Beyond rise over run:
A local instructional theory for slope

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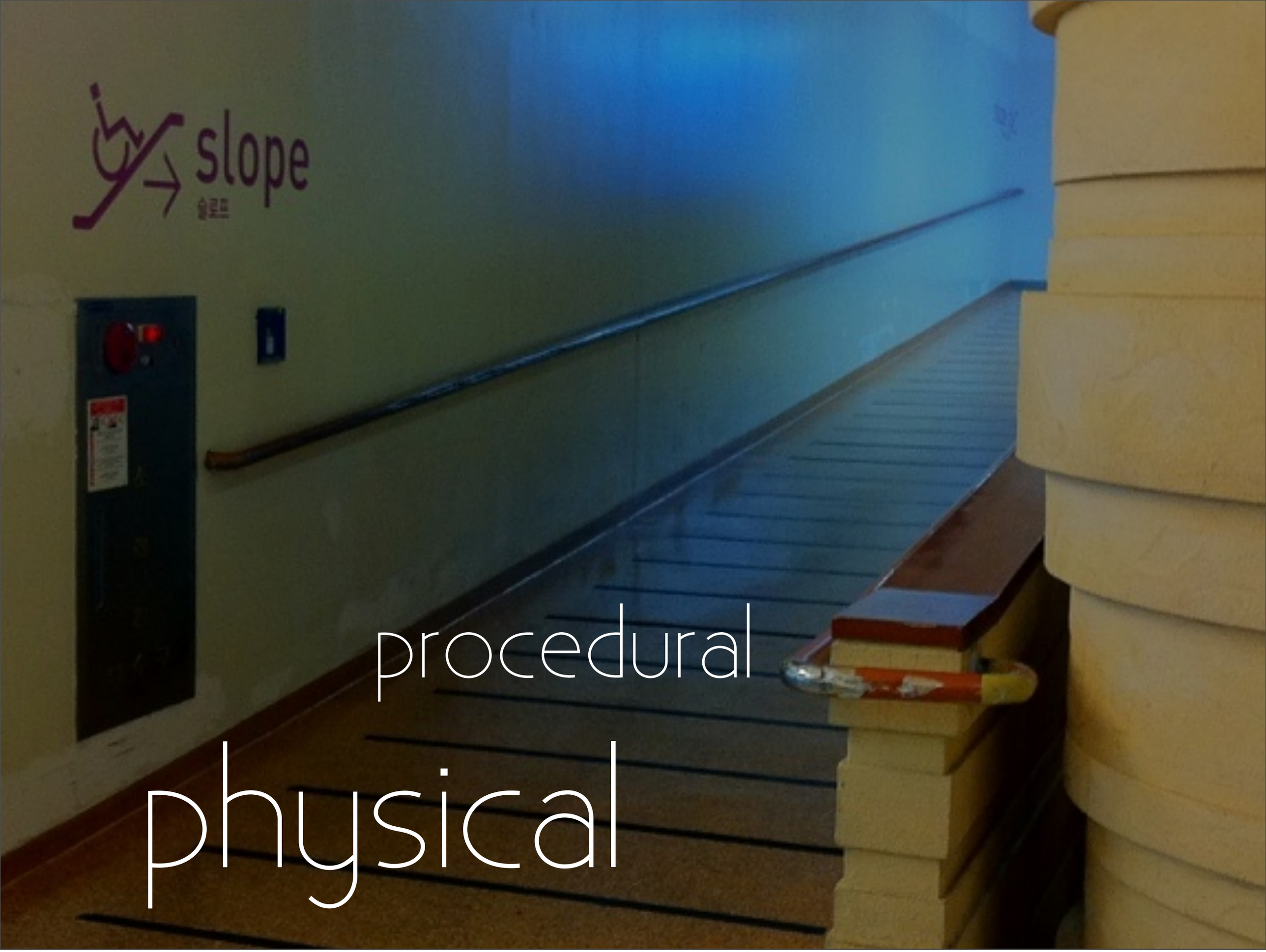
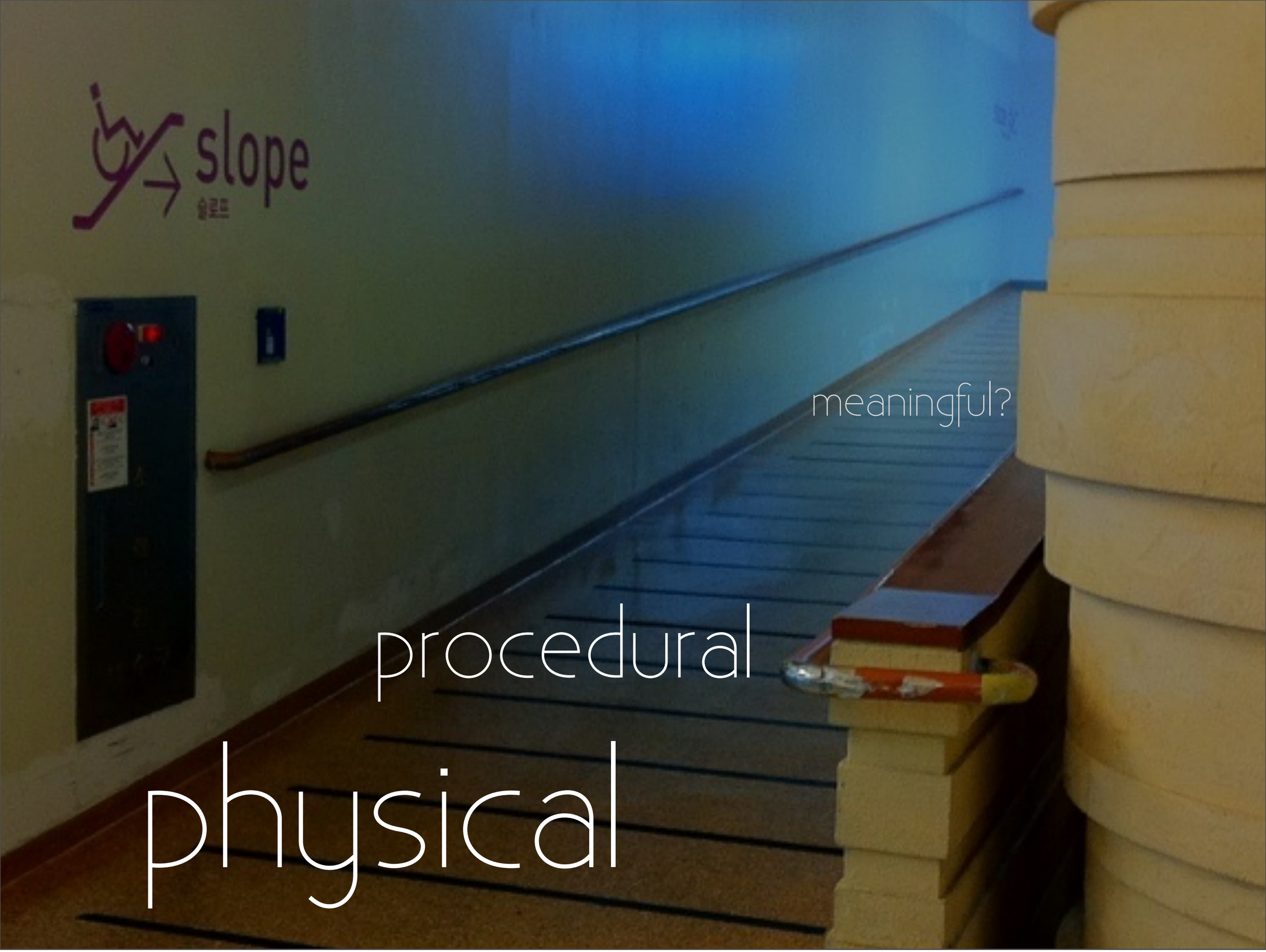
slope
슬로프


slope
슬로프
physical


slope
procedural
physical


slope
meaningful?
procedural
physical


slope
meaningful?

How do students make slope meaningful?

Method

- Design experiment in HS algebra I classroom
- Data: Student work, field notes, video & audio
- 15 days; 19 students; I was the teacher
- Outcome: Local instructional theory

local instructional theory

local instructional theory

- Progression of learning

local instructional theory

- Progression of learning
- Activities

local instructional theory

- Progression of learning
- Activities
- Rationale

learning

learning

Culture

Culture

Culture

"the collection through time of partial solutions to frequently encountered problems"

Culture

... process ...

"the collection through time of partial solutions to frequently encountered problems"

Culture

"the collection through time of partial solutions to frequently encountered problems"

[product]

Culture

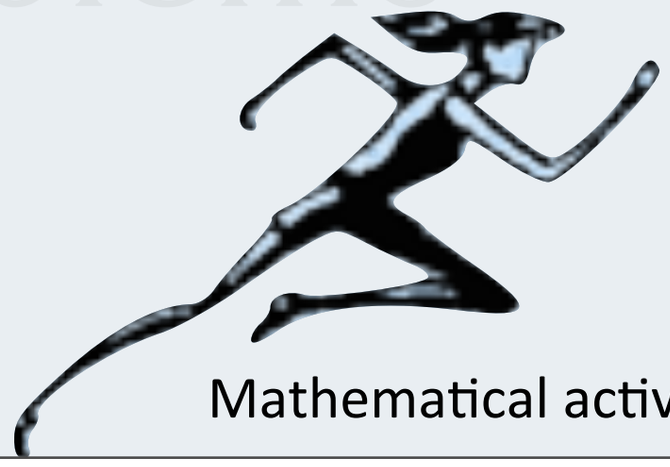
"the collection through time of partial solutions to frequently encountered problems"

[artifacts]

Culture

"the collection through time of partial solutions to frequently encountered problems"

[artifacts]



Mathematical activity

Artifacts

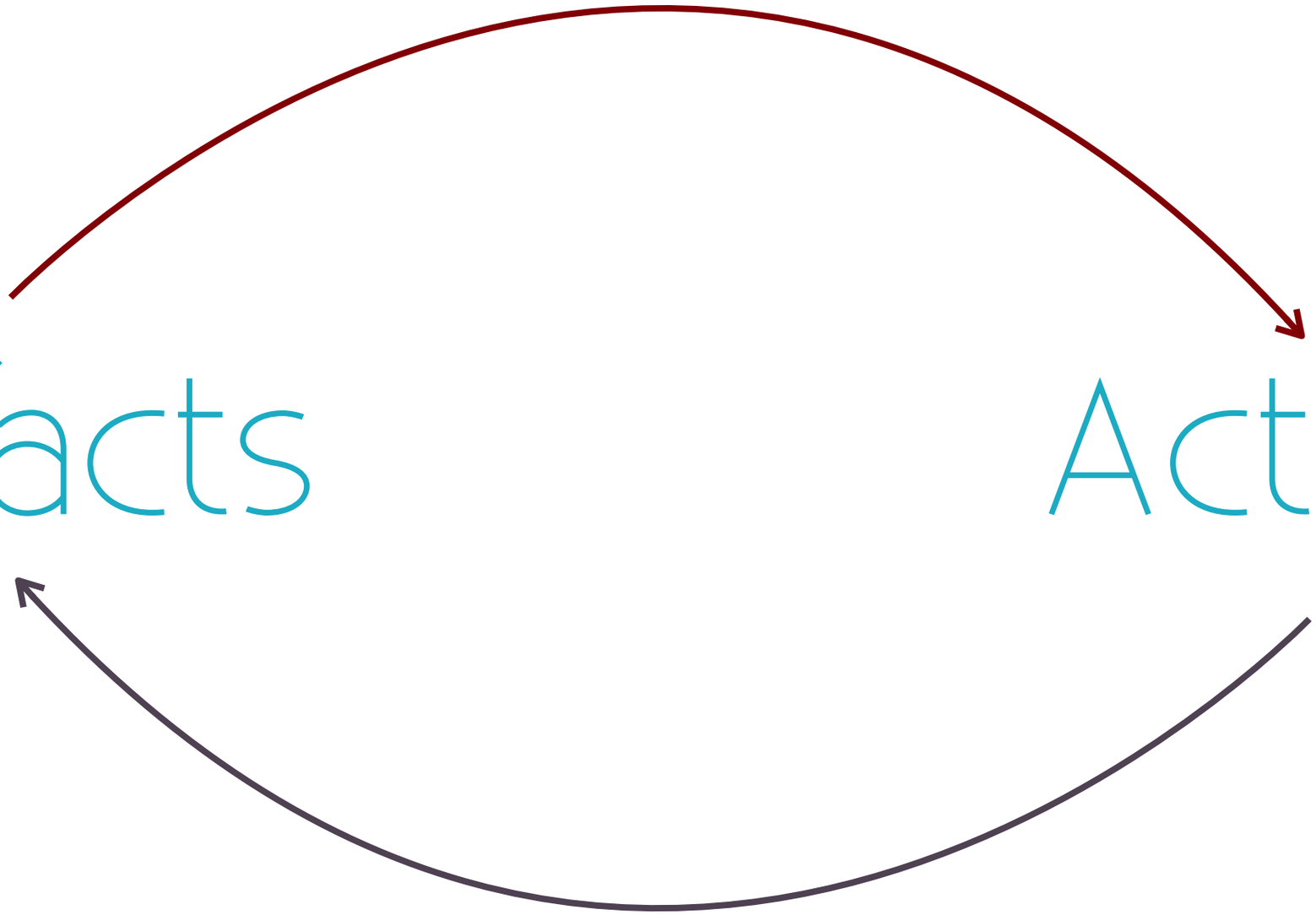
Activity

Artifacts

Activity

artifacts are the residue of historic activity

artifacts mediate
current activity



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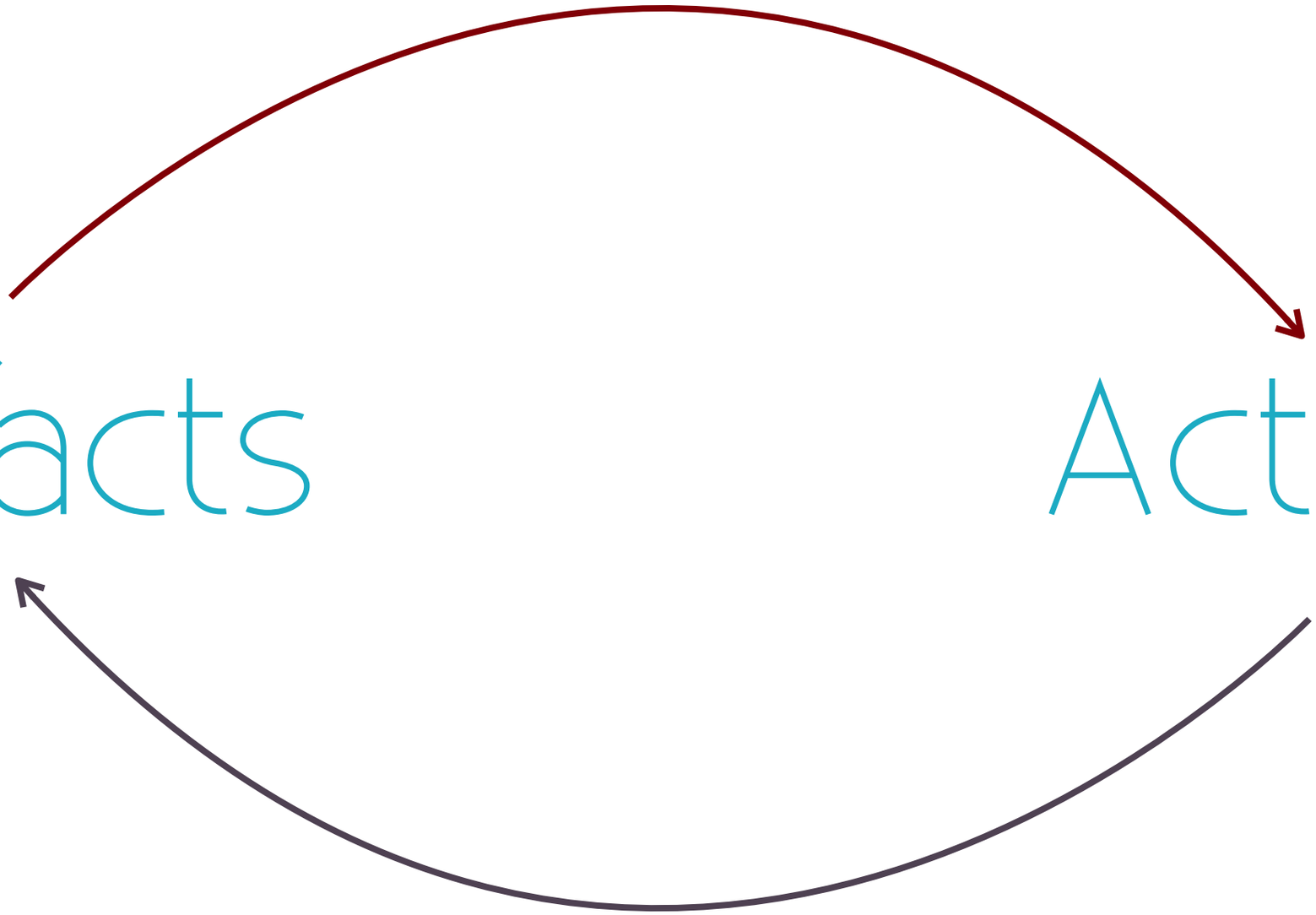

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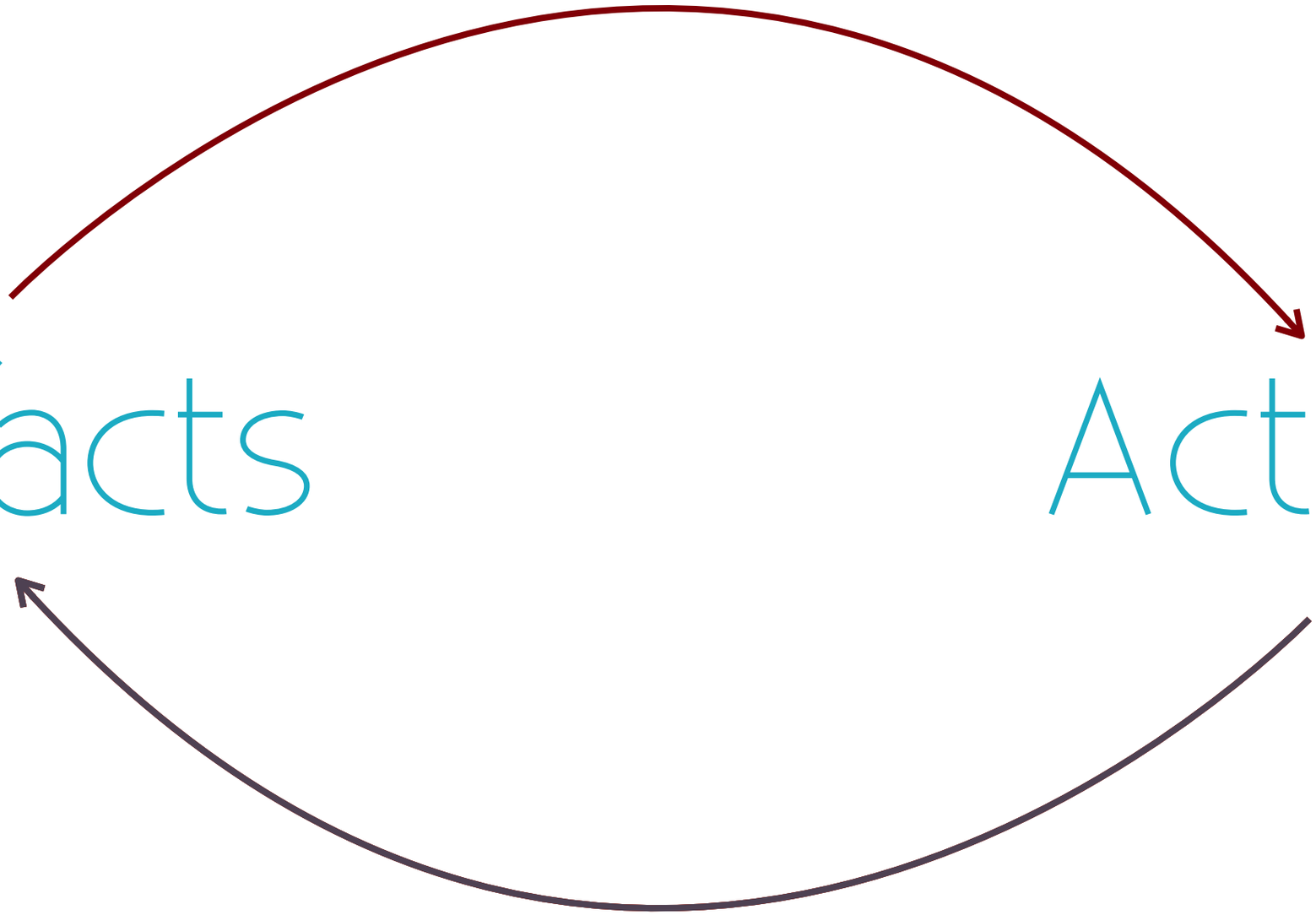


Artifacts

Activity

artifacts are the residue of historic activity

artifacts become meaningful through activity

artifacts are the residue of historic activity

artifacts become meaningful through activity

learning

meaningful

learning

making artifacts meaningful

learning

making artifacts meaningful

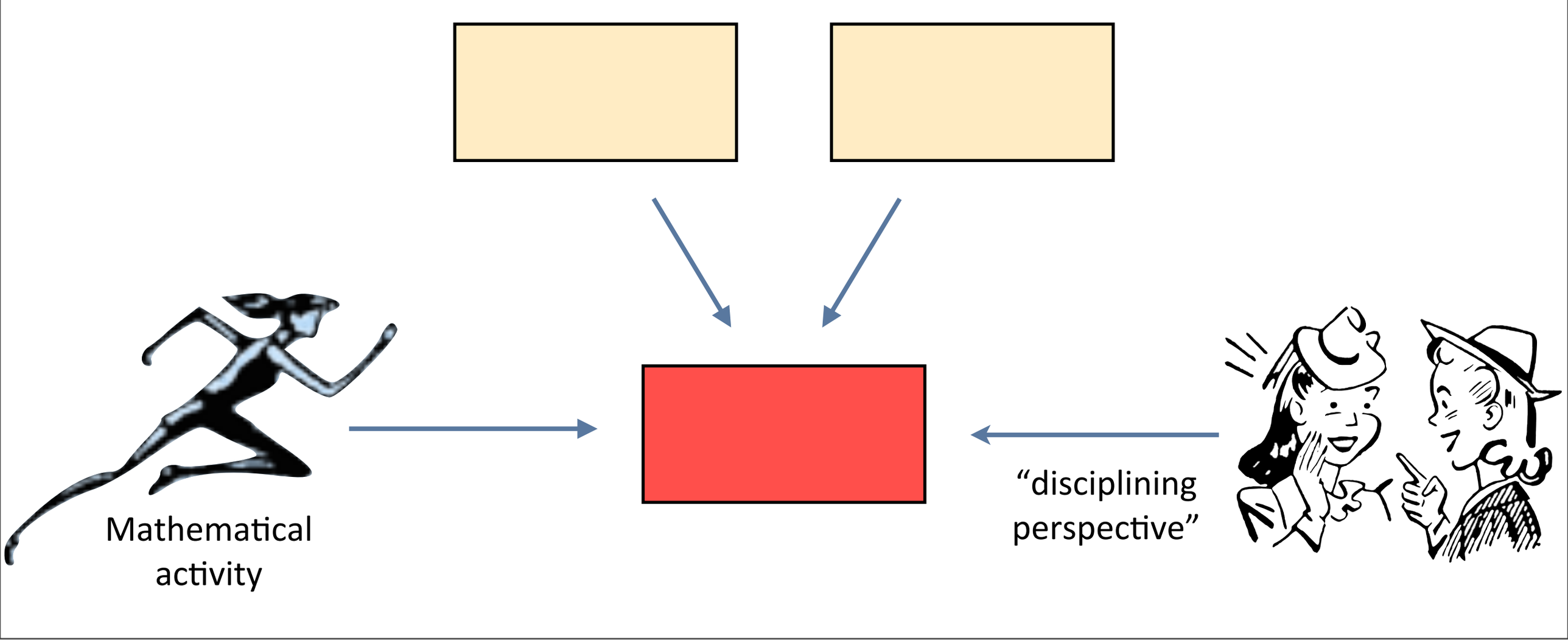
reinvention & objectification

learning as

reinvention & objectification

learning as

reinvention & objectification



slope

slope

- Parametric coefficient
- Algebraic ratio
- Rate of change
- Geometric ratio
- Physical property

slope

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

Physical property

$$y = ax + b$$

slope

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

Physical property

$$\frac{y_2 - y_1}{x_2 - x_1}$$

slope

- Parametric coefficient
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slope

- Parametric coefficient
- Algebraic ratio
- Rate of change
- **Geometric ratio**
- Physical property

$$\frac{\text{rise}}{\text{run}}$$

slope

Parametric coefficient

Algebraic ratio

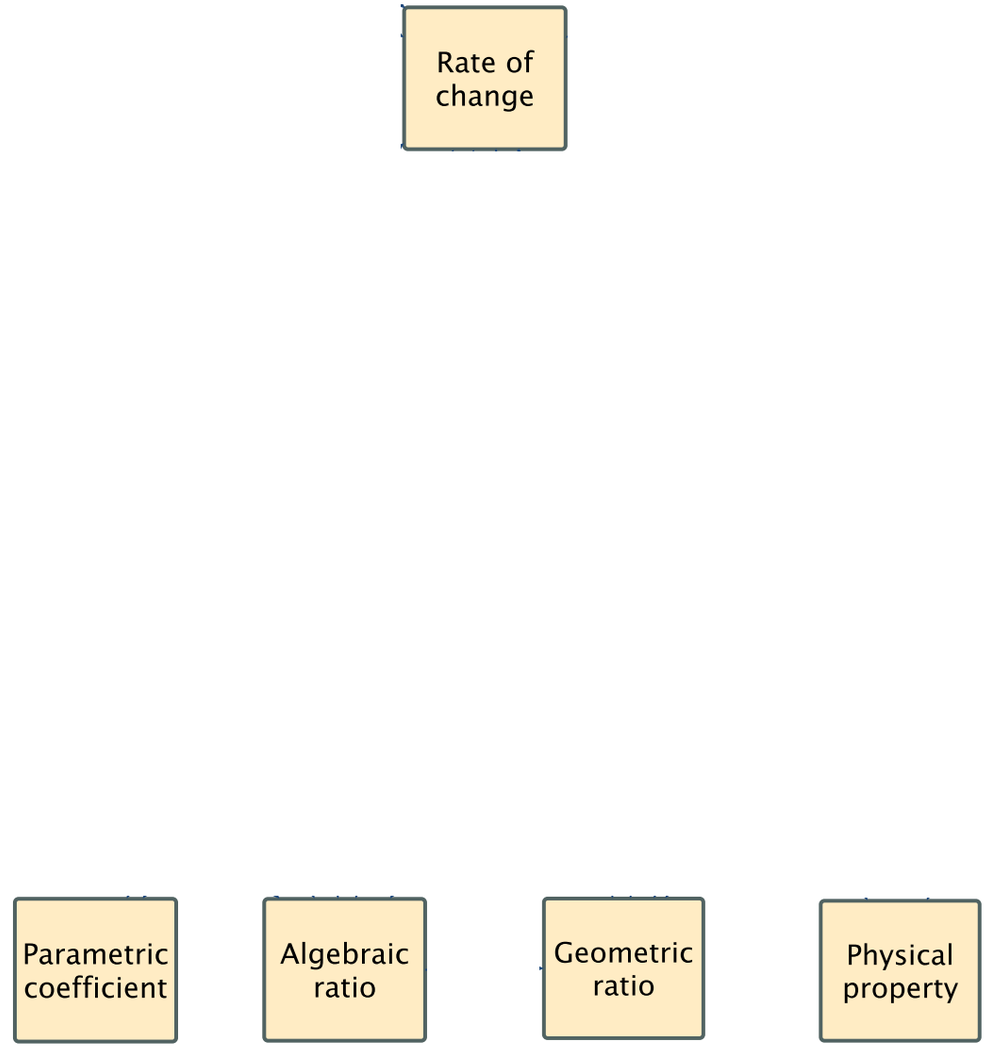
Rate of change

Geometric ratio

Physical property

“steepness”

cascade of artifacts



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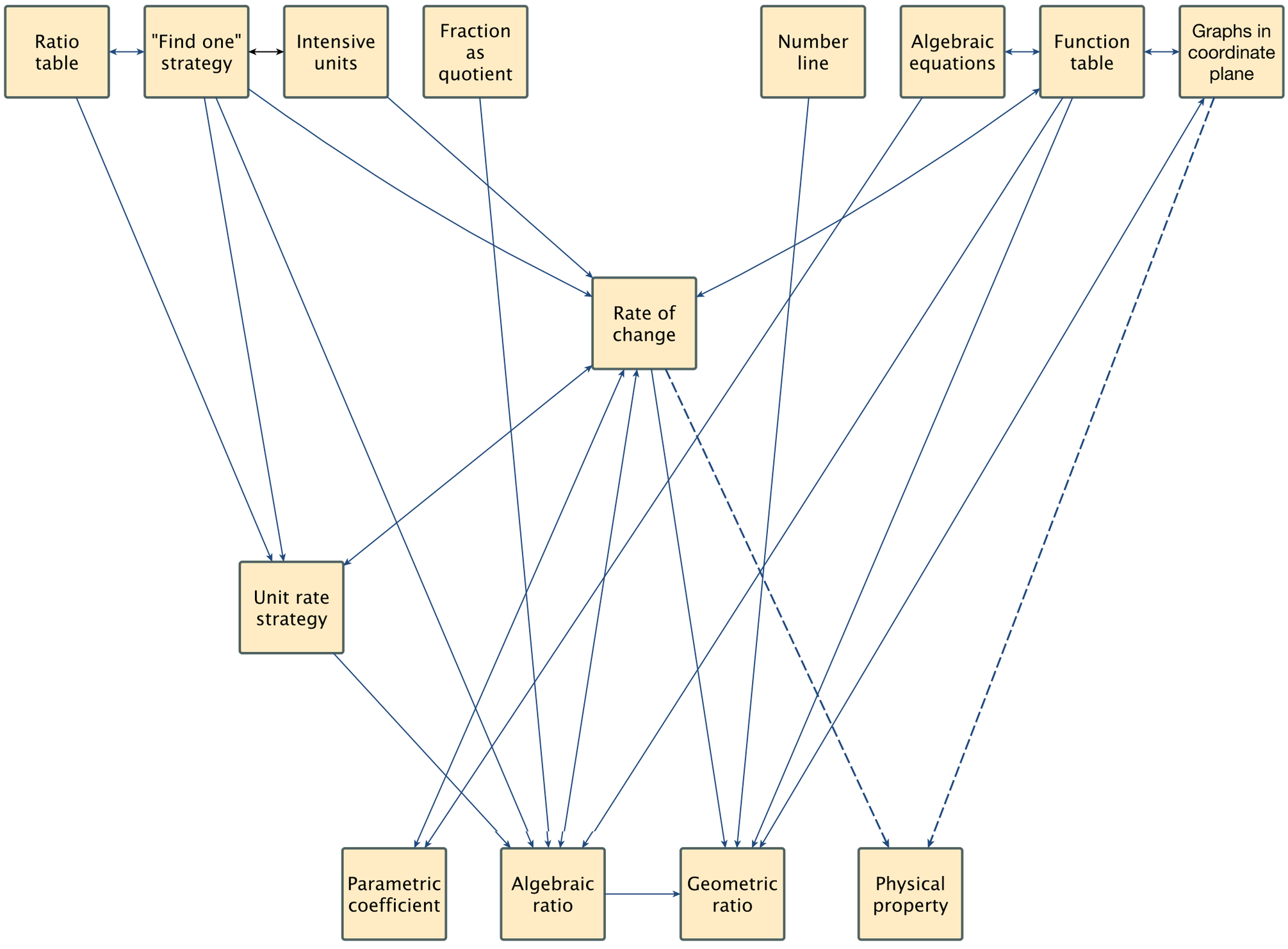

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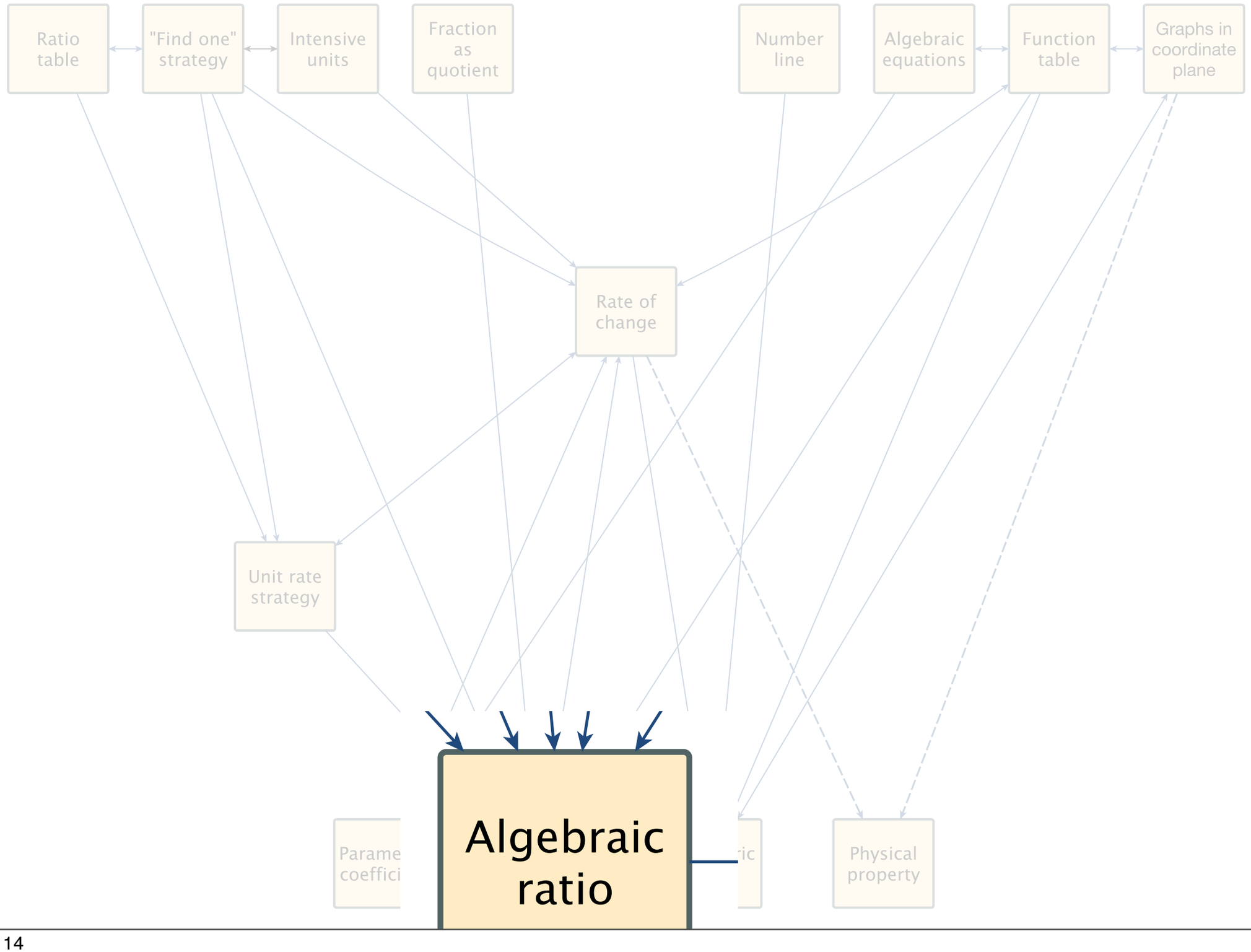

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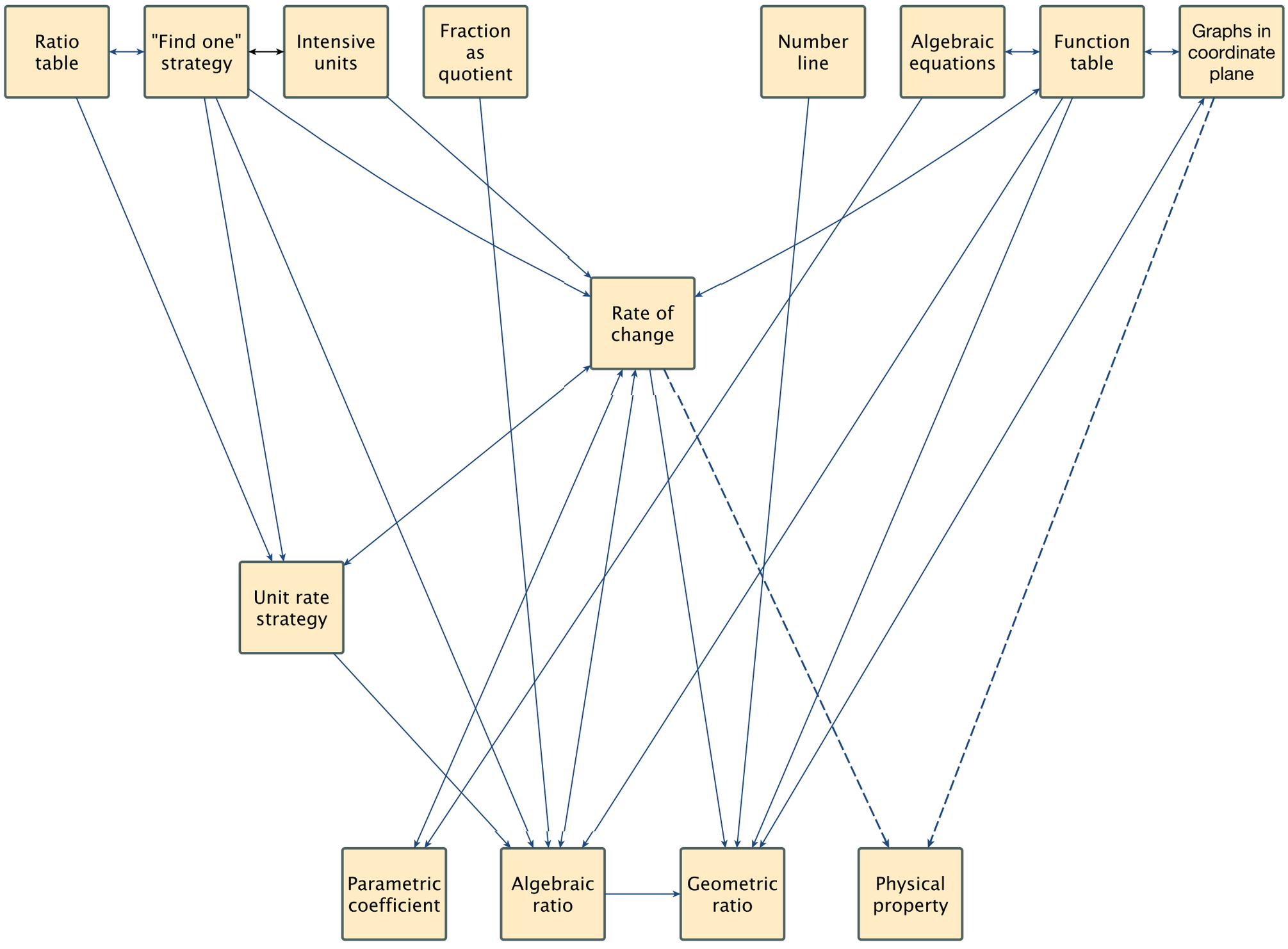

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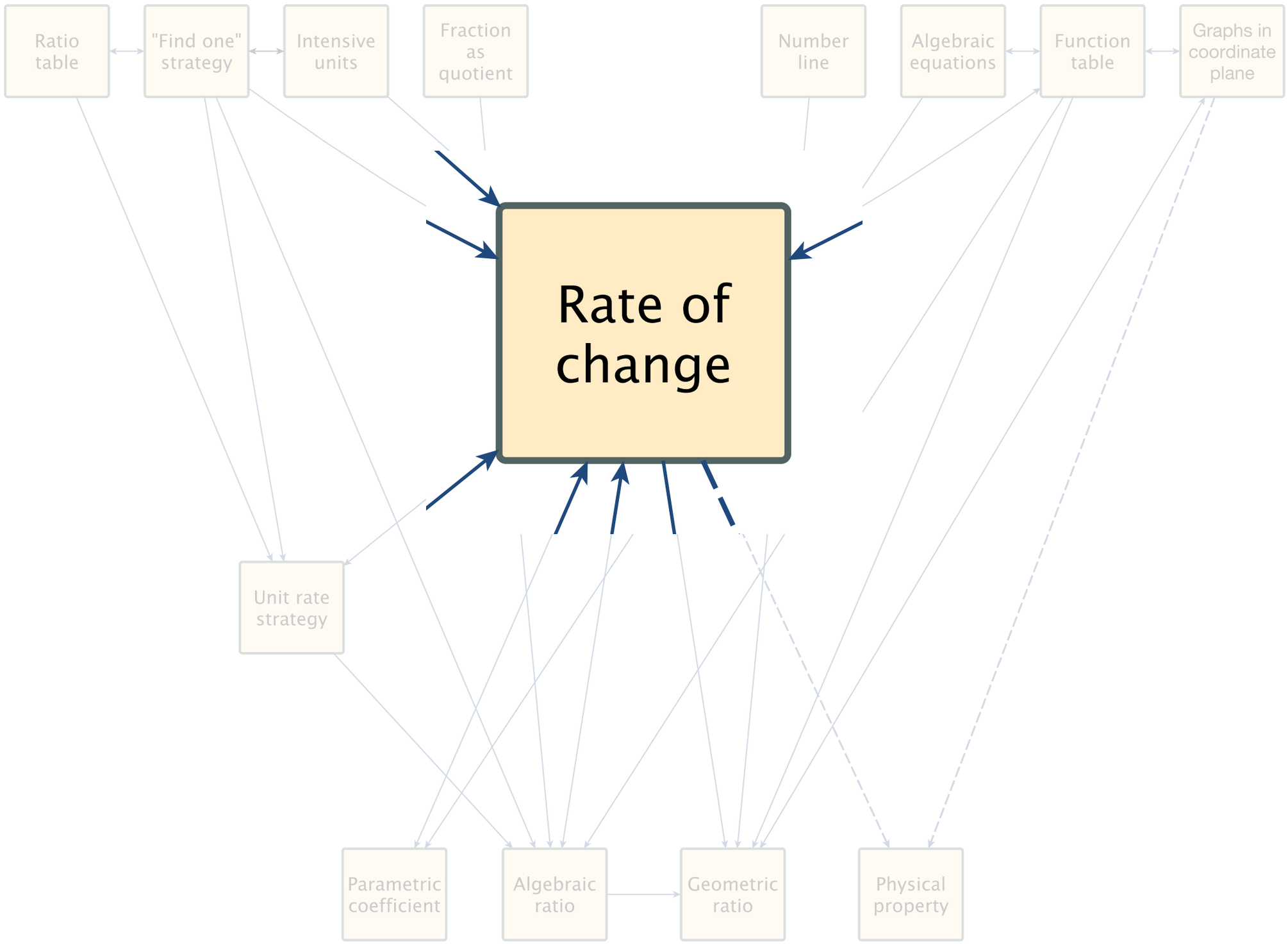

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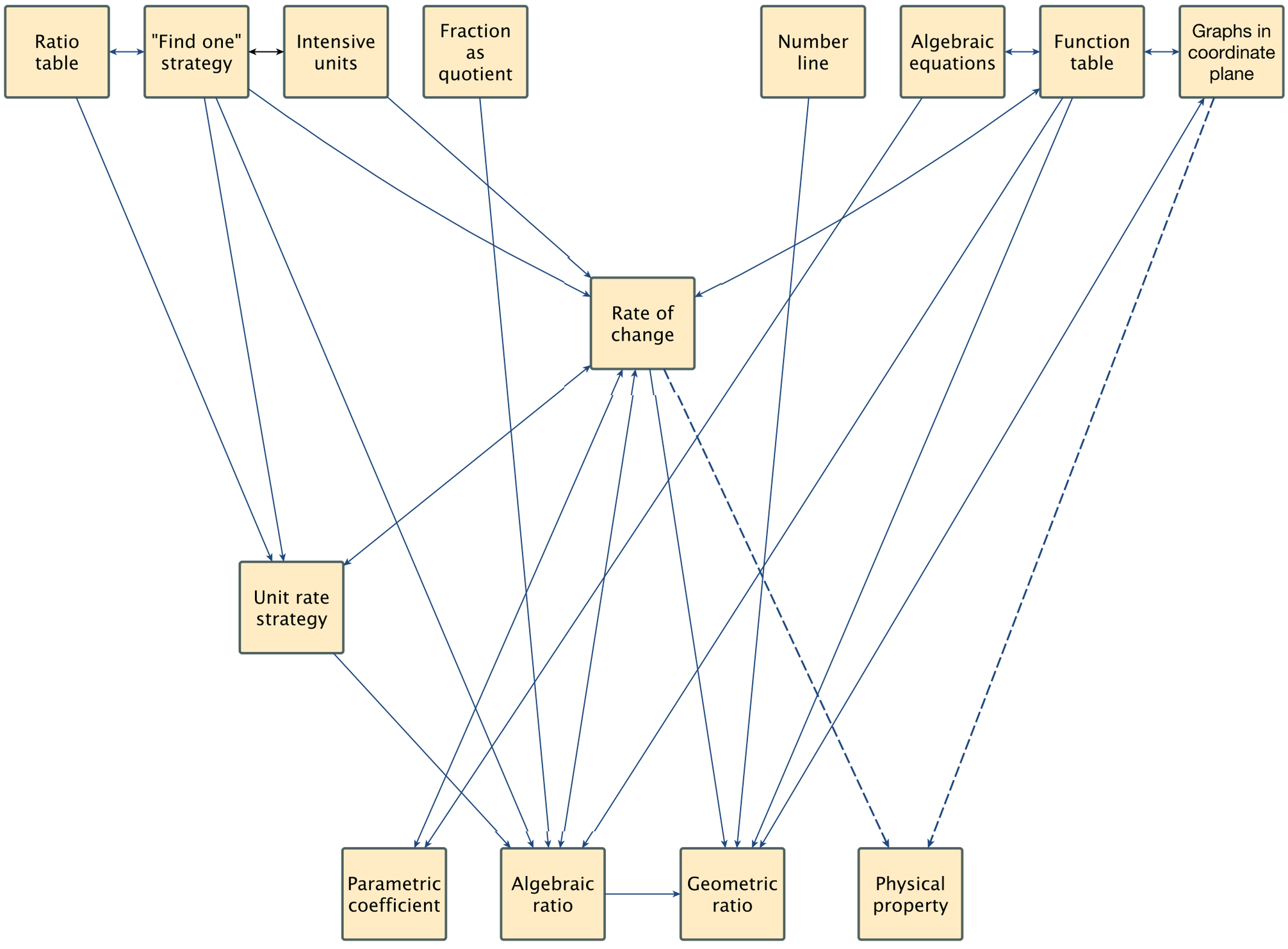
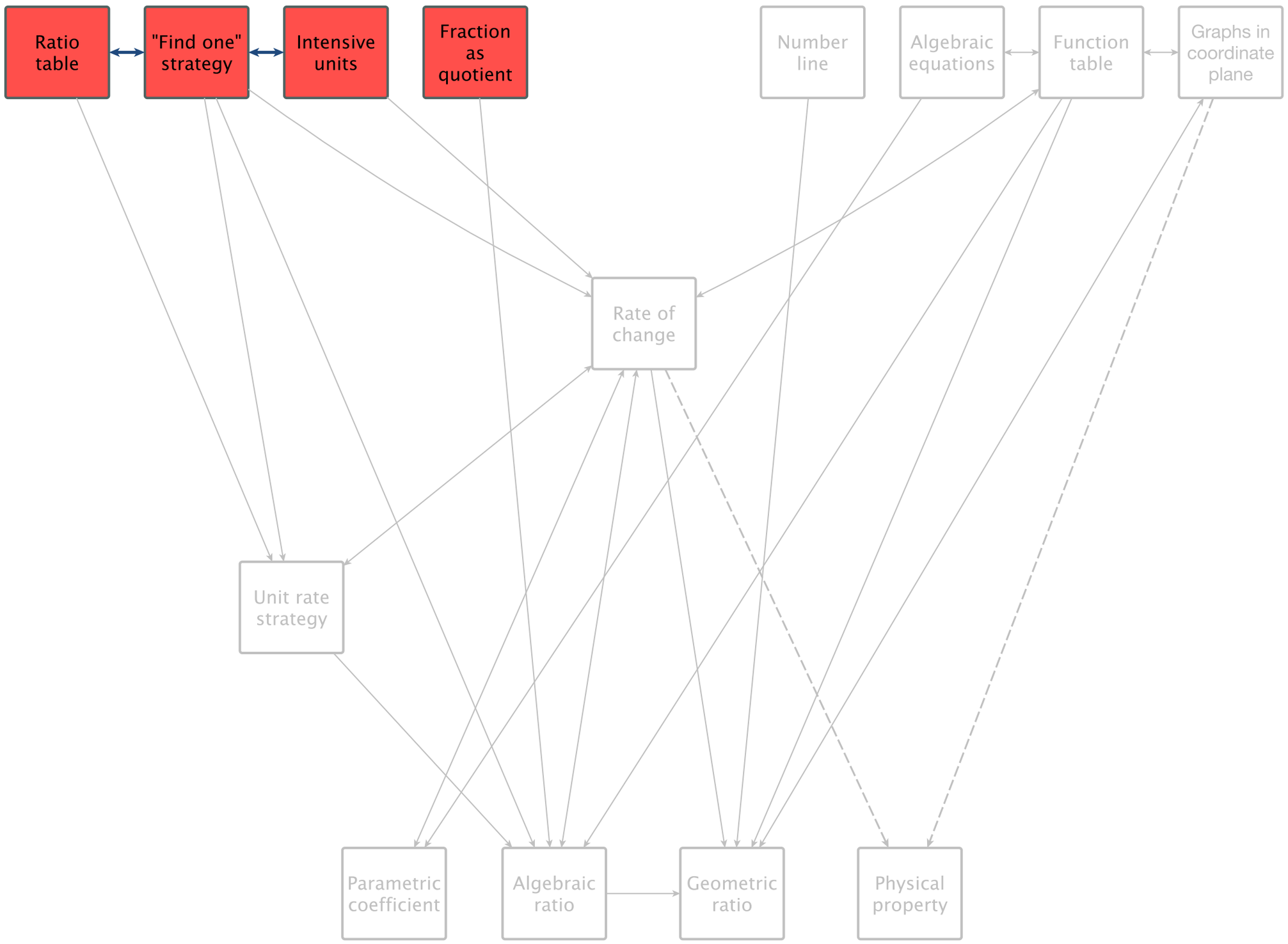


stage 1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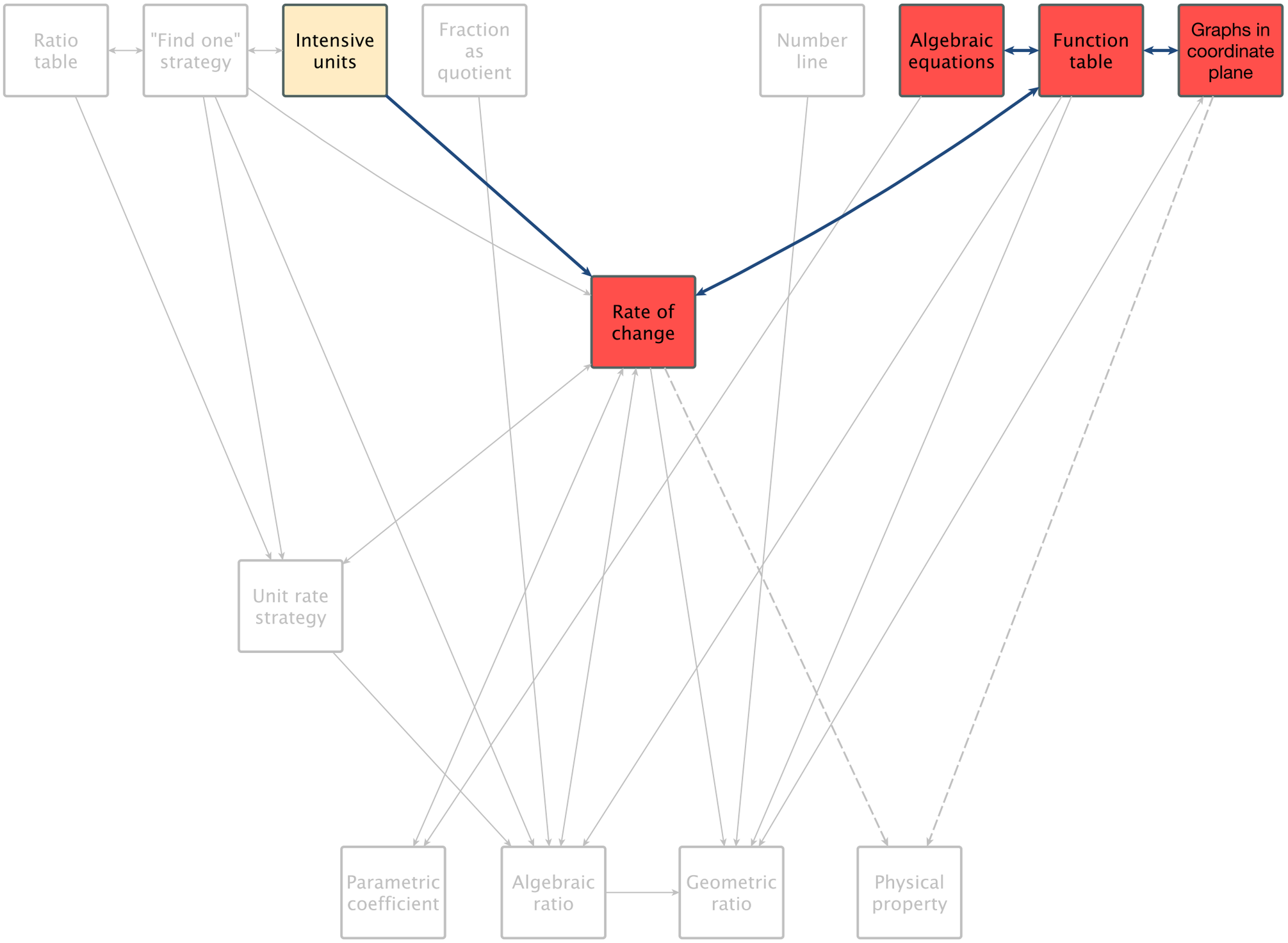
"Find one" strategy
Intensive units
Fraction as quotient
Number line
Algebraic equations
Function table
Graphs in coordinate plane
Rate of change
Unit rate strategy
Parametric coefficient
Algebraic ratio
Geometric ratio
Physical property

stage 2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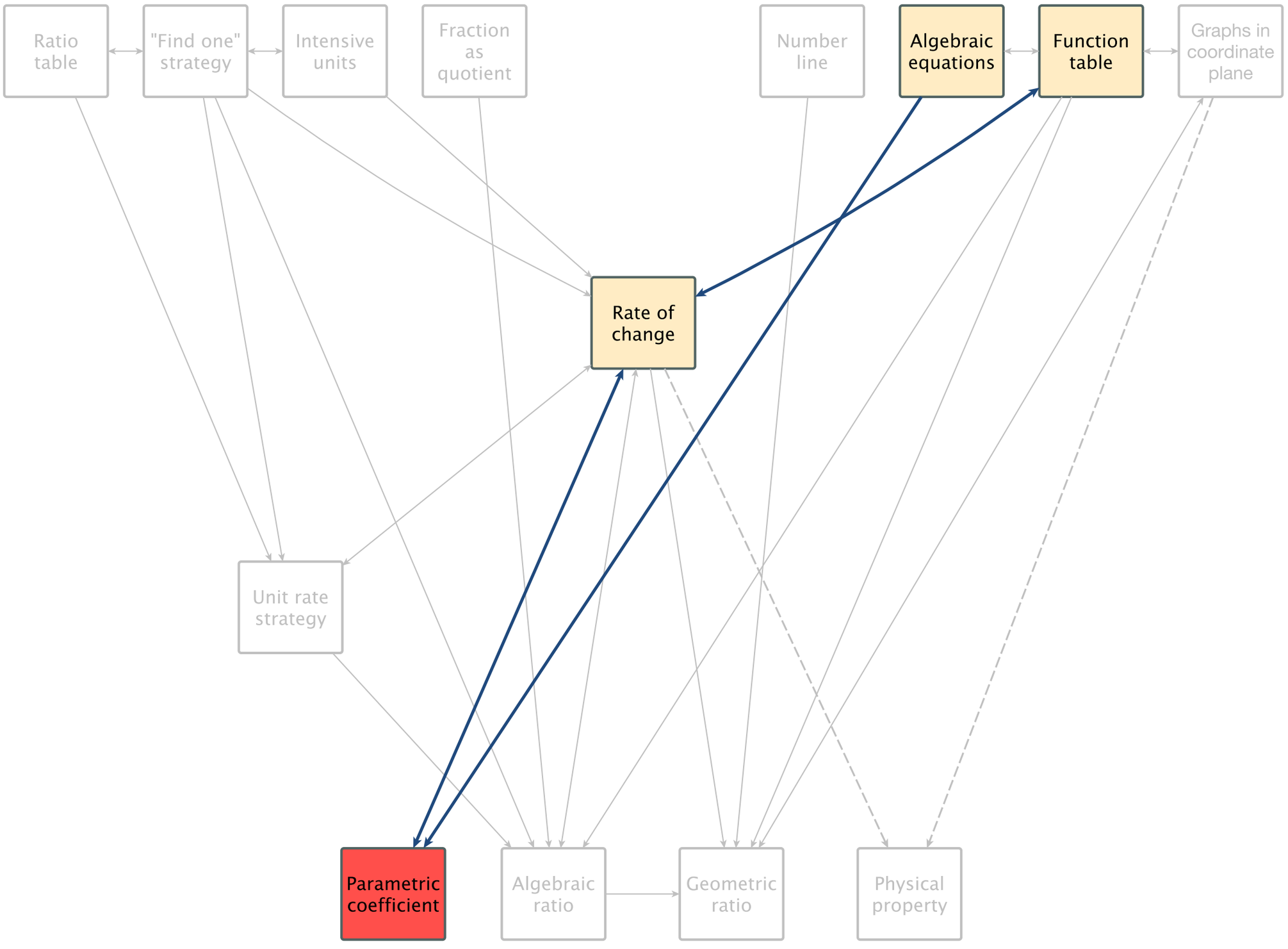
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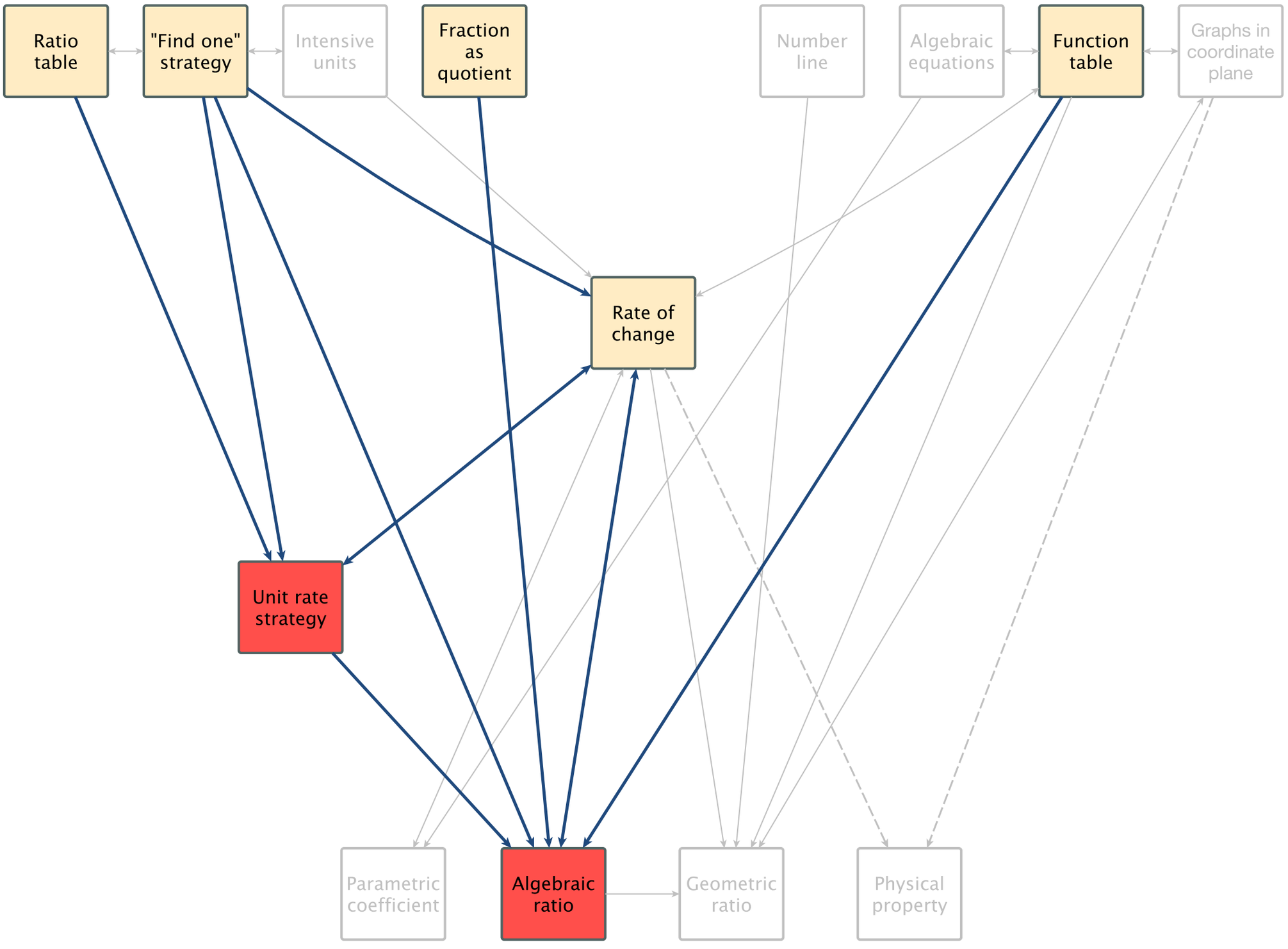
stage 4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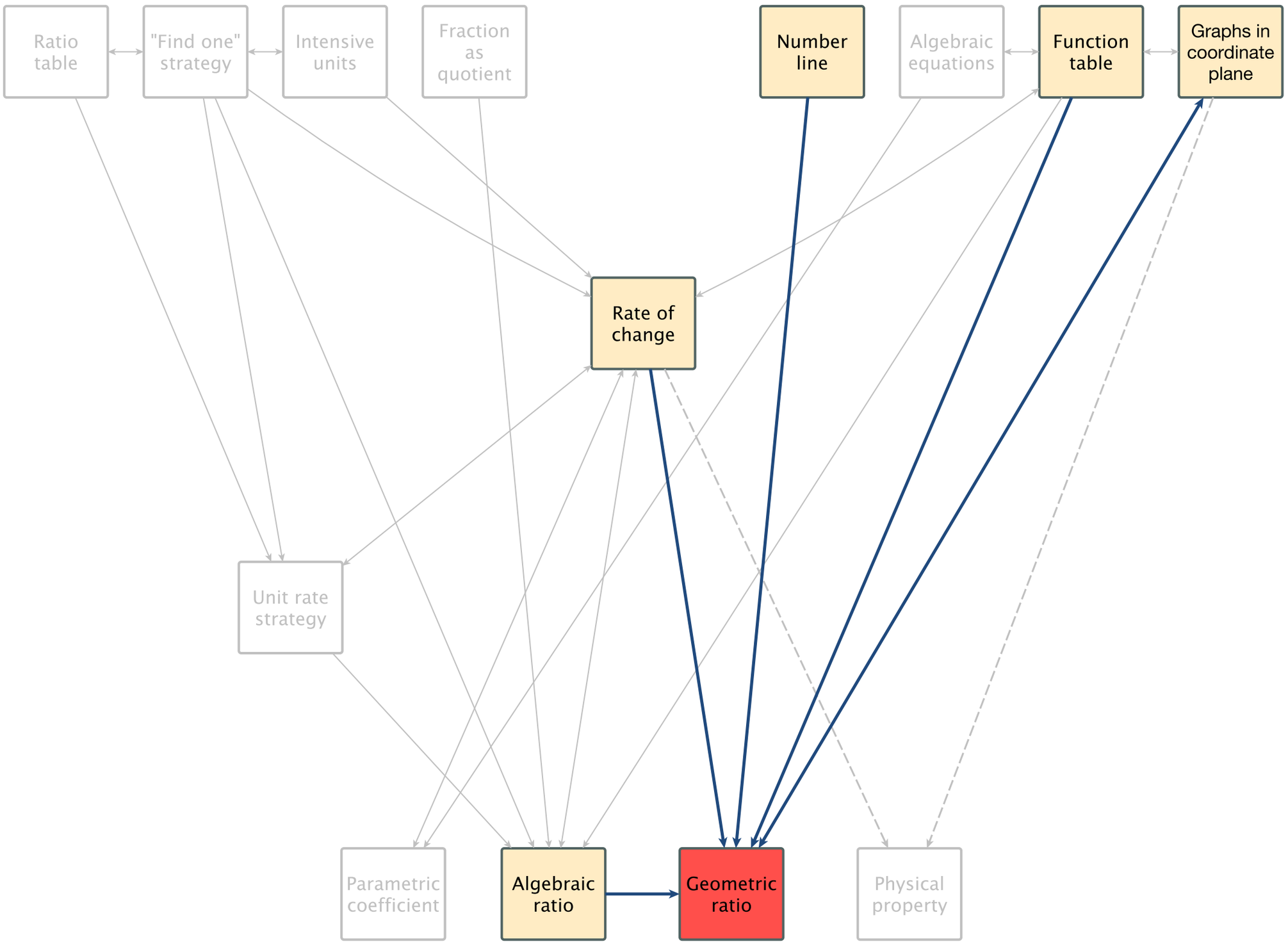
stage 5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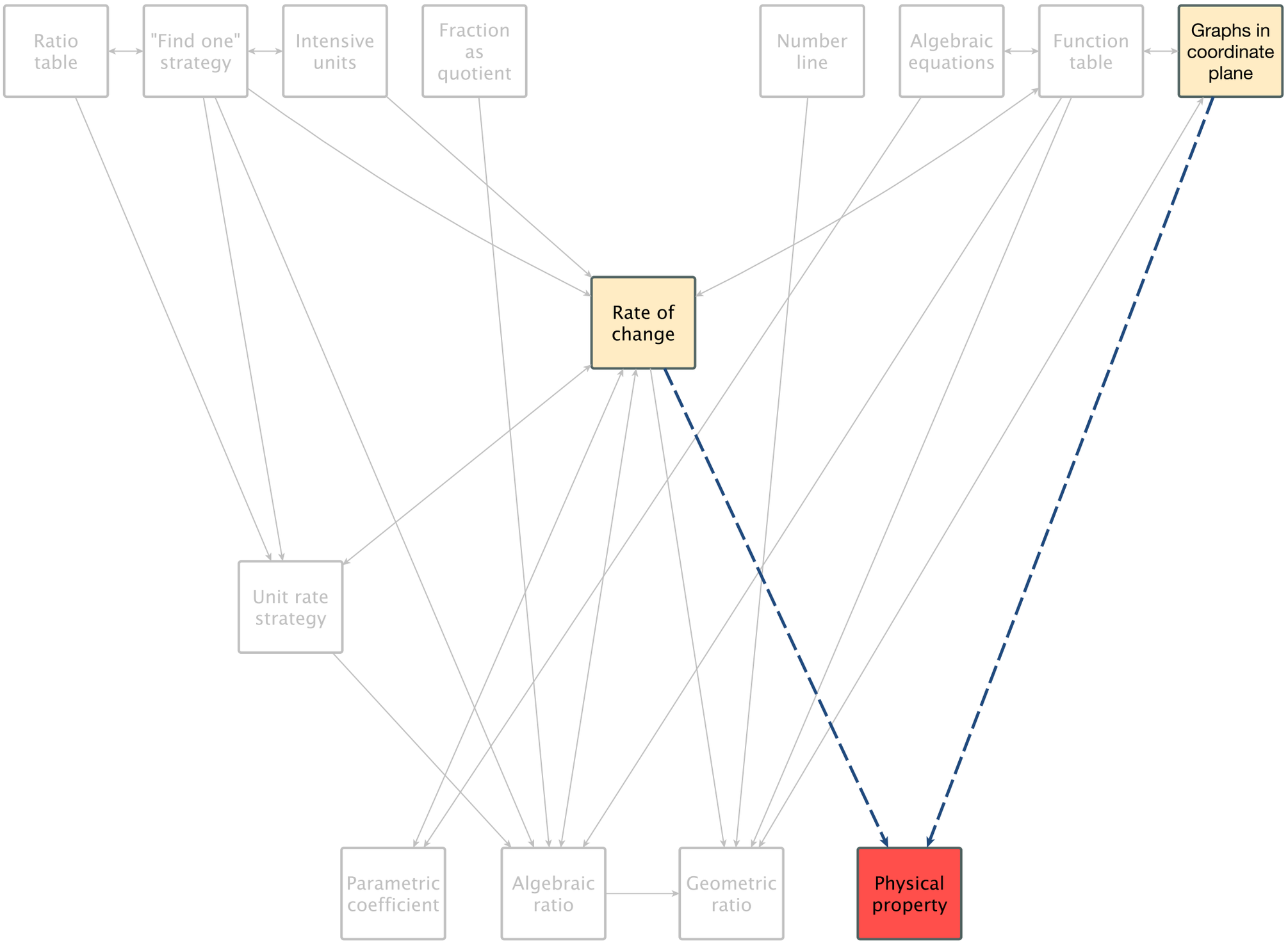
stage 6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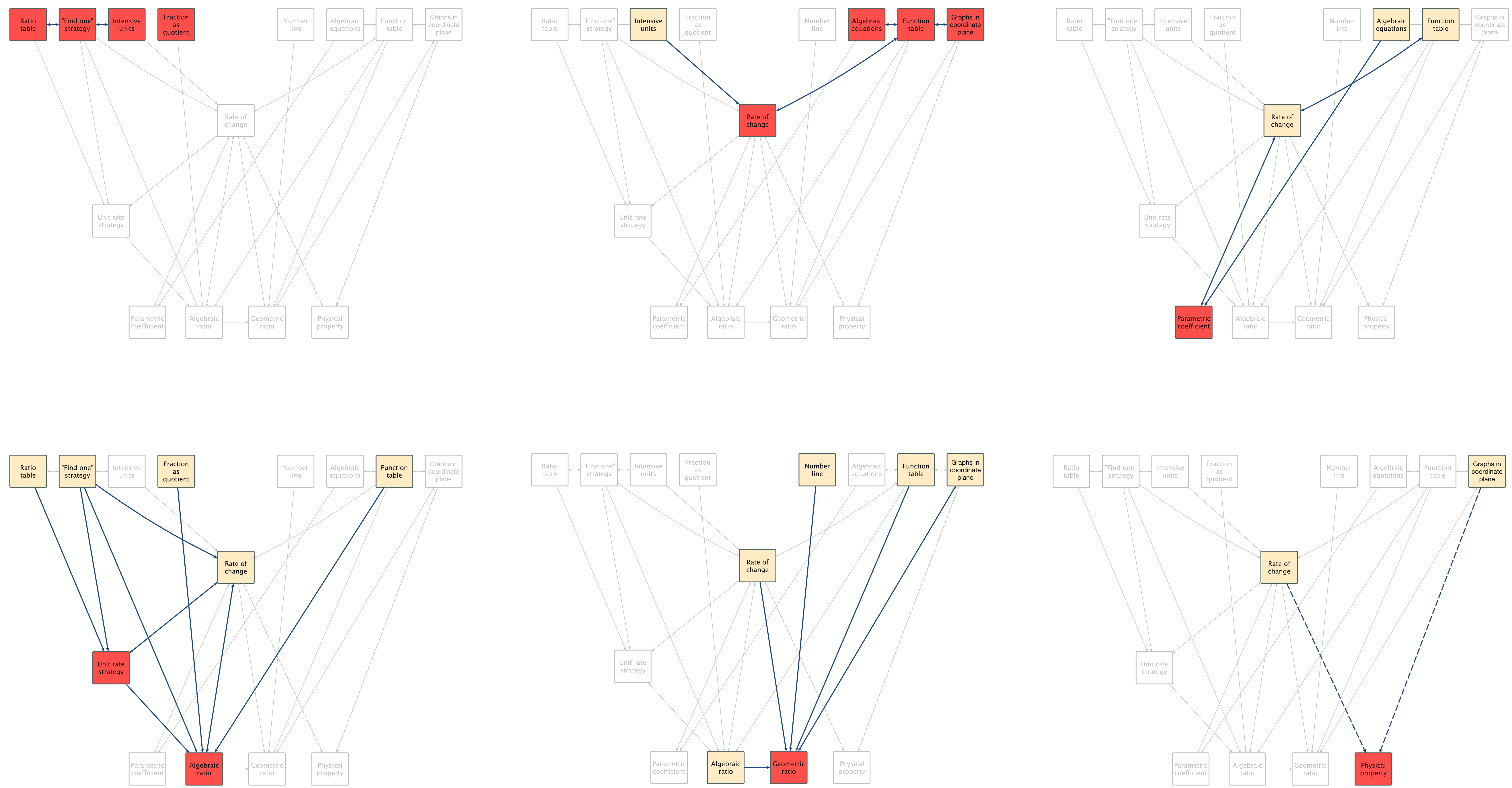
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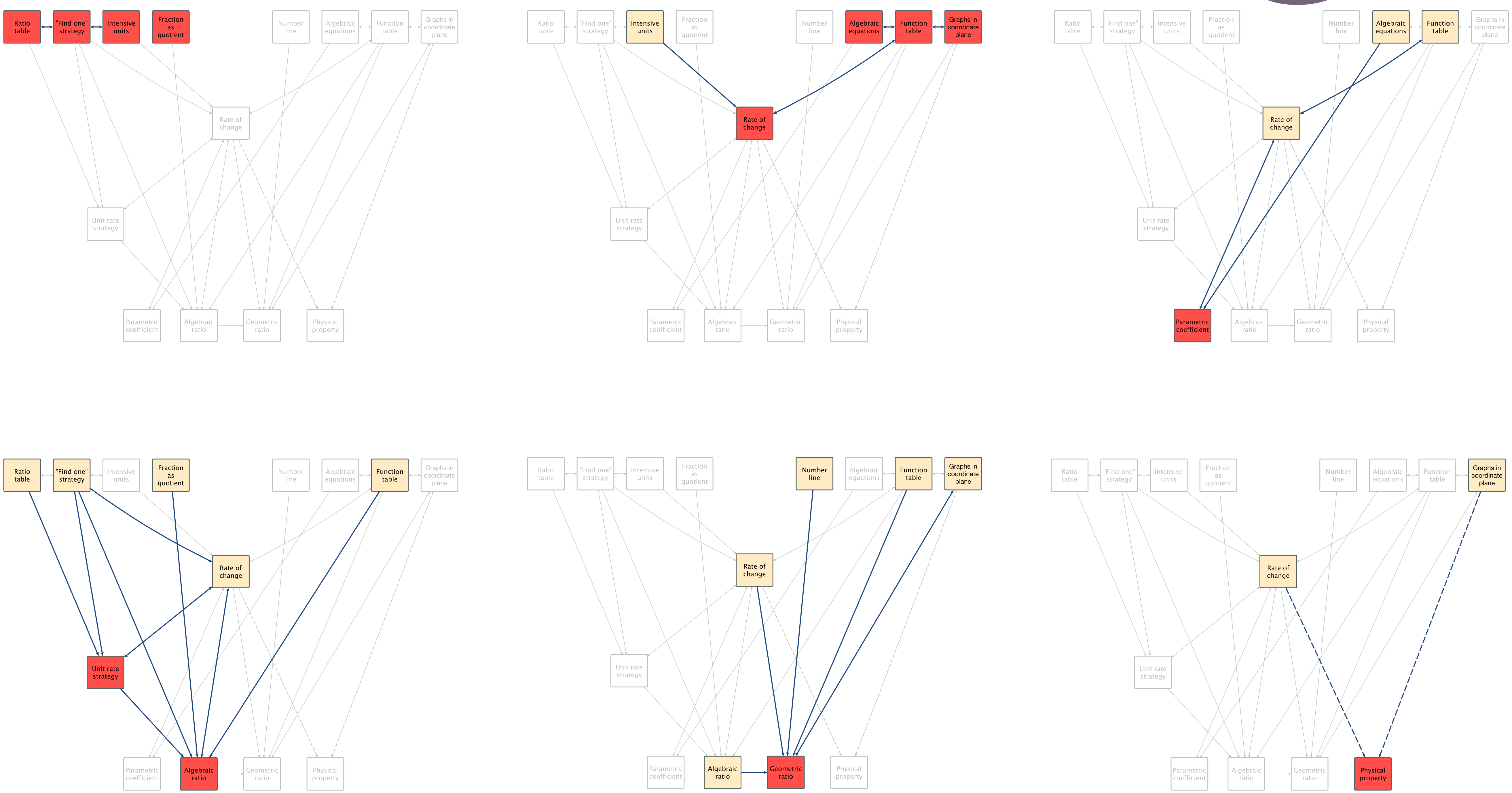
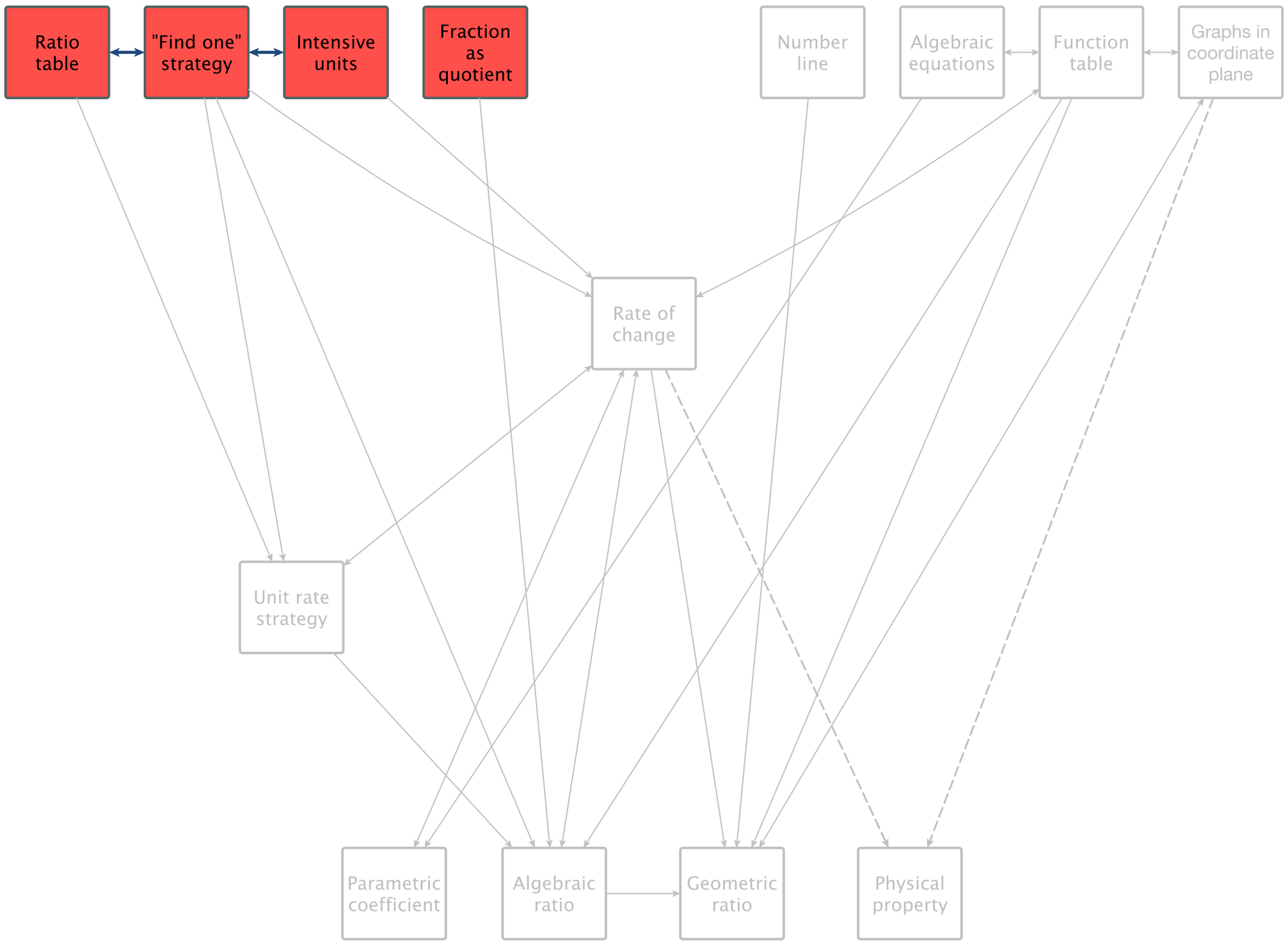
progression of learning



stage 1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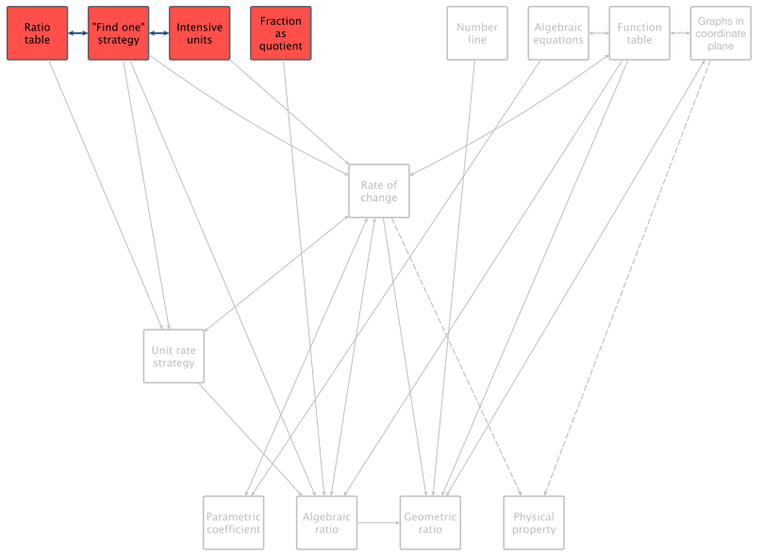
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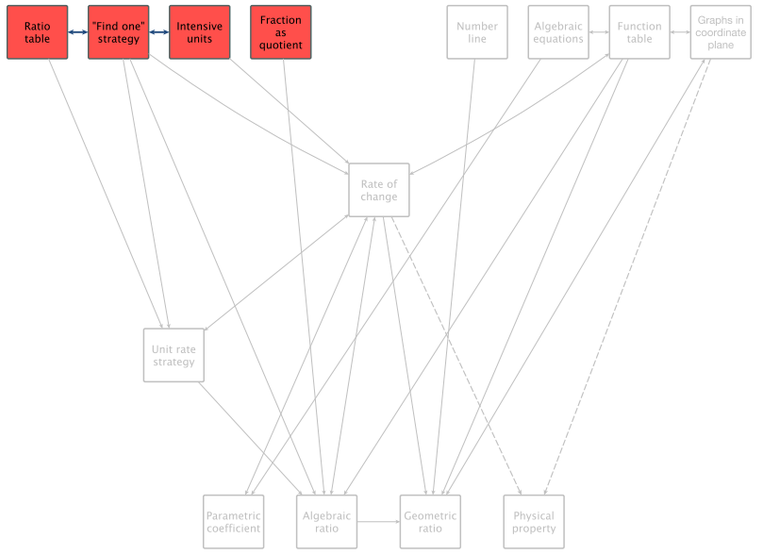
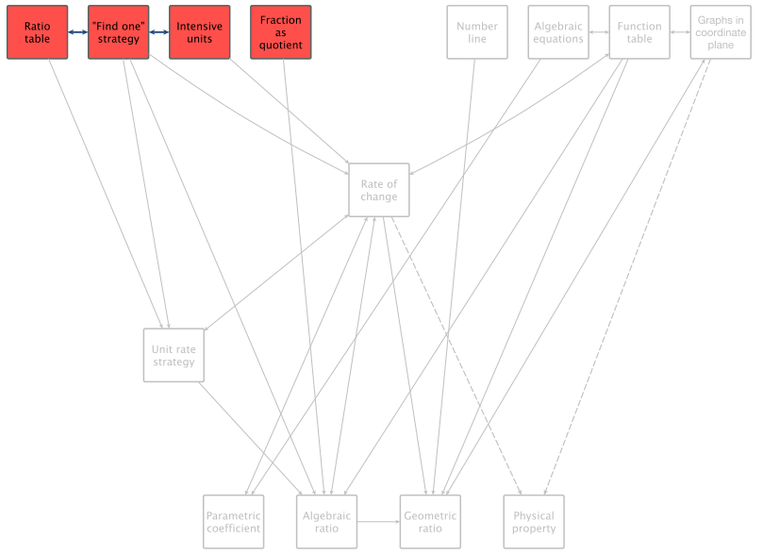


stage 1

Reinvented & objectified

- ratio table
- "find one" strategy
- intensive units (many-to-one)
- fraction-as-quotient



stage 1

Reinvented & objectified

- ratio table
- "find one" strategy
- intensive units (many-to-one)
- fraction-as-quotient

Activities

"partitive division" situations

- fair sharing
- find unit values given many-to-many

stage 1

7 tomatoes weigh **3** pounds

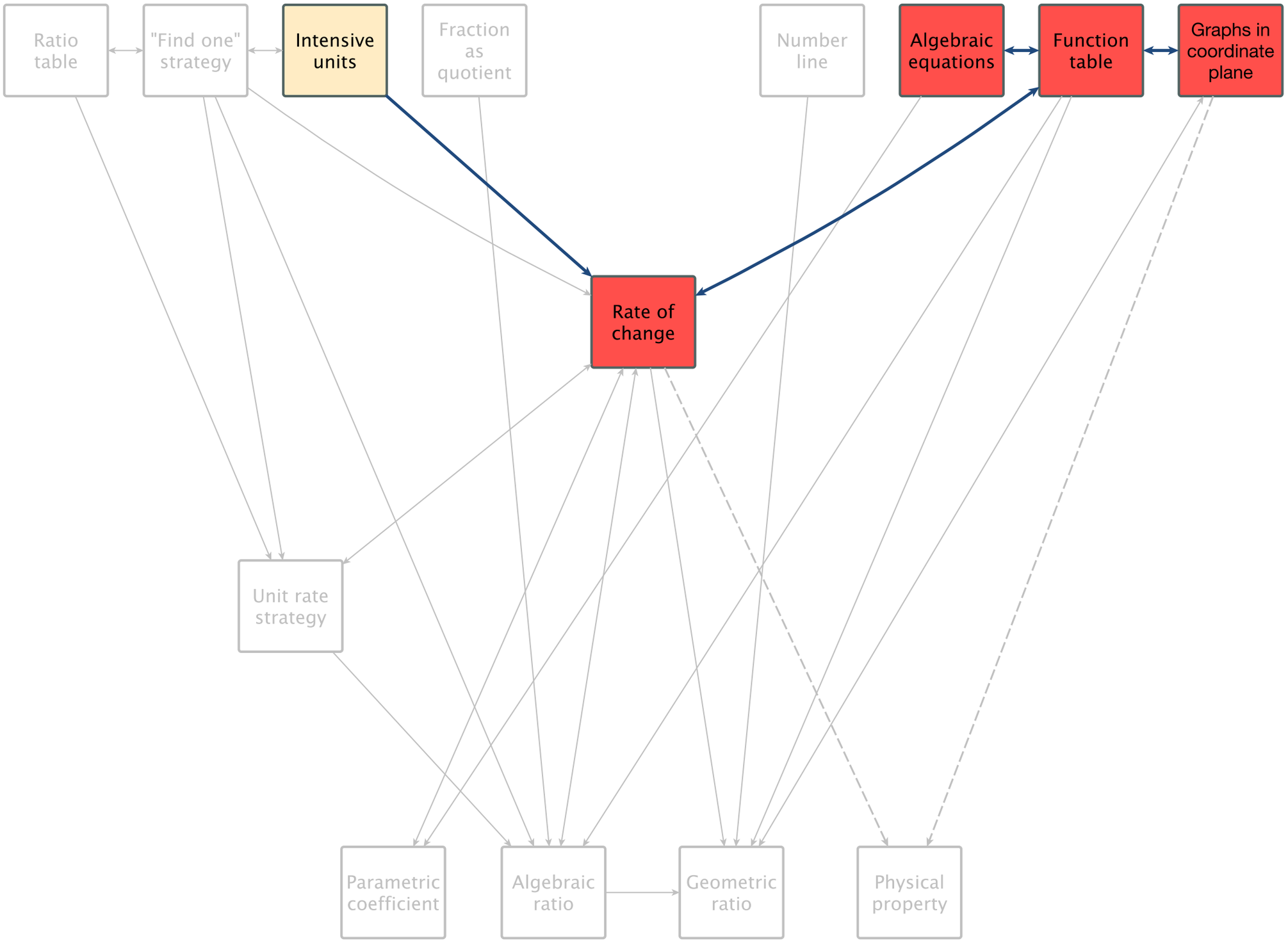
1 tomato weighs _____ pounds

Show your work or explain your reasoning:

State your final answer using units: _____ *per* _____.

Activities
- fair sharing
- find unit values given many-to-many

stage 2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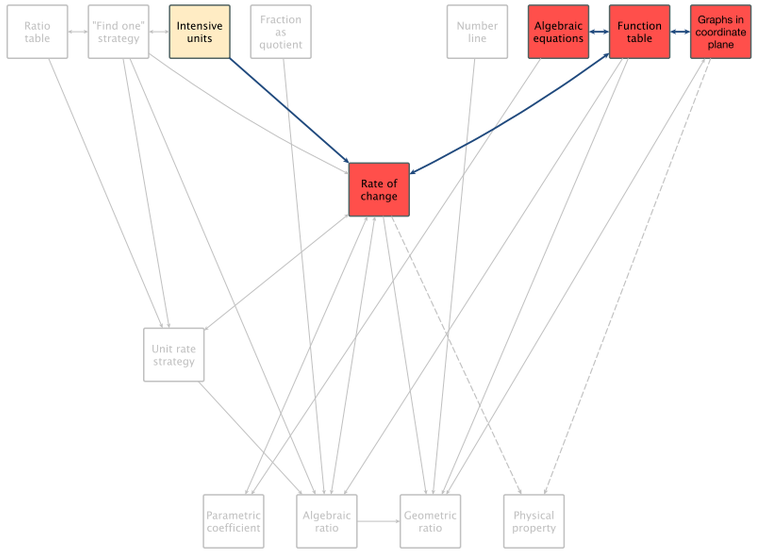
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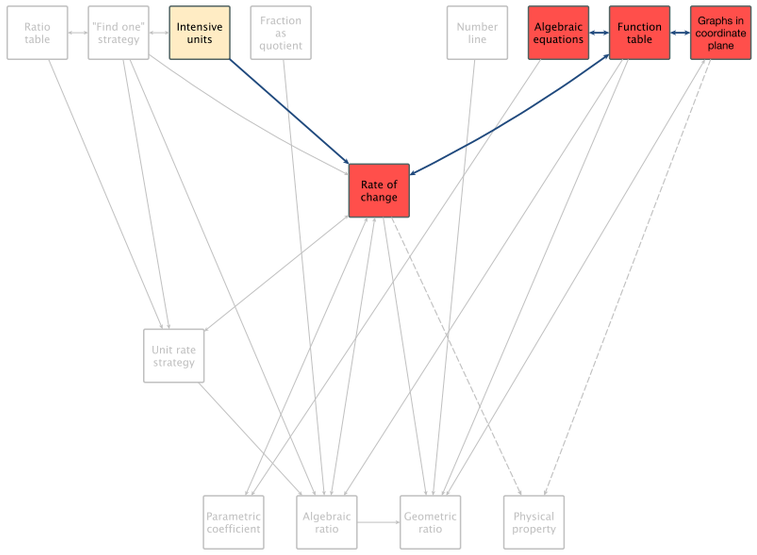
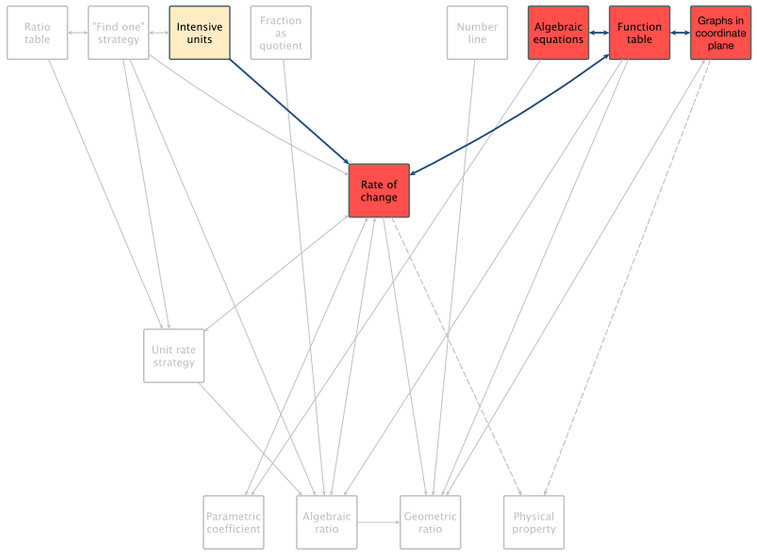


stage 2

Reinvented & objectified

- function tables
- algebraic equations
- graphs in coord. plane
- rate of change



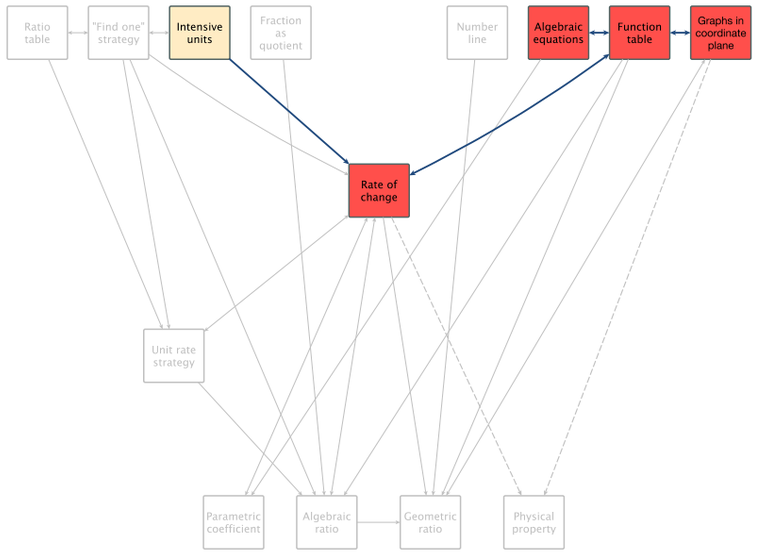
stage 2

Reinvented & objectified

- function tables
- algebraic equations
- graphs in coord. plane
- rate of change

Assembled & coordinated

- intensive units



stage 2

Reinvented & objectified

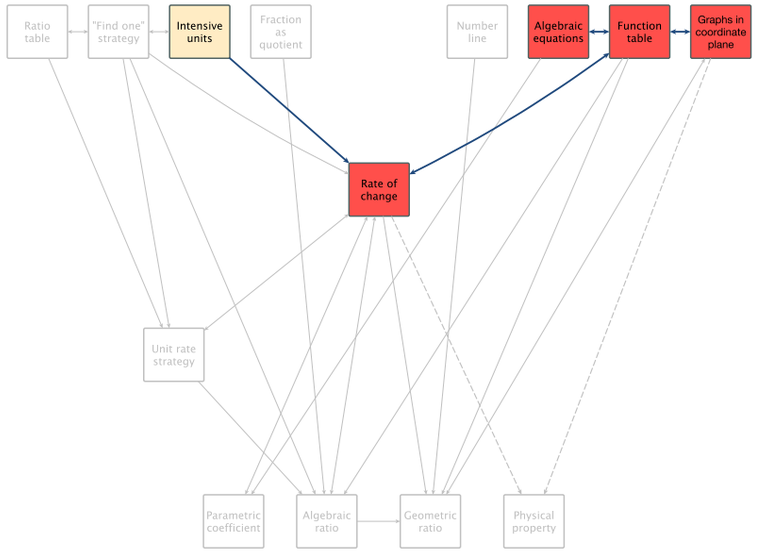
- function tables
- algebraic equations
- graphs in coord. plane
- rate of change

Assembled & coordinated

- intensive units

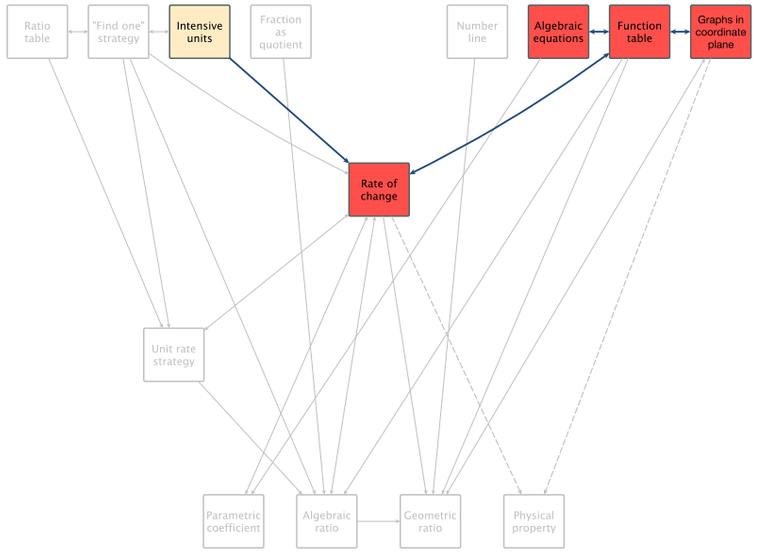
Activities

- find and continue patterns
- convert between multiple representations of functions



stage 2

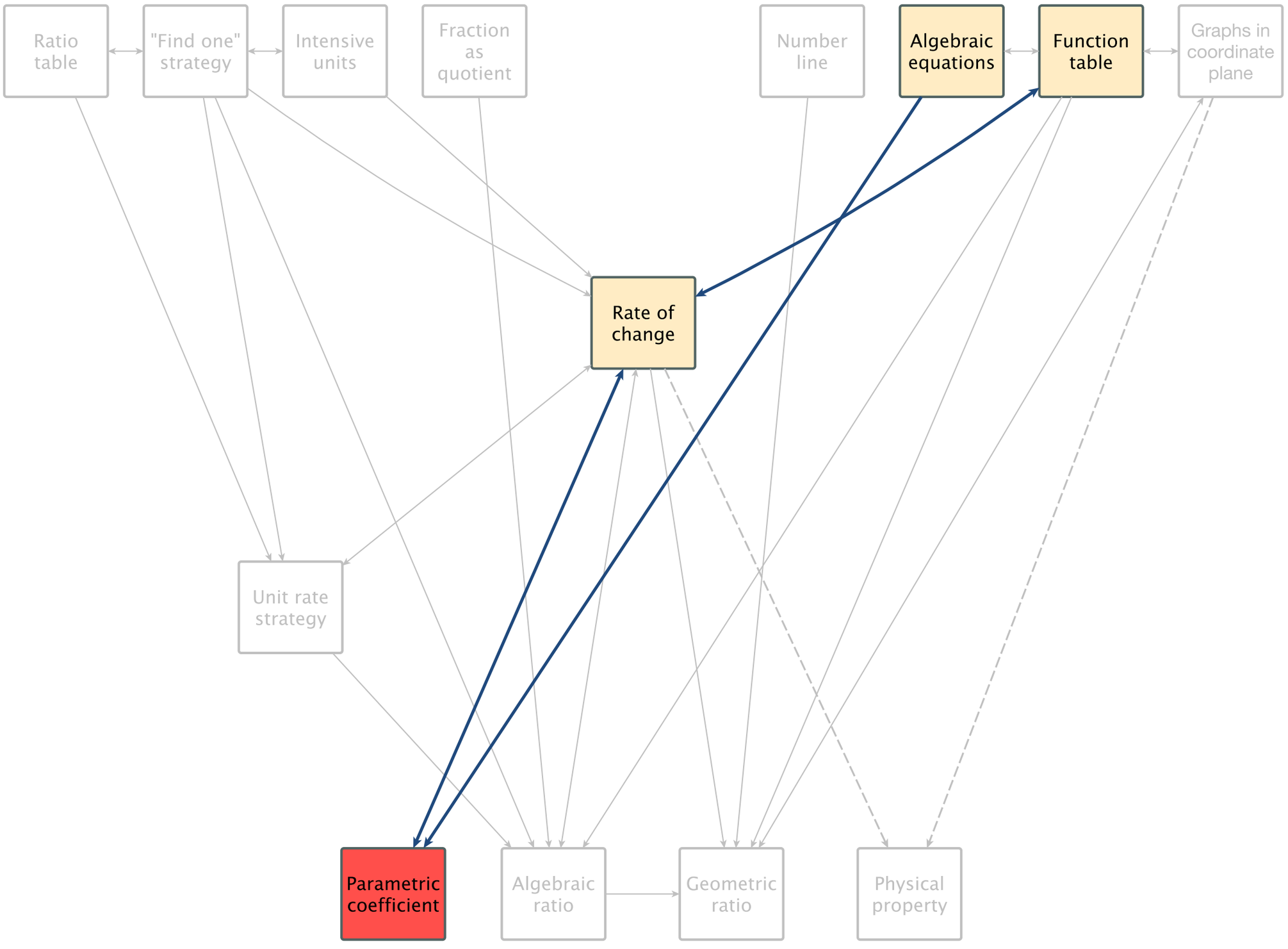
- function tables
- algebraic equations
- graphs in coord. plane
- rate of change



stage 2

- function tables
- algebraic equations
- graphs in coord. plane
- rate of change
  - "the amount that the output changes by when the input increases by 1"
  - "exchanger"

stage 3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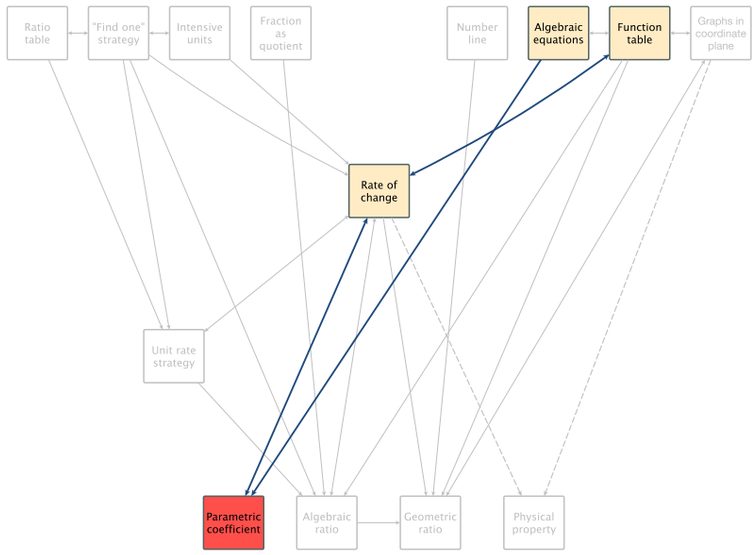
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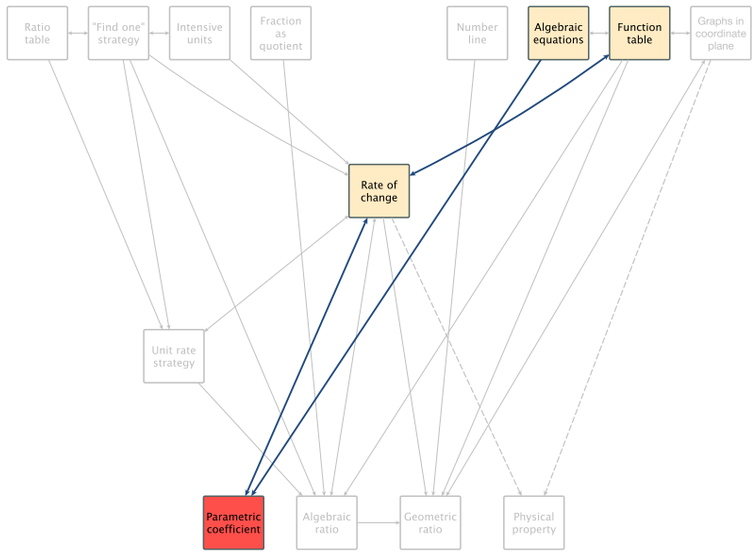
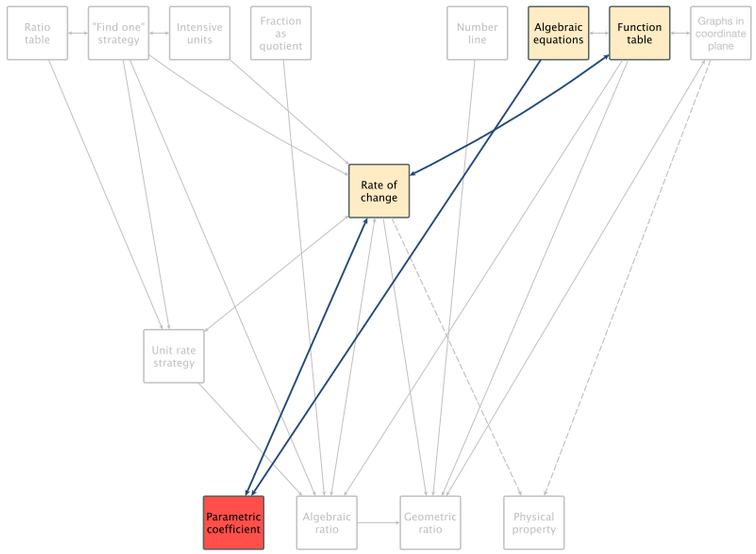


stage 3

Reinvented & objectified {

- parametric coefficient



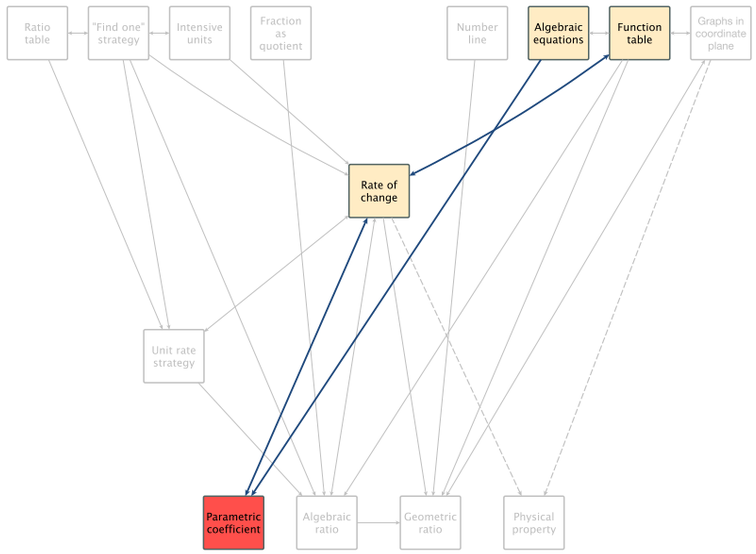
stage 3

Reinvented & objectified

- parametric coefficient

Assembled & coordinated

- algebraic equations
- function tables
- rate of change



stage 3

Reinvented & objectified

- parametric coefficient

Assembled & coordinated

- algebraic equations
- function tables
- rate of change

Activities

make predictions given:

- rate and start
- well-ordered function table ($\triangle x = 1$)

Objectifying rate in a prediction

Monday, August 04, 2008, 07:00 am PT (10:00 am ET)

+ A -

Apple already building iPhones at rate of 40 million a year?

By Slash Lane

Apple is reportedly testing the limits of its overseas manufacturing facilities in order to keep up with demand for the new iPhone 3G, with production already cranked nearly sevenfold compared to the first-generation model.

Foxconn, the company's Taiwanese handset and iPod manufacturer, has recently ramped production of the new iPhone to 800,000 units per week, says *TechCrunch*, citing a person "close to Apple with direct knowledge of the numbers."

The build rate is said to be "above current full capacity" for the Foxconn facilities alloted to Apple's handset business, which has led to concerns that quality control may suffer. At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months.

That paces well ahead of analysts