Making Algebra Meaningful

Fred Peck

frederick.peck@umontana.edu

www.RMEintheclassroom.com

Montana Educator Conference
Billings, MT
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UNIVERSITY OF MONTANA

Agenda

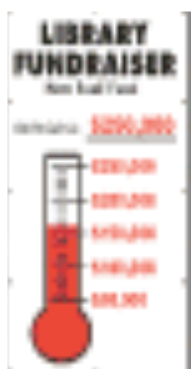
Do some math

Talk about what we did

On the one hand…

Lived experiences

- Contain mathematical principles
- Context-bound
- Models of a situation



On the other hand…

Formal mathematics
- Potentially very general
- Far removed from context

$$\frac{3}{4}$$

Lived experiences
- Contain mathematical principles
- Context-bound
- Models <u>of</u> a situation

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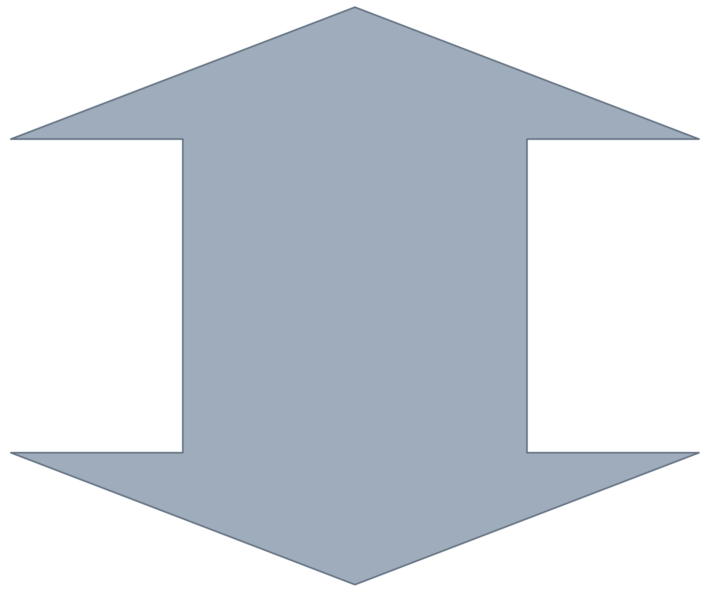

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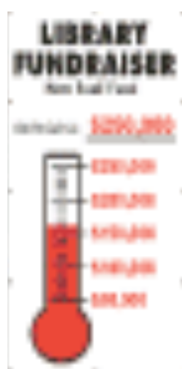

Formal mathematics

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How to "connect" formal mathematics with students' lived experiences?

Lived experiences

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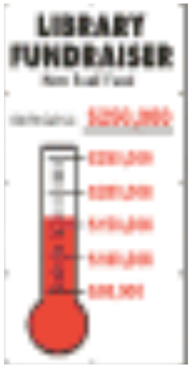
- Models of a situation



“traditional” sequence

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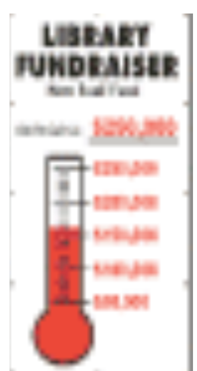

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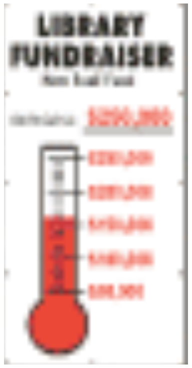
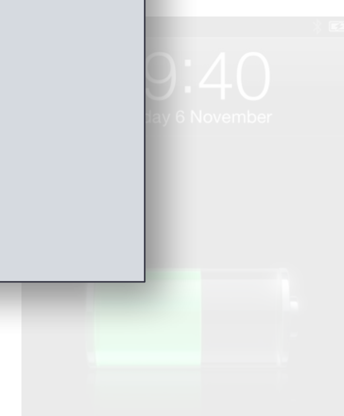


“traditional” sequence

- Structure without structuring
- Mathematics is disconnected from lived reality
- Math is seen as meaningless
- Little opportunity to participate in mathematical practices

- Context-bound
- Models *of* a situation



“discovery” sequence

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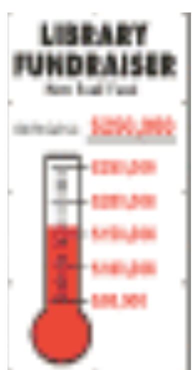

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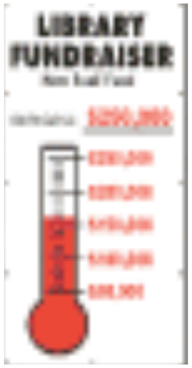

“discovery” sequence

Form…

- Pote…
- Far…

Better! But still…

- Not enough structuring
- There is a big jump from lived experiences to formal mathematics – often too big
- Ultimately, formal mathematics is the only tool that students have to solve problems

Lived…

- Con… prin…
- Context-bound
- Models <u>of</u> a situation

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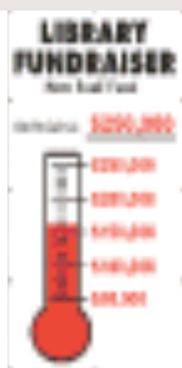

the "pre-formal" layer

Formal mathematics

- Potentially very general
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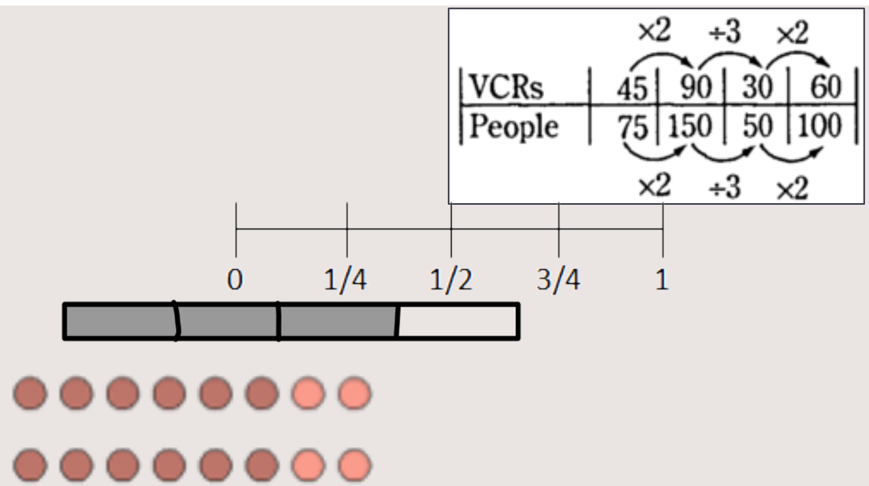
"Pre-formal" models

- Generalizable, but still retain contextual cues
- Models for mathematics



Lived experiences

- Contain mathematical principles
- Context-bound
- Models of a situation

the iceberg model

Formal mathematics
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“Pre-formal” models
- Generalizable, but still retain contextual cues
- Models <u>for</u> mathematics

Lived experiences
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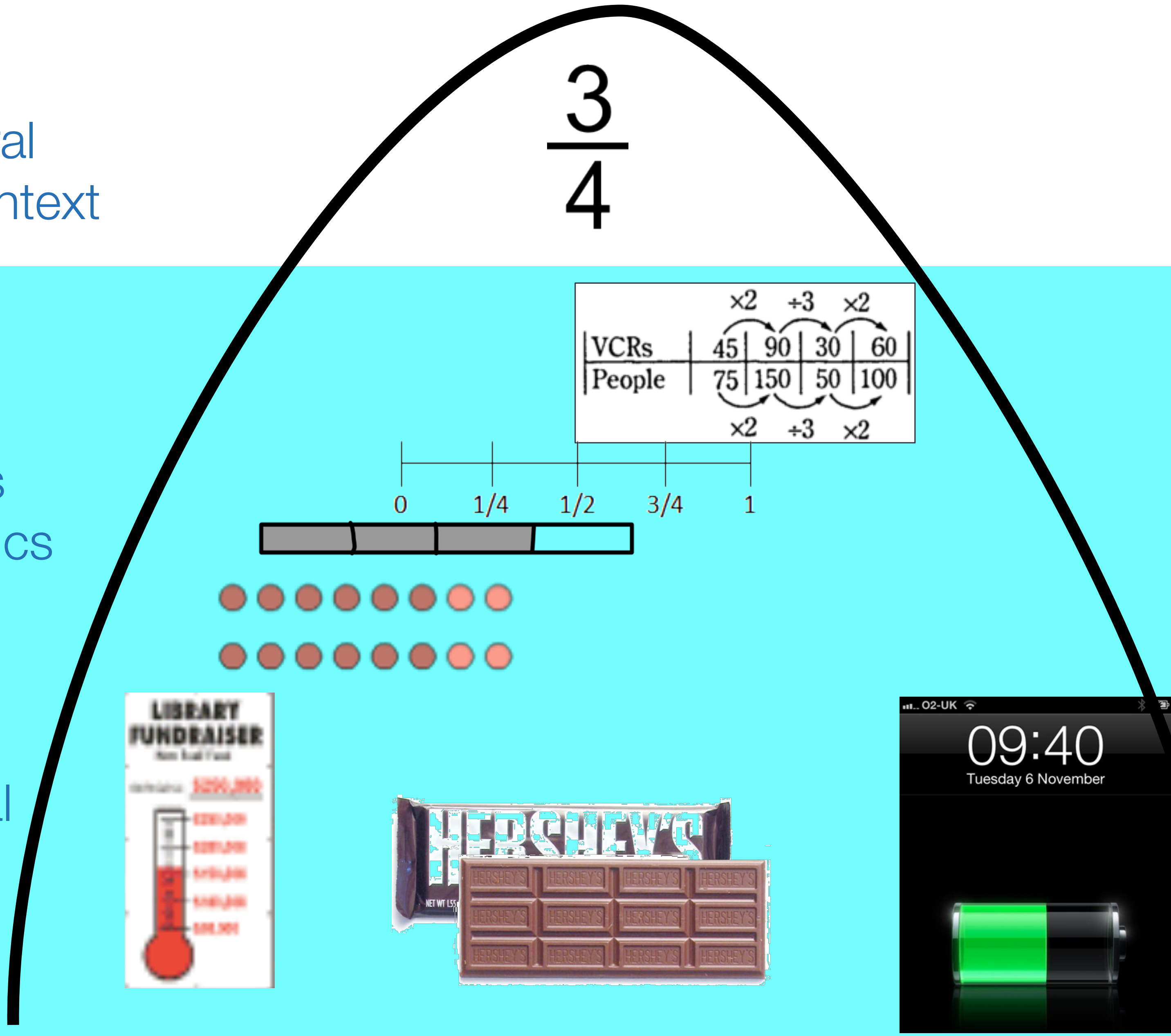

