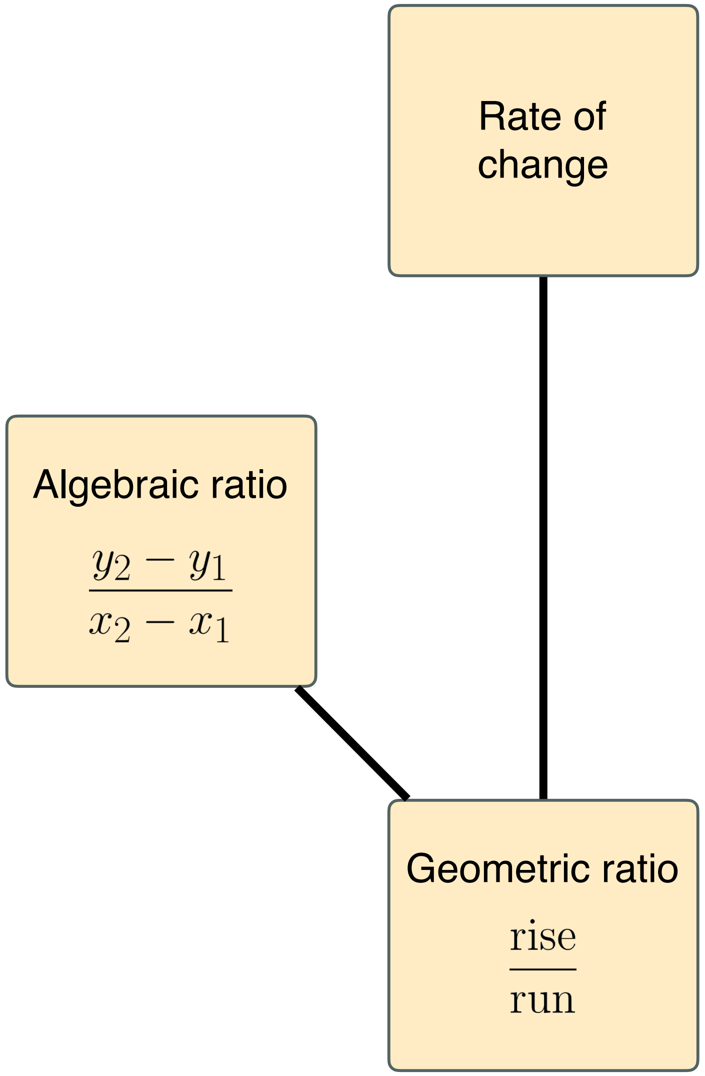



ph.

2 points of the
racted output,
t2, and then ÷
which was
put1.






# phase 5

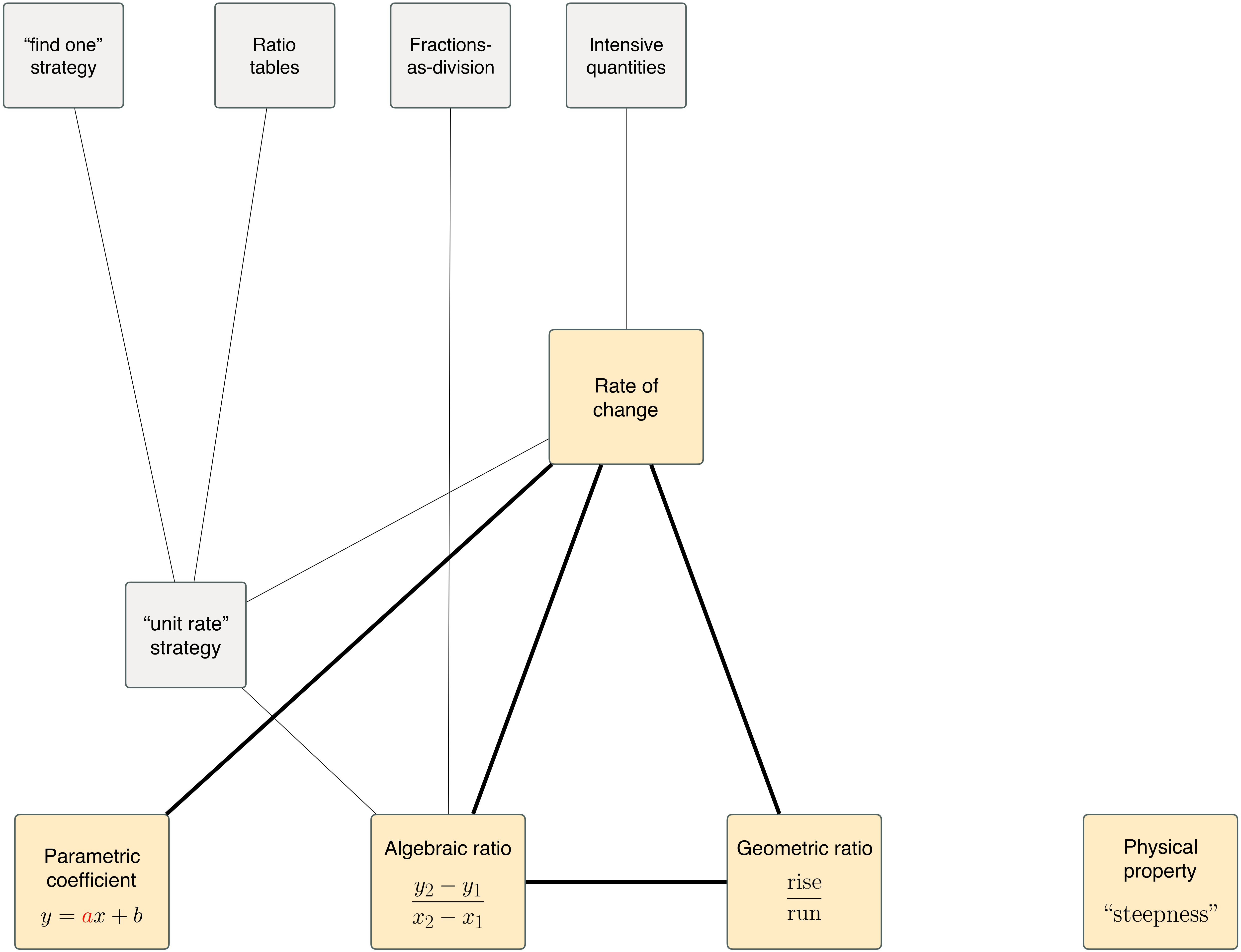
Students **reinvent** and **learn**

geometric ratio

$$\frac{\text{rise}}{\text{run}}$$

by engaging in these **activities**

- Show change on number-line diagrams.
- Make predictions in linear situations, given a graph of a function in a coordinate plane.



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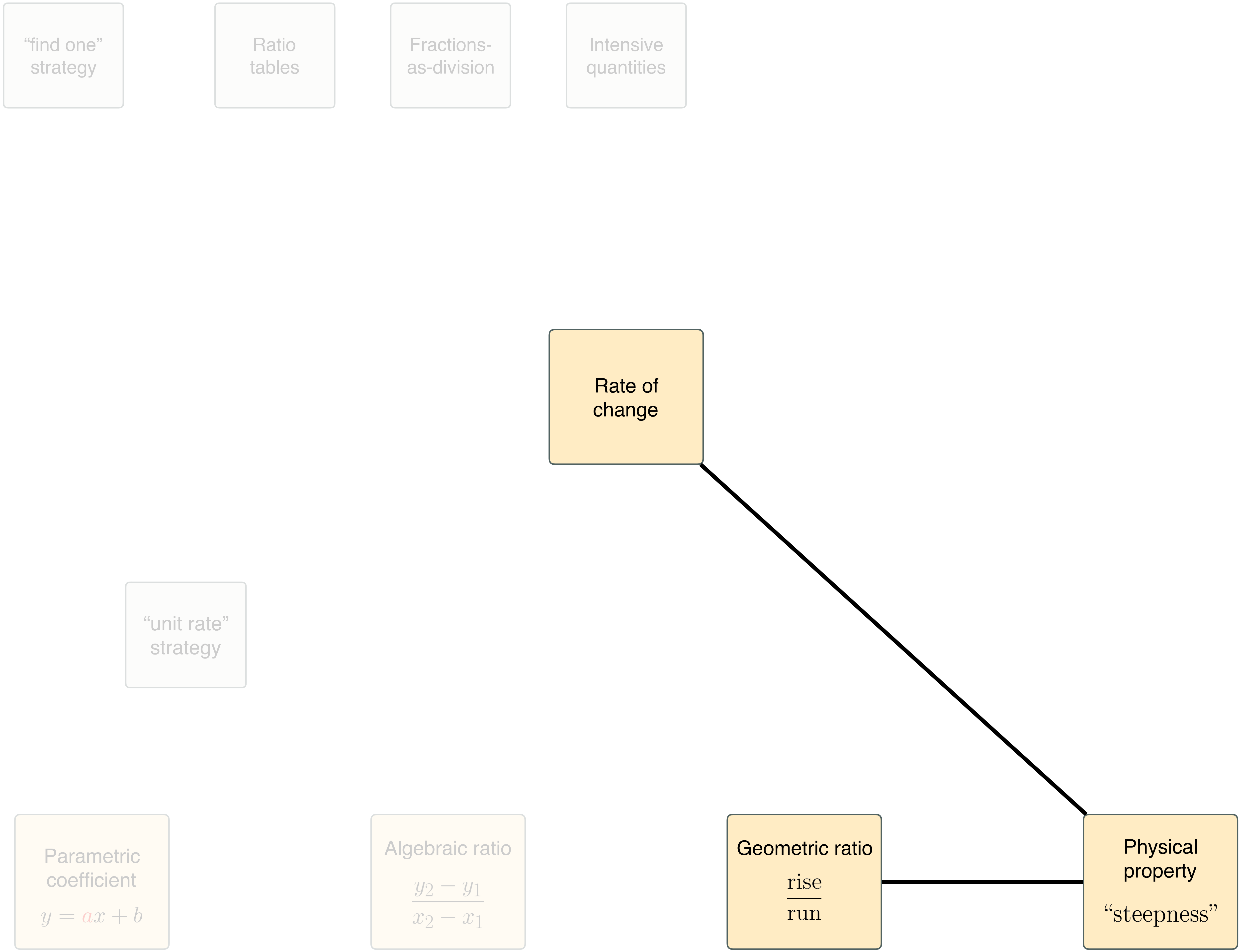

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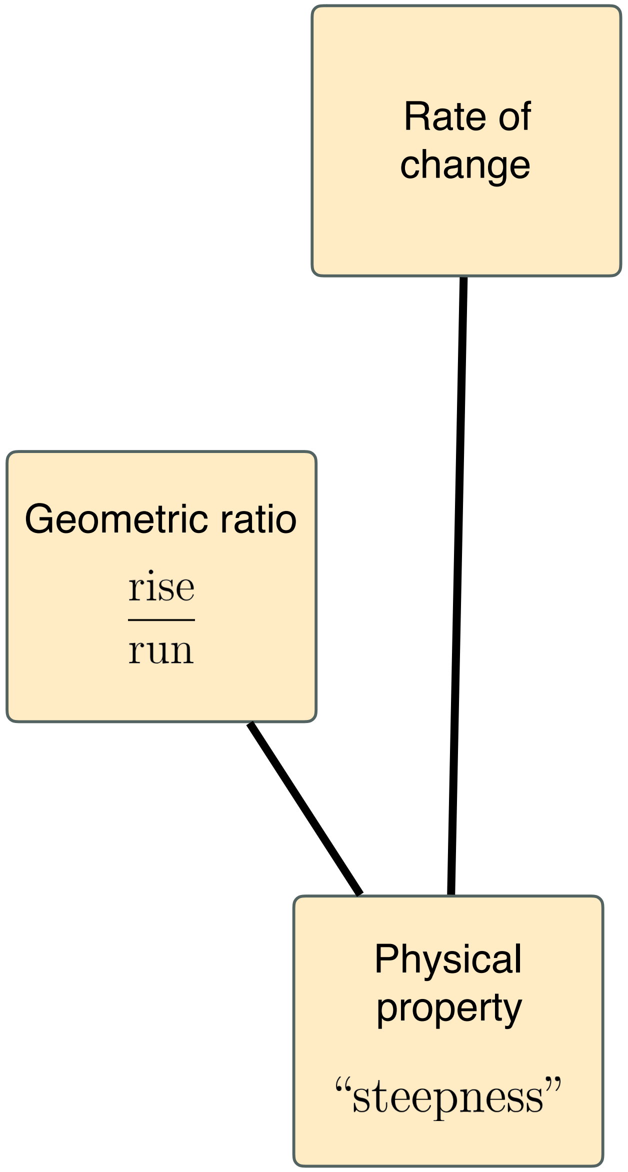
# phase 6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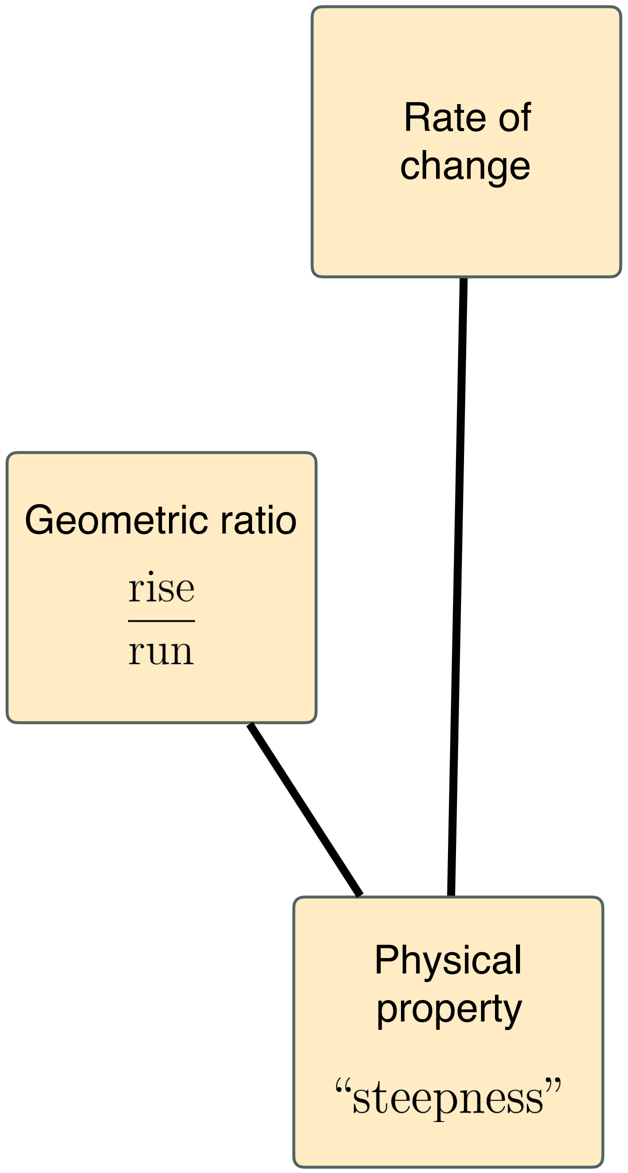
# phase 6



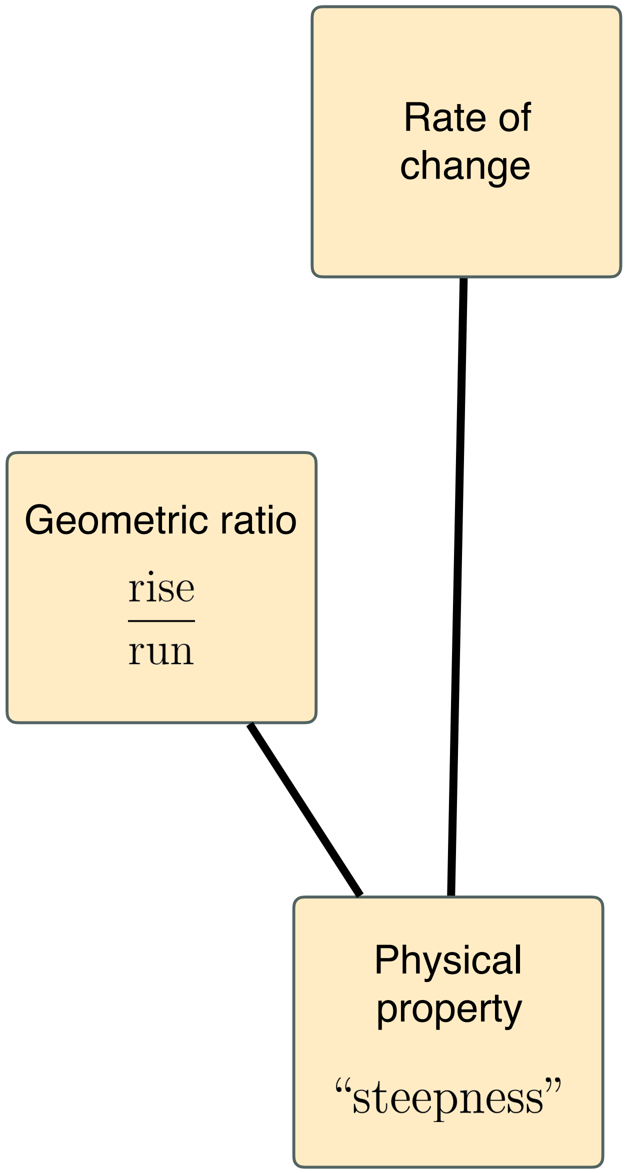

Students **reinvent** and **learn**

physical property

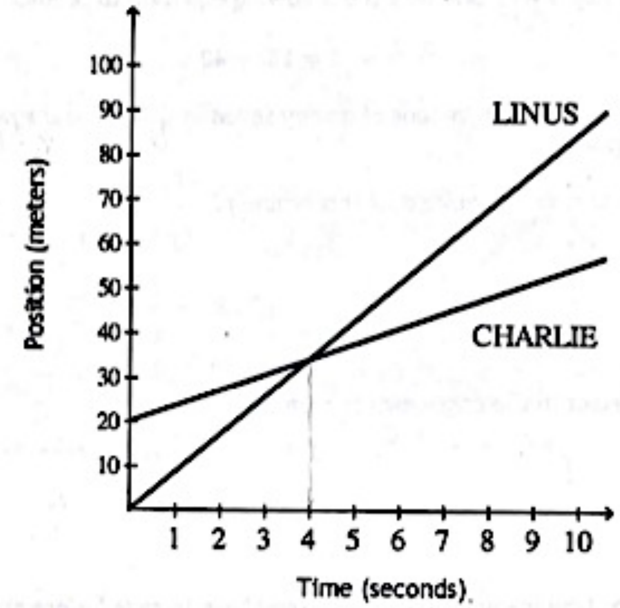
"steepness"

by engaging in these **activities**

- compare rates given graph of two intersecting linear functions
- measure steepness of objects




**8.** Charlie and Linus are running along a straight track. A position vs. time graph for both runners is shown below.




a. At the instant, $t = 2$ sec, who is running faster, Charlie or Linus? linus

*Explain your reasoning*

linuses line is steeper so he is running faster

b. Do Linus and Charlie ever have the same speed? If so, at what time?

*Explain your reasoning.*

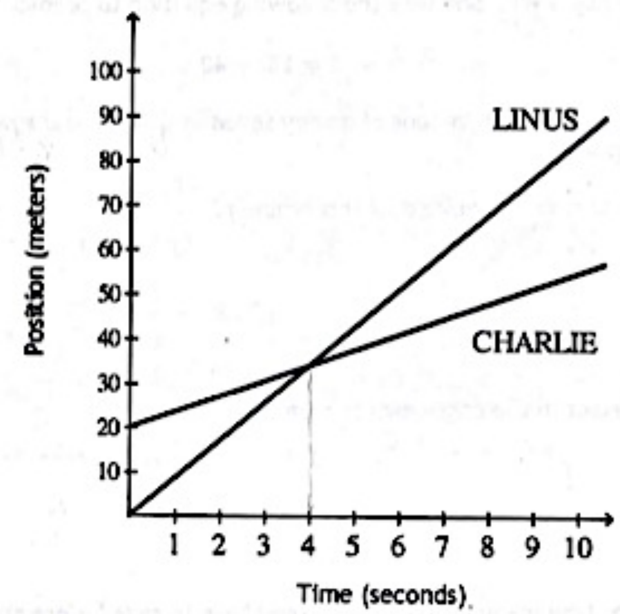
yes at 4 seconds they are going at the same speed

Rate of change

Geometric ratio

Connecting **rate of change** and **steepness**

8. Charlie and Linus are running along a straight track. A position vs. time graph for both runners is shown below.

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*Explain your reasoning*

linuses line is steeper so he is running faster

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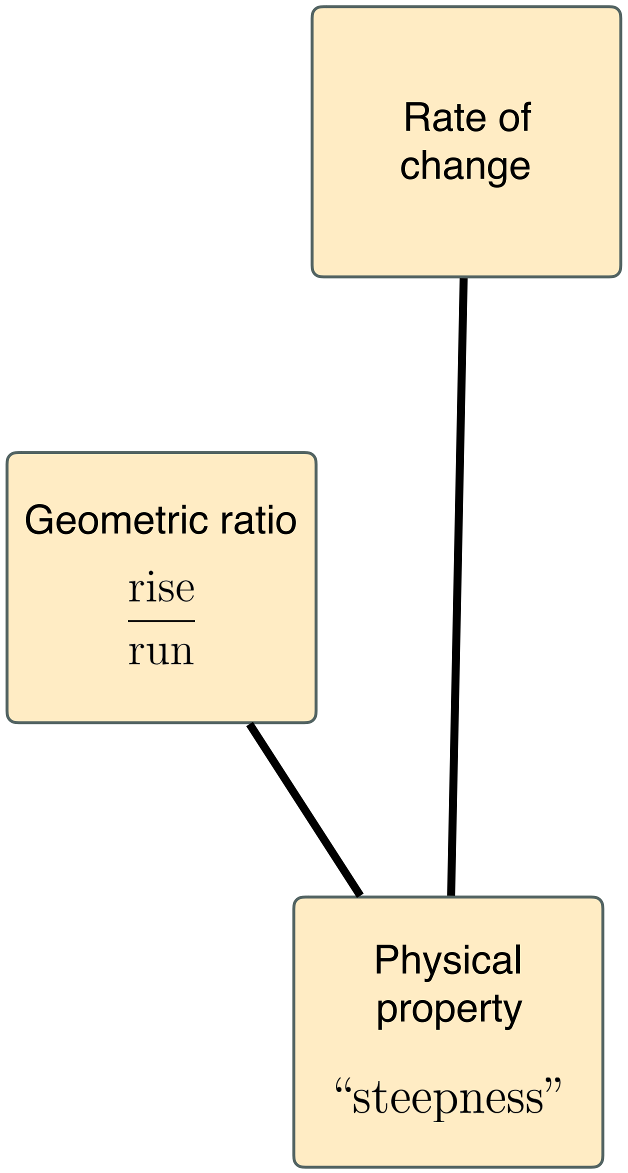
*Explain your reasoning.*

yes at 4 seconds they are going at the same speed

Steepness is not the only salient feature of the graph

# phase 6



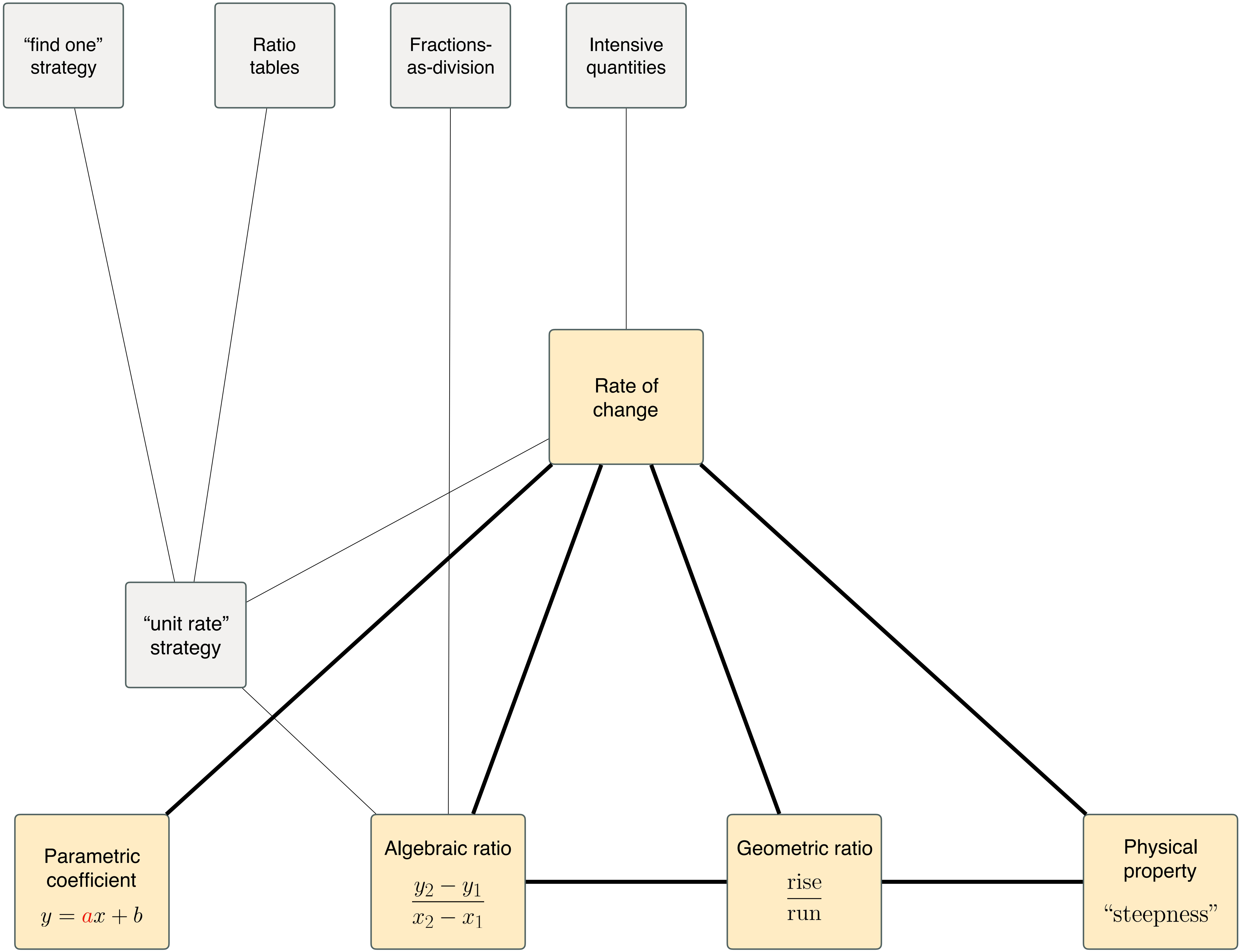

Students **reinvent** and **learn**

physical property

"steepness"

by engaging in these **activities**

- compare rates given graph of two intersecting linear functions
- measure steepness of objects



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# summary

# How do students make slope meaningful?

# slope

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

Physical property

# slope

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

**Physical property**

Not robust, not motivating

# slope

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

Physical property

Not meaningful

# slope

Parametric coefficient

Algebraic ratio

**Rate of change**

Geometric ratio

Physical property

Motivating meaningful, and robust

# slope

- Students learn all five faces of slope with *meaning* by engaging in meaningful activity in meaningful contexts
- Unit is organized around rates and predictions
- Use **tables** as the key representation to help students see covariation
- Save graphs and steepness until the end

# Questions and discussion

Frederick Peck

University of Montana
Department of Mathematical Sciences

frederick.peck@umontana.edu
www.RMEInTheClassroom.com

UNIVERSITY OF MONTANA