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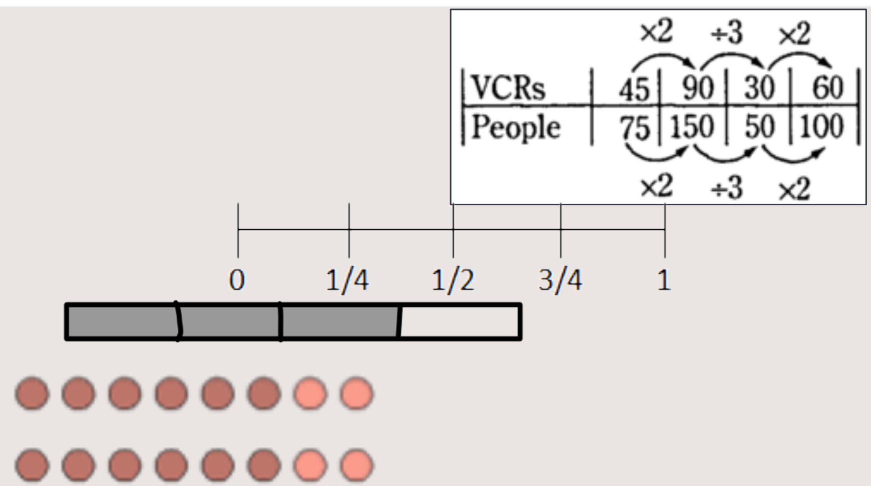

Lived experiences
- Contain mathematical principles
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- Models <u>of</u> a situation

“Pre-formal” models

. . . help students ***learn*** mathematics

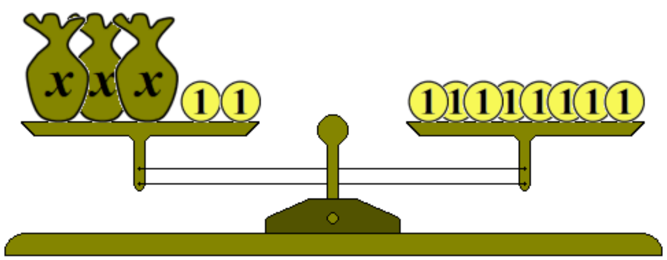
Structure and meaning of algebra equations

. . . are tools that students can use to ***do*** math

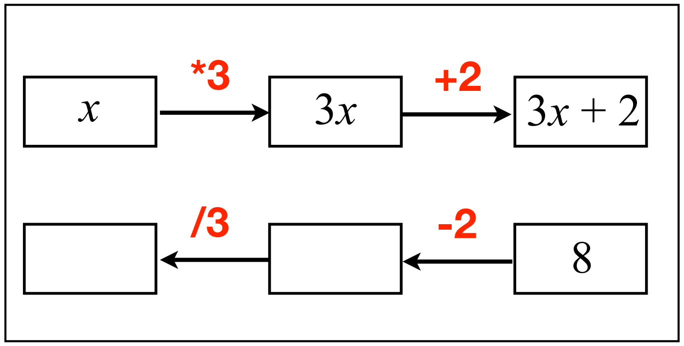
Strategies to solve algebra equations

Two models for algebra equations

$$3x + 2 = 8$$



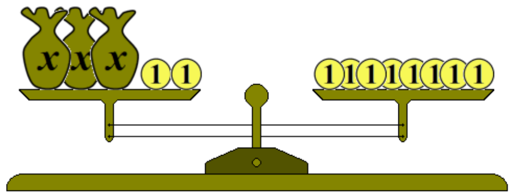
Balance model



Arrow chain model

$$3x + 2 = 8$$

Models



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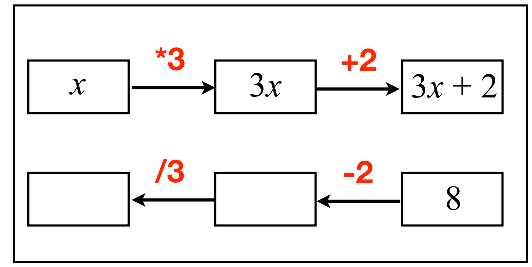


Arrow chain model

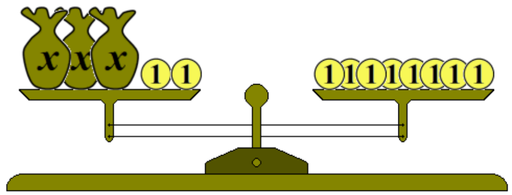
Structure and meaning

The equation represents
objects
that are
grouped & compared

The equation represents
a process
that describes how an
unknown is changed

$$3x + 2 = 8$$

Models



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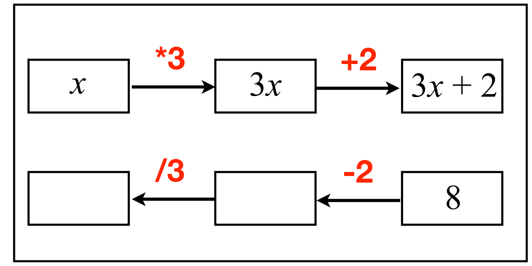

Arrow chain model

Structure and meaning

The equation represents
objects
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The equation represents
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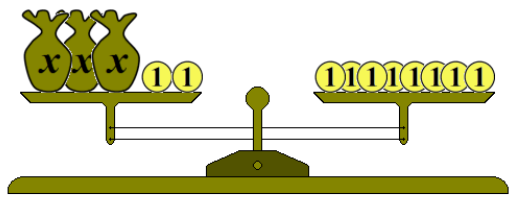
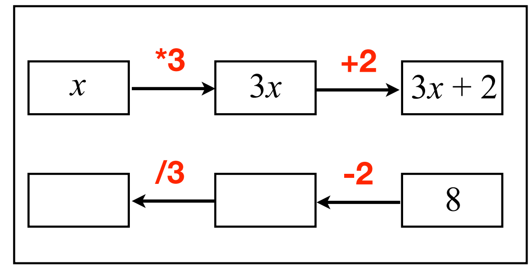
"Three x's and two ones are the same as eight ones."

"Start with a number. Multiply it by three. Add two. You end up with 8. What number did you start with?"

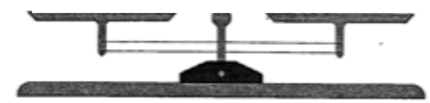
$$3x + 2 = 8$$

Models	 **Balance model**	 **Arrow chain model**
Structure and meaning	The equation represents **objects** that are **grouped & compared**	The equation represents **a process** that describes how an **unknown is changed**
Strategy	**Balance:** Make simpler equations by doing the same thing to both sides	**Backtrack:** Work backwards from the end, using opposite operations to undo.

What do "pre-formal" models do for students?

1. Use the balance scale on the right to model the equation,

$$2x + 9 = 5x + 2$$

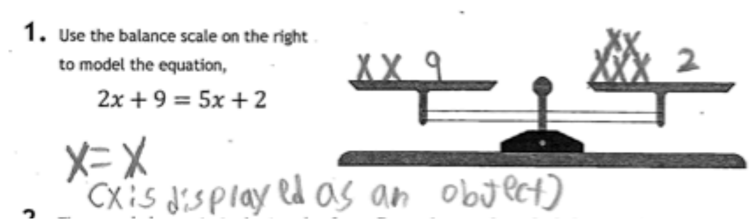


Models

What do "pre-formal" models do for students?

Models

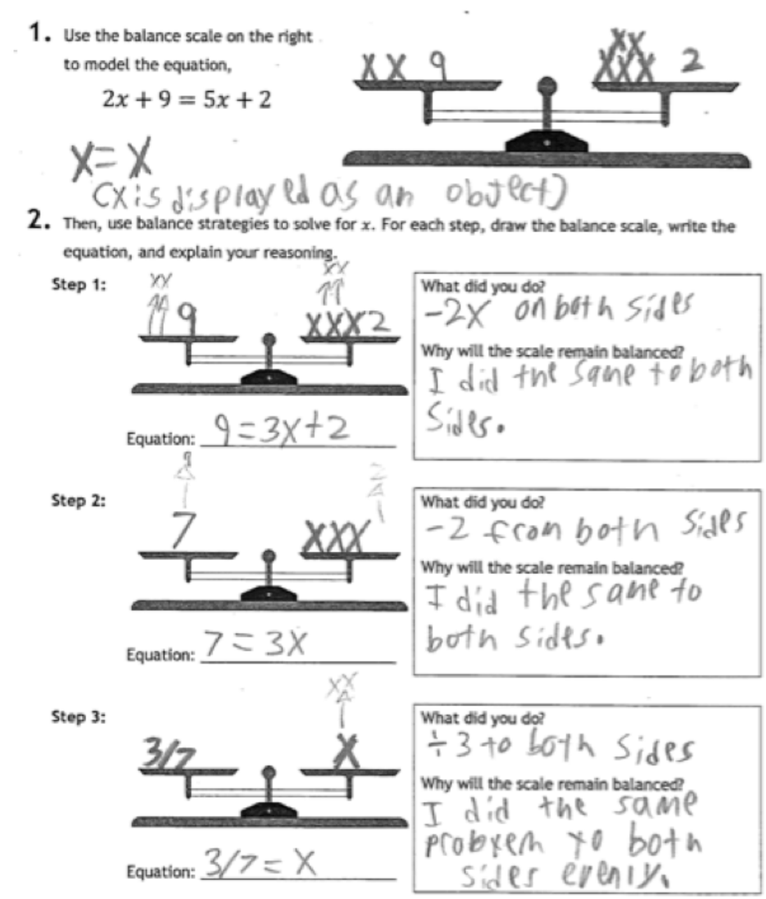
Structure and meaning



What do “pre-formal” models do for students?

Models → Structure and meaning

Models → Strategy


1. Use the balance scale on the right to model the equation,

$2x + 9 = 5x + 2$

XX 9 | XXXXX 2

X= X

(X is displayed as an object)

2. Then, use balance strategies to solve for x. For each step, draw the balance scale, write the equation, and explain your reasoning.

Step 1:

9 | XXX 2

Equation: 9=3X+2

What did you do?

-2X on both sides

Why will the scale remain balanced?

I did the same to both sides.

Step 2:

7 | XXX

Equation: 7=3X

What did you do?

-2 from both sides

Why will the scale remain balanced?

I did the same to both sides.

Step 3:

3/7 | X

Equation: 3/7=X

What did you do?

÷3 to both sides

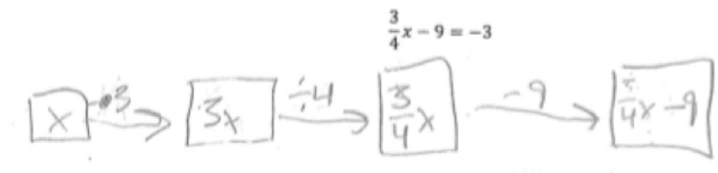
Why will the scale remain balanced?

I did the same problem to both sides evenly.

What do “pre-formal” models do for students?

Models

Structure and meaning



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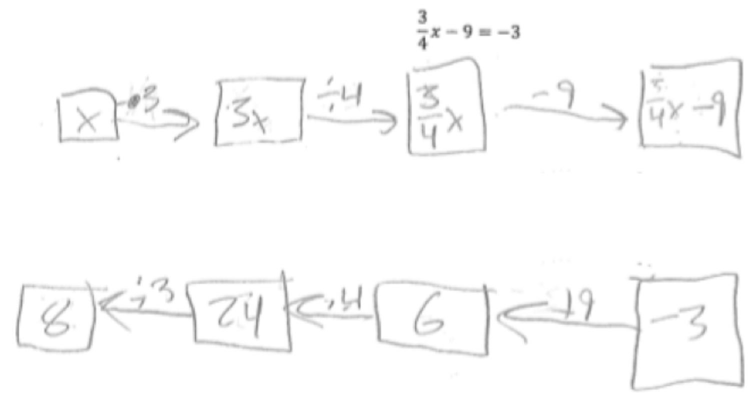
Models

Structure and meaning

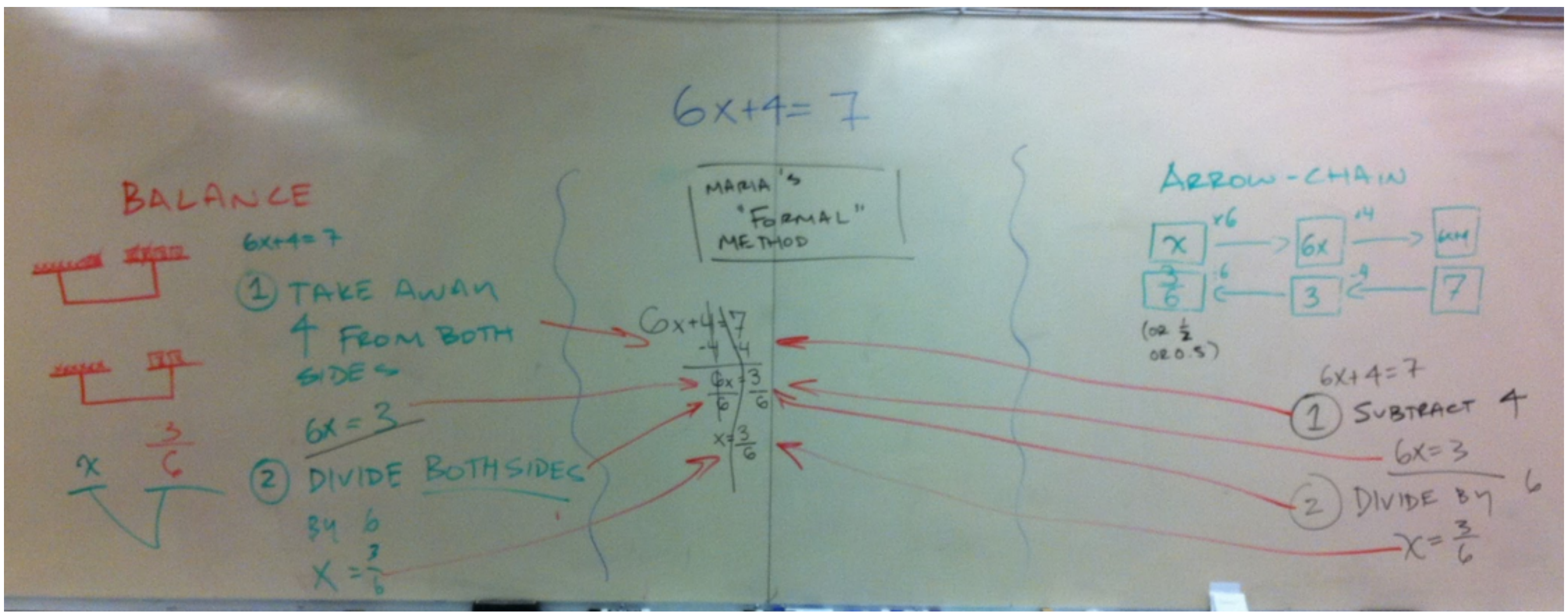
Strategy



Models help students *learn* math

Models are tools that students can use to *do* math

What do “pre-formal” models do for students?



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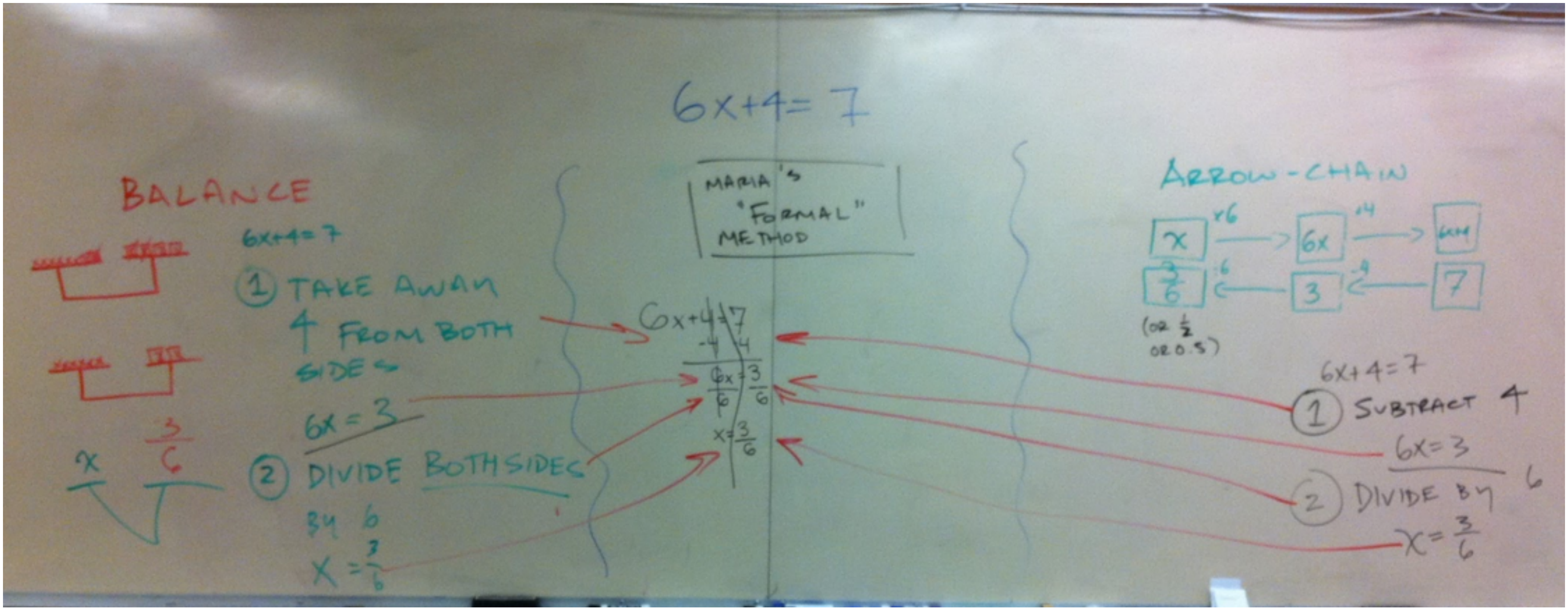

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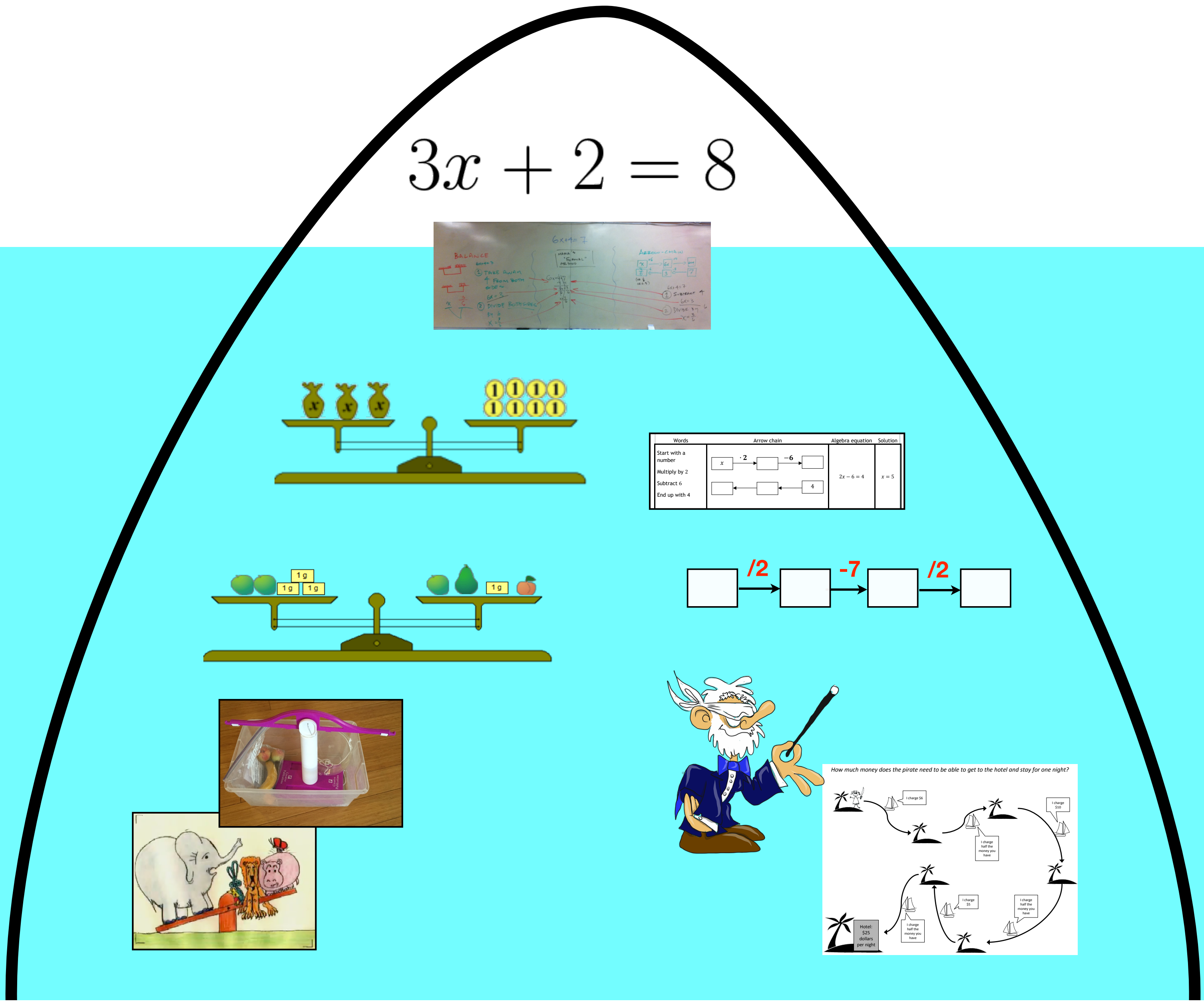


$3x + 2 = 8$
Words
Arrow chain
Algebra equation
Solution
Start with a number
Multiply by 2
Subtract 6
End up with 4
x
$\cdot 2$
-6
4
$2x - 6 = 4$
$x = 5$
x
1
1 g
/2
-7
/2
How much money does the pirate need to be able to get to the hotel and stay for one night?
I charge $6
I charge half the money you have
I charge $10
I charge half the money you have
I charge $5
I charge half the money you have
Hotel: $25 dollars per night

Instructional sequences

Connect to formal

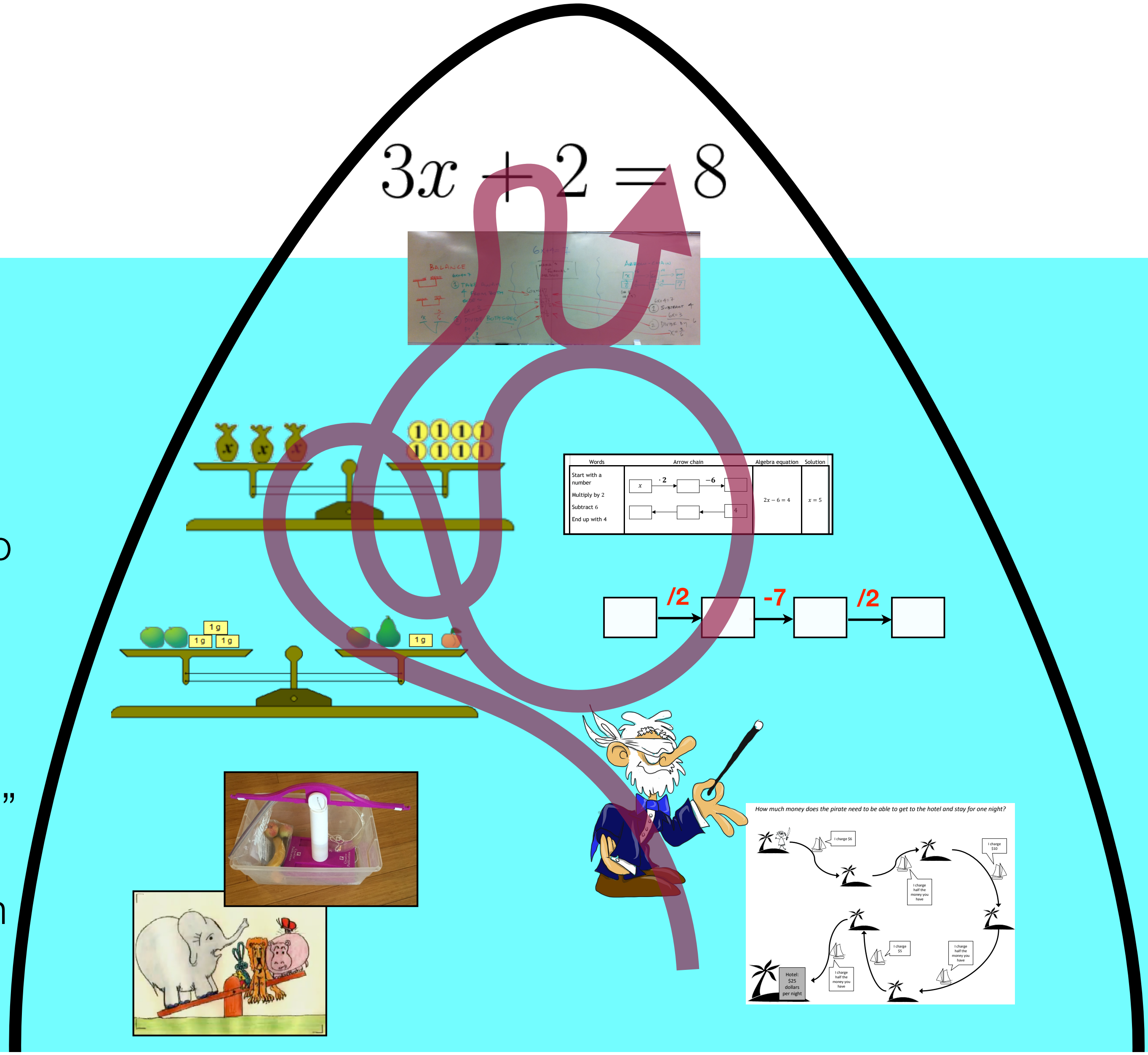
- Squiggle between models and formal

The “pre-formal” layer

- Spend lots of time solving problems using models
- Goal: Models become “tools to think with”

Start

- Activities are “experientially real” for students
- Informal models *of* the situation can become “preformal” models *for* mathematics



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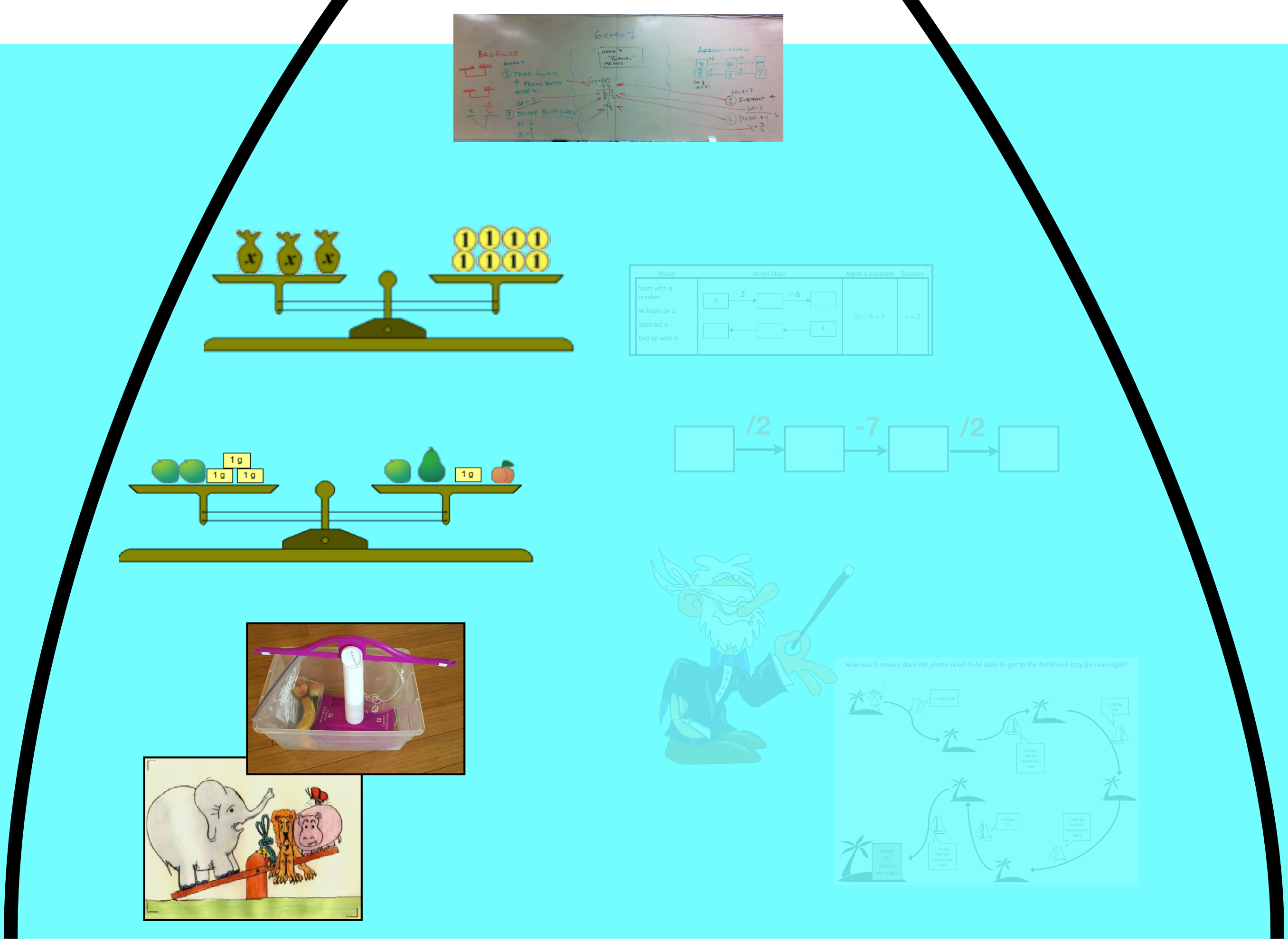
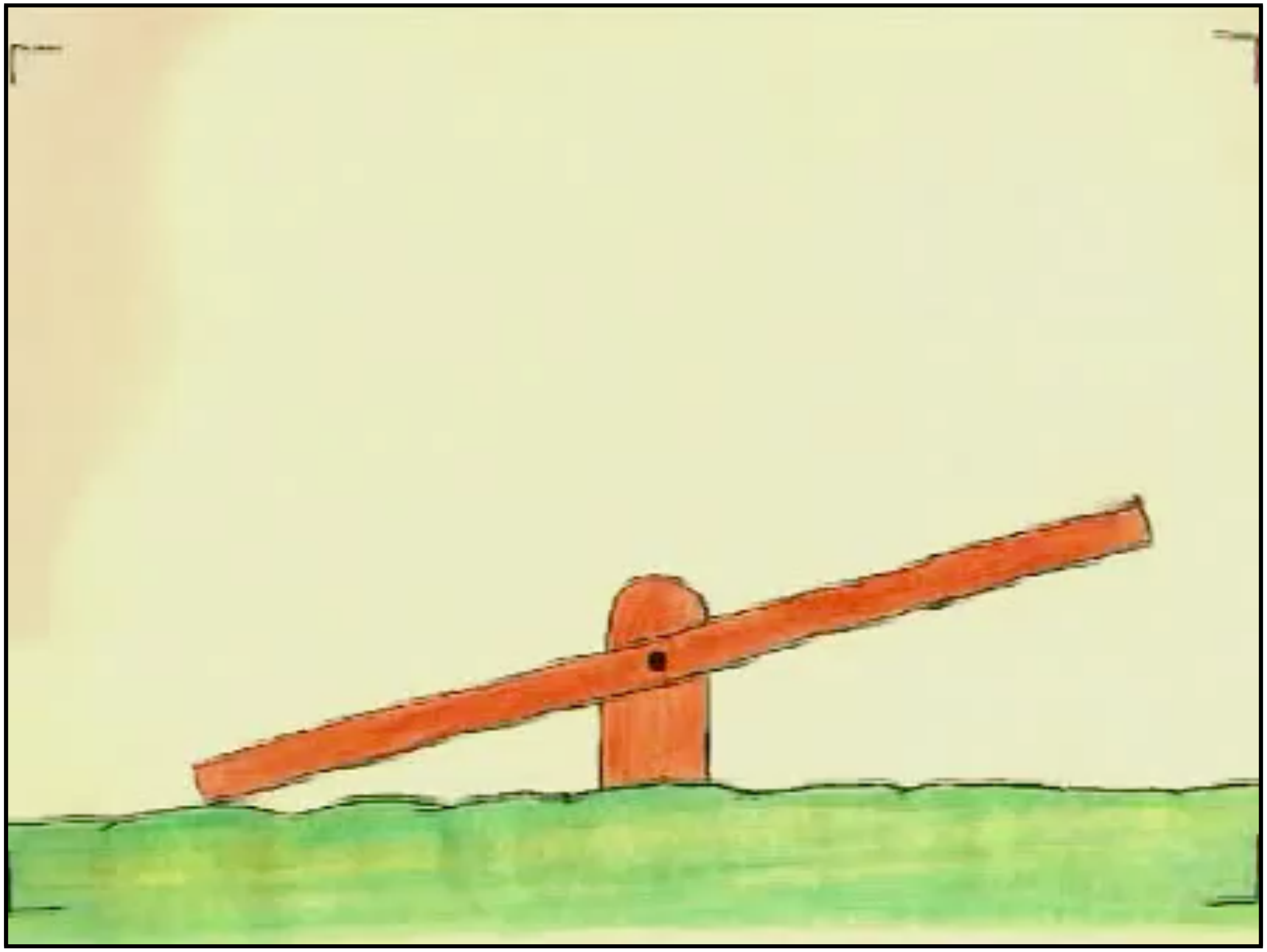
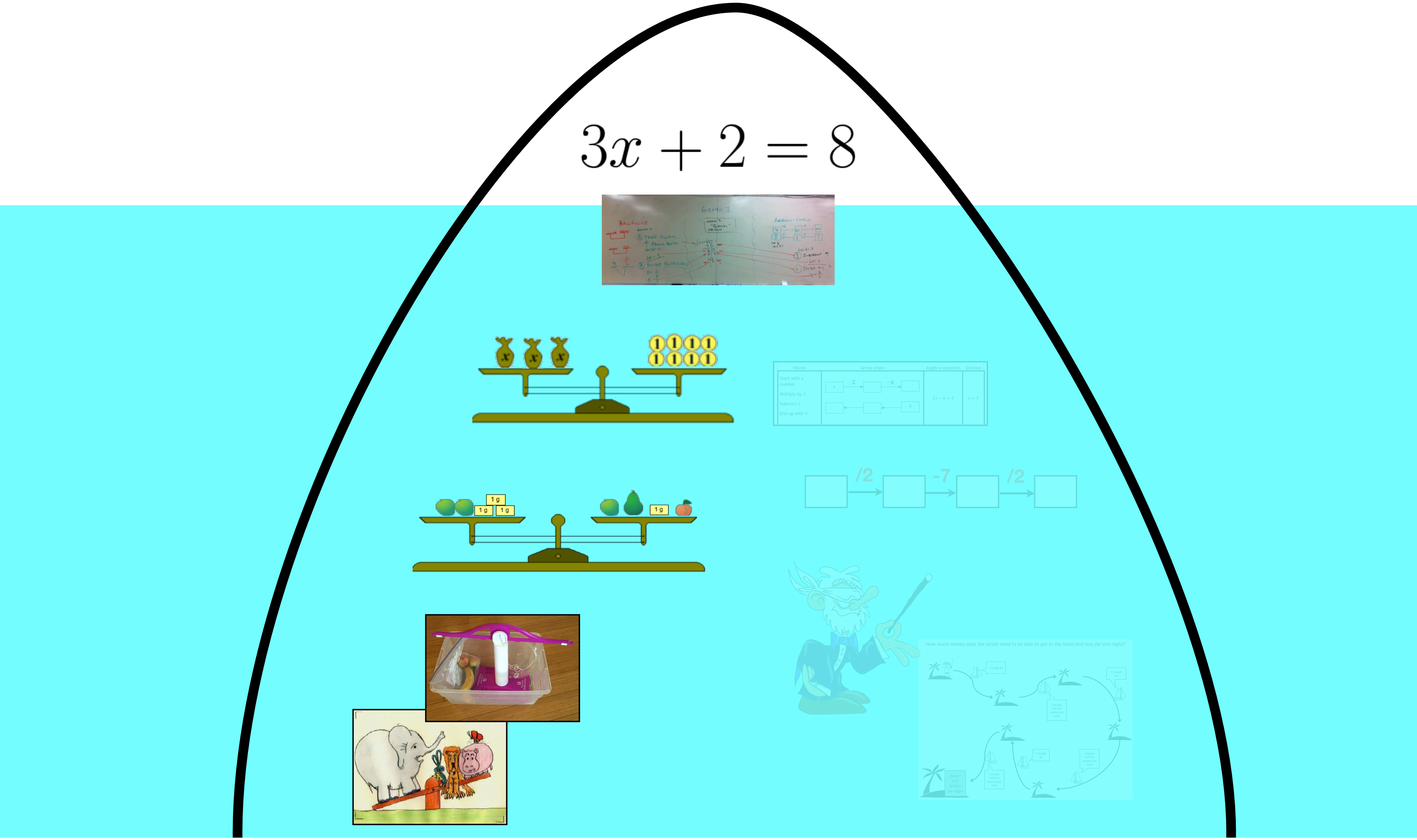


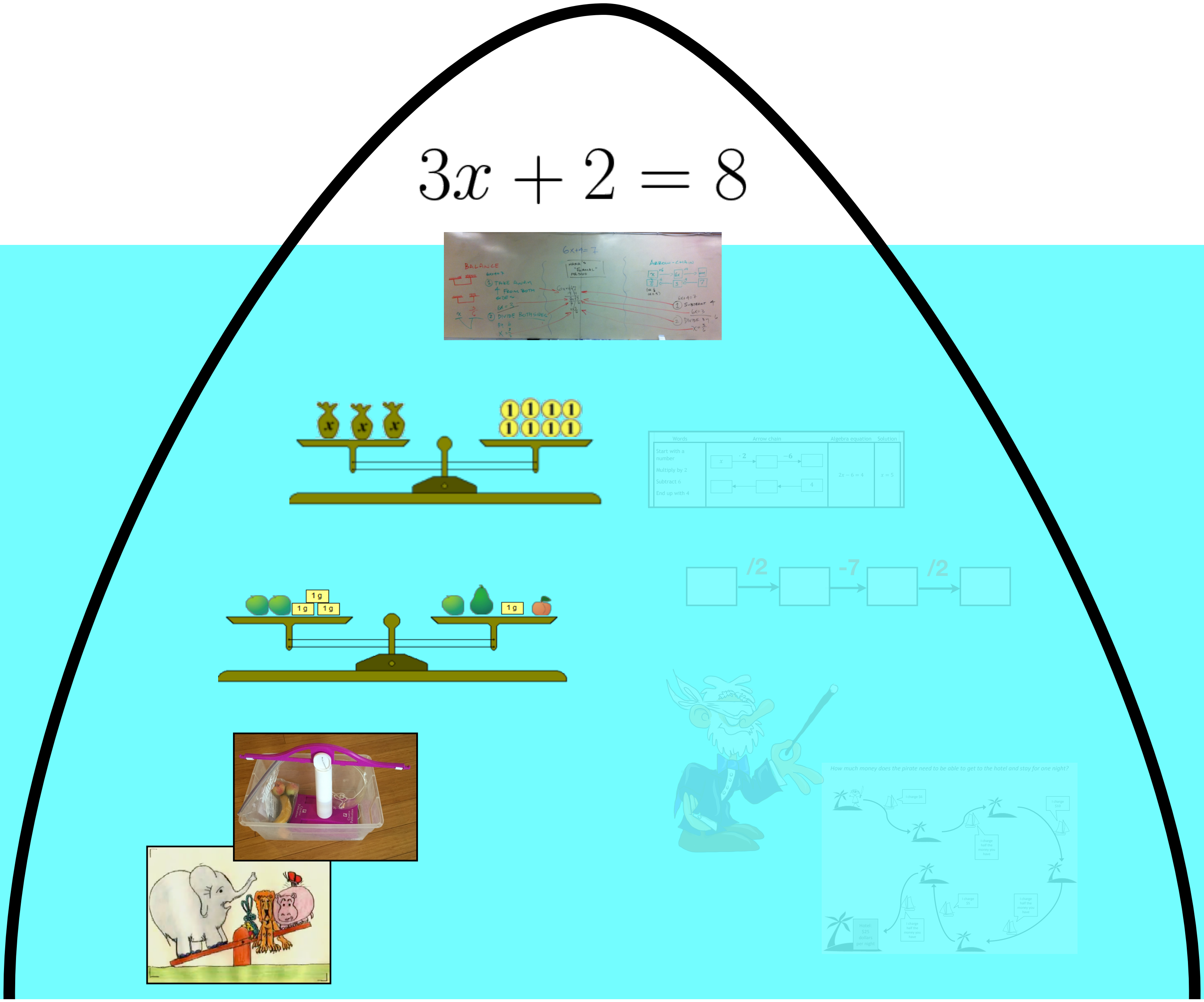



$3x + 2 = 8$
1 g
1 g
1 g
1 g
/2
-7
/2

One piece of fruit can be weighed directly

One piece can’t be weighted directly, but multiple pieces can.

Need to use weights on both sides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/2
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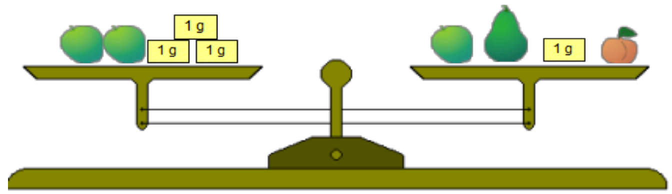
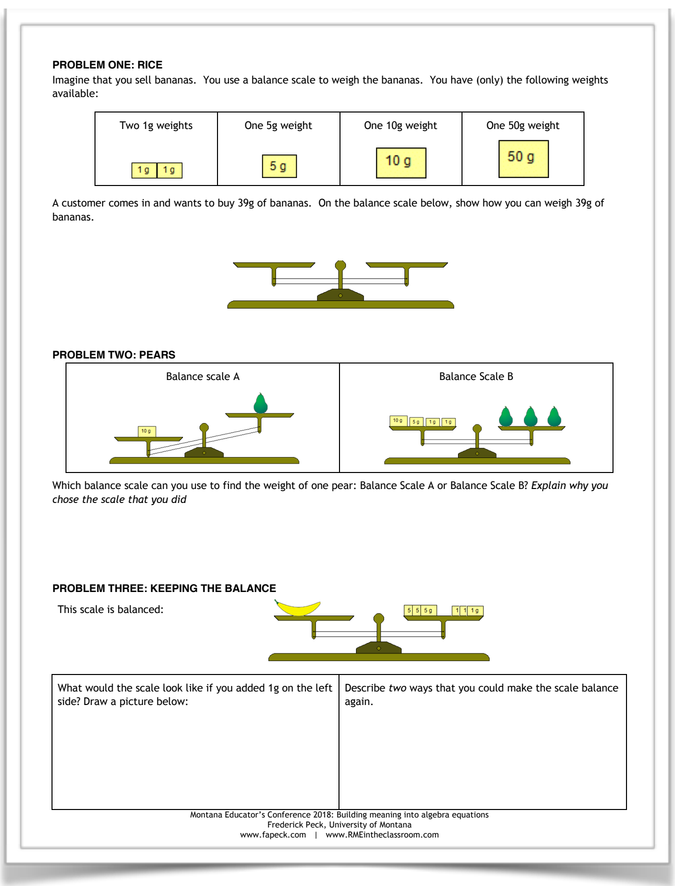
/2





PROBLEM ONE: RICE

Imagine that you sell bananas. You use a balance scale to weigh the bananas. You have (only) the following weights available:

Two 1g weights	One 5g weight	One 10g weight	One 50g weight
1 g 1 g	5 g	10 g	50 g

A customer comes in and wants to buy 39g of bananas. On the balance scale below, show how you can weigh 39g of bananas.

PROBLEM TWO: PEARS

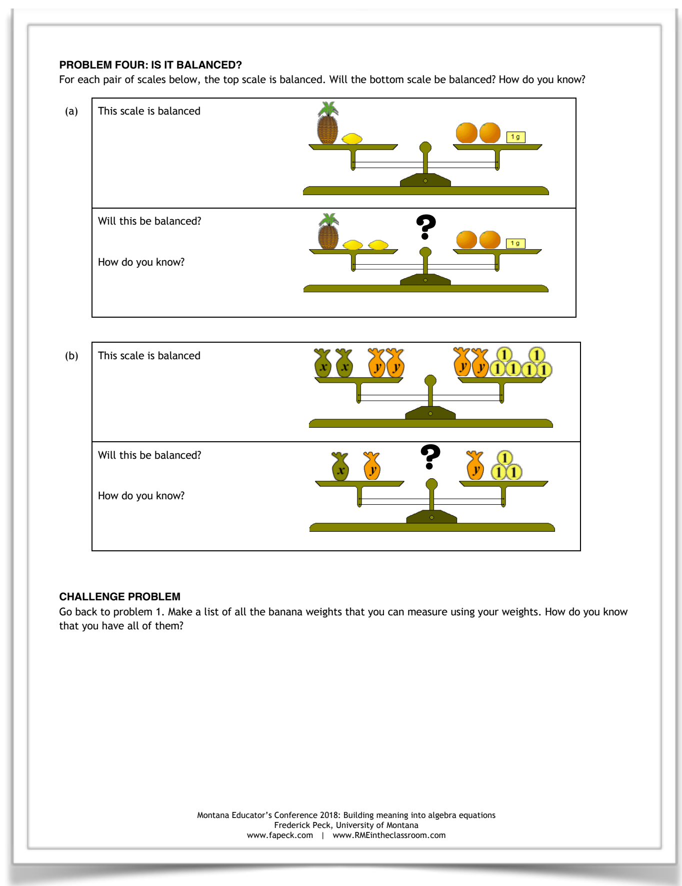
Balance scale A	Balance Scale B
10 g	10 g 5 g 1 g 1 g

Which balance scale can you use to find the weight of one pear: Balance Scale A or Balance Scale B? *Explain why you chose the scale that you did*

PROBLEM THREE: KEEPING THE BALANCE

This scale is balanced:

What would the scale look like if you added 1g on the left side? Draw a picture below:	Describe *two* ways that you could make the scale balance again.



PROBLEM FOUR: IS IT BALANCED?

For each pair of scales below, the top scale is balanced. Will the bottom scale be balanced? How do you know?

(a)

This scale is balanced
Will this be balanced? How do you know?

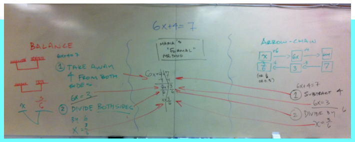
(b)

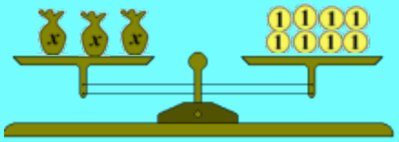
This scale is balanced
Will this be balanced? How do you know?

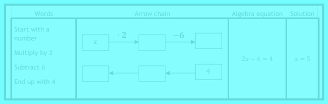
CHALLENGE PROBLEM

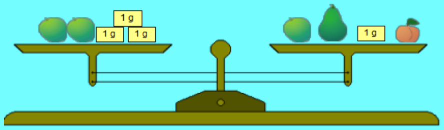
Go back to problem 1. Make a list of all the banana weights that you can measure using your weights. How do you know that you have all of them?

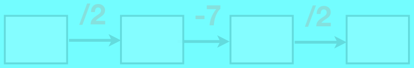
$$3x + 2 = 8$$

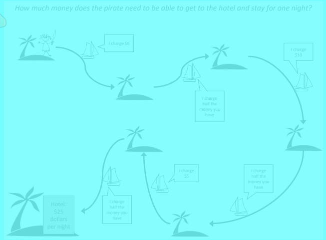


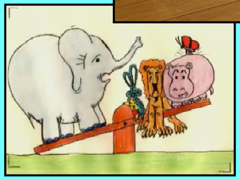


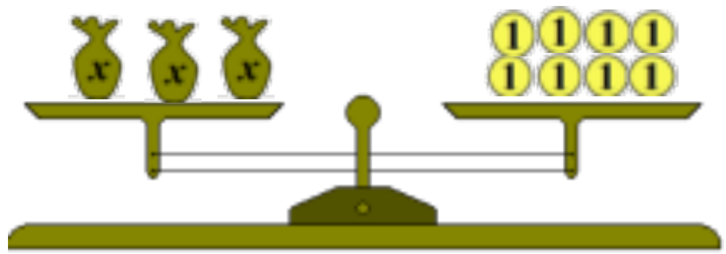


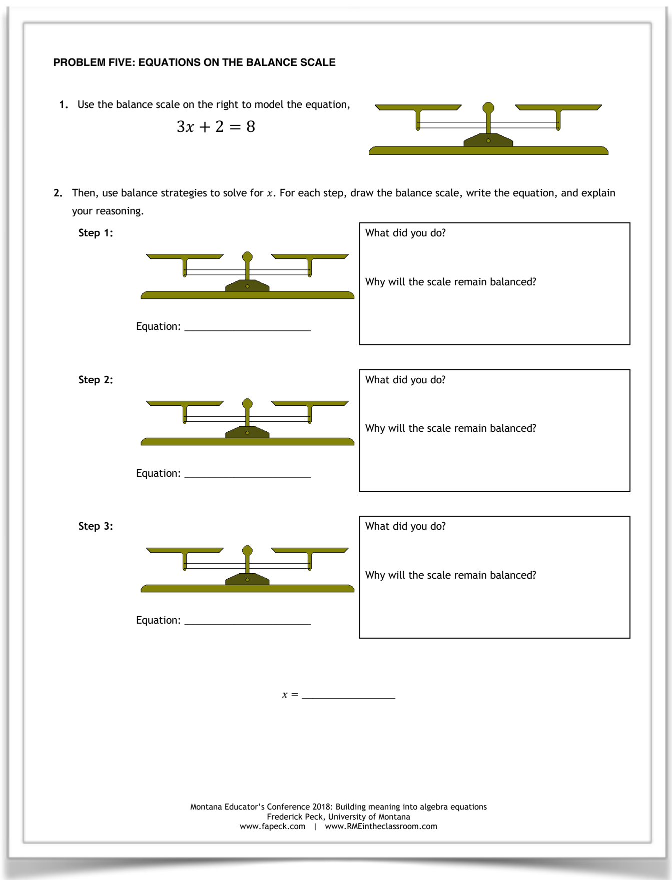












PROBLEM FIVE: EQUATIONS ON THE BALANCE SCALE

1. Use the balance scale on the right to model the equation,

$$3x + 2 = 8$$

2. Then, use balance strategies to solve for x. For each step, draw the balance scale, write the equation, and explain your reasoning.

Step 1:

Equation: ____________________

What did you do?

Why will the scale remain balanced?

Step 2:

Equation: ____________________

What did you do?

Why will the scale remain balanced?

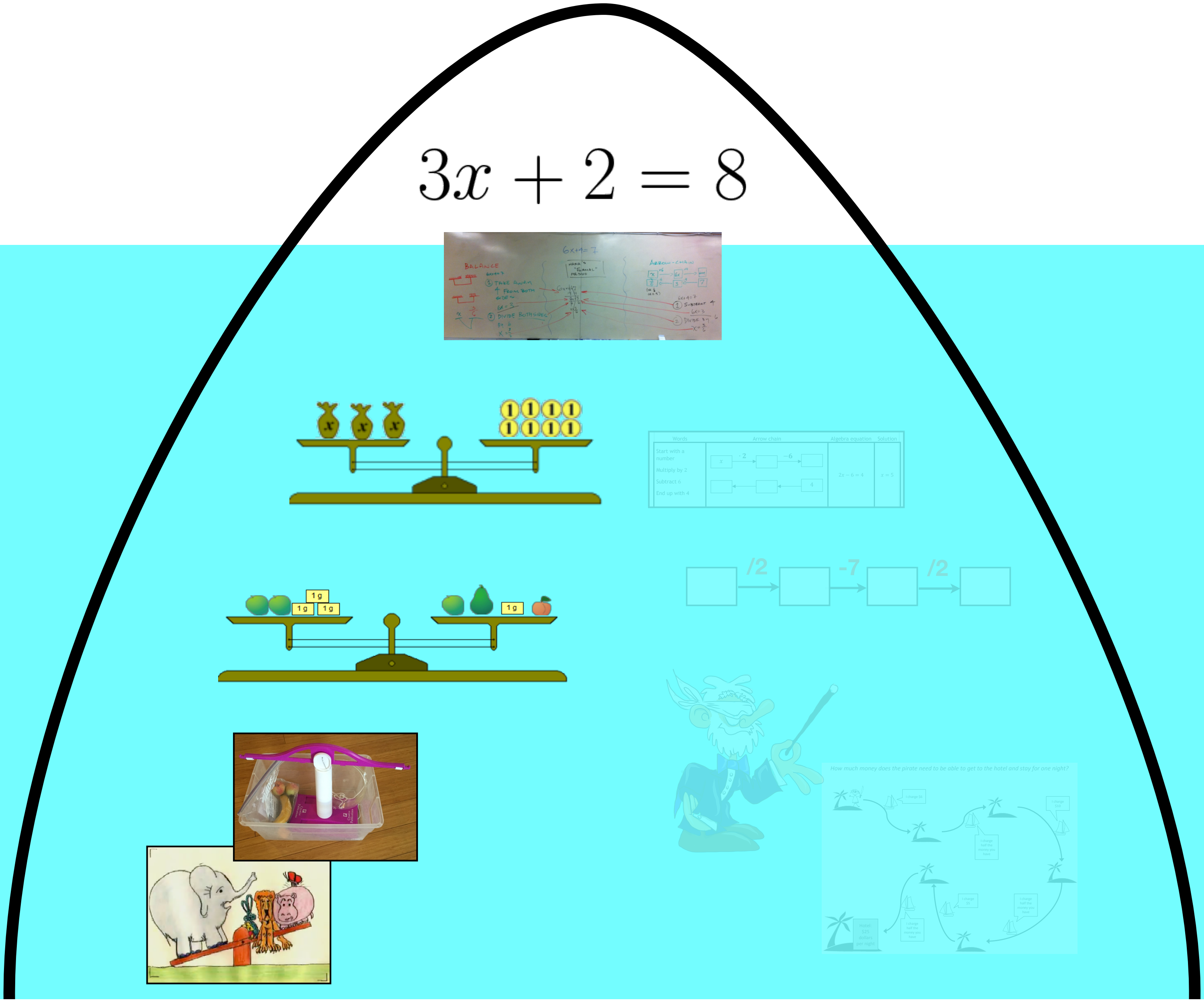
Step 3:

Equation: ____________________

What did you do?

Why will the scale remain balanced?

$x =$ _______________



$3x + 2 = 8$
1 g
/2
-7
/2

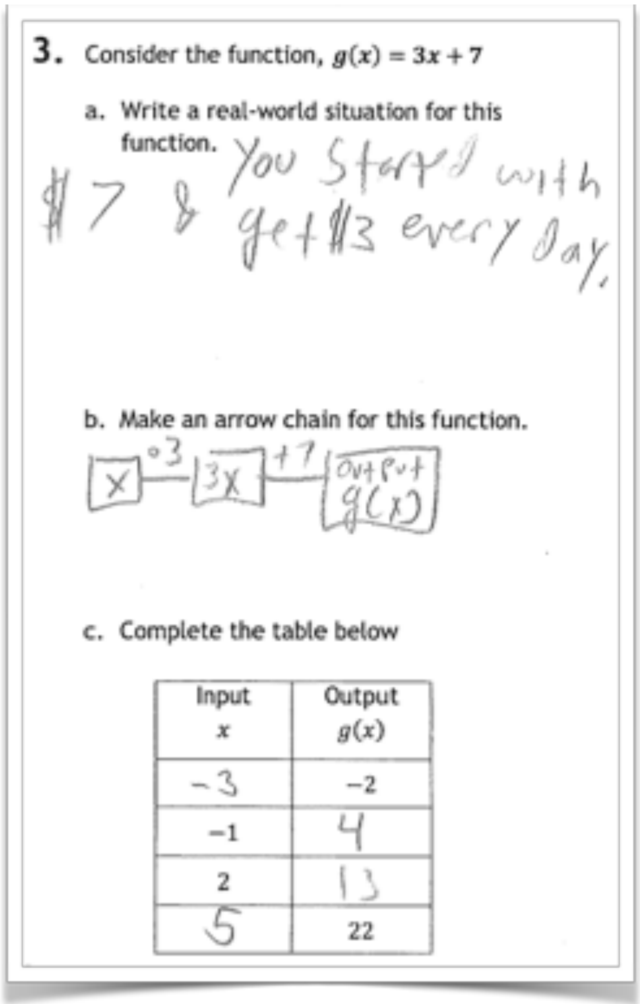
Extensions

Draw an arrow chain for each equation

$$16x^2 - 8x + 11 = 9$$

$$(4x - 2)^2 + 7 = 9$$

Extensions



3. Consider the function, $g(x) = 3x + 7$

a. Write a real-world situation for this function. You started with \$7 & get \$3 every day.

b. Make an arrow chain for this function.

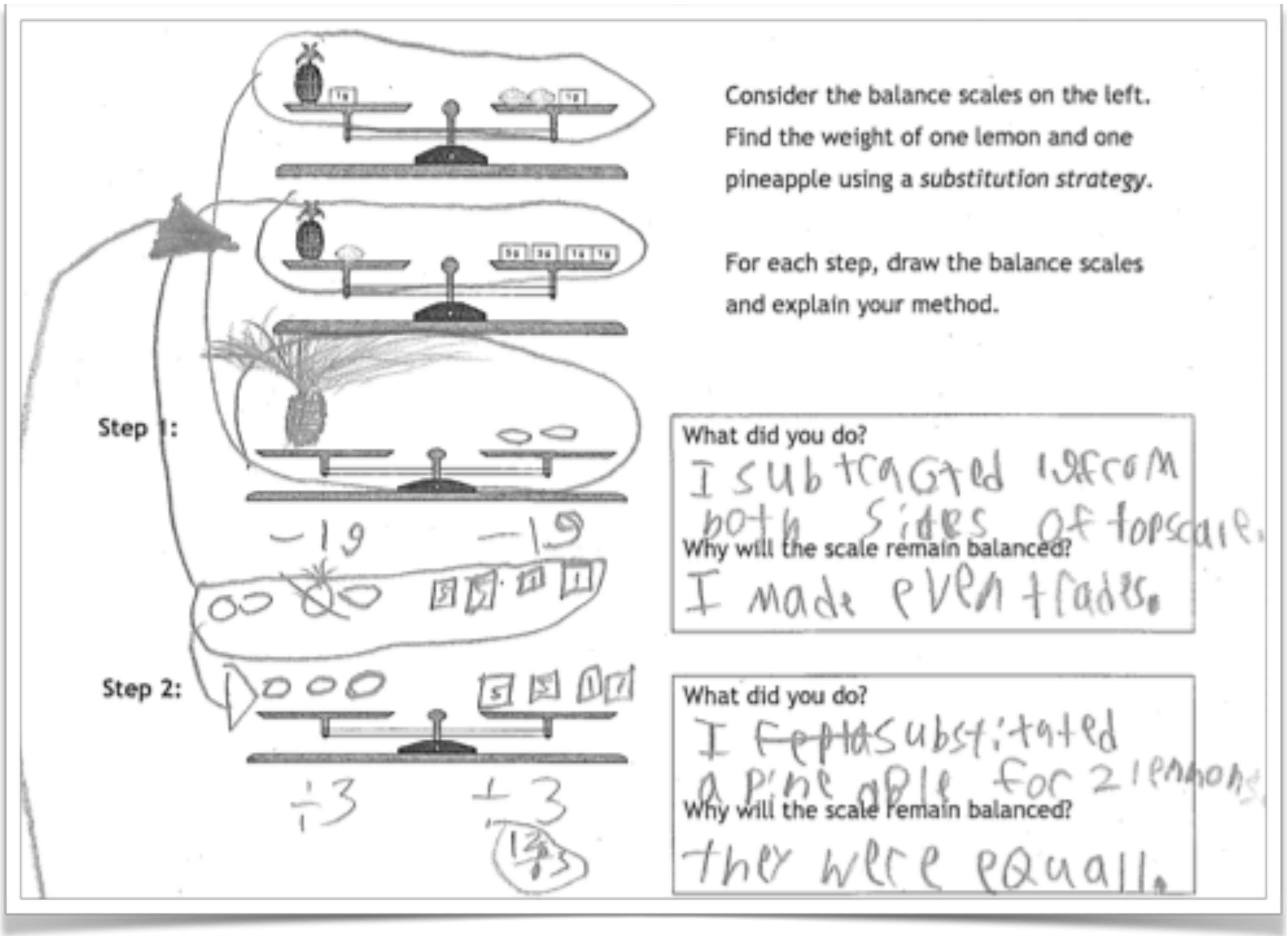
$$\boxed{x} \xrightarrow{\cdot 3} \boxed{3x} \xrightarrow{+7} \boxed{\text{output } g(x)}$$

c. Complete the table below

Input x	Output $g(x)$
−3	−2
−1	4
2	13
5	22

Arrow chains for functions

Extensions



Balance models for systems of equations

Thank-you!

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UNIVERSITY OF MONTANA

Cool! An online MA degree just for math teachers!

MCTM trivia night:

Craft Local

7pm doors

8:15 Trivia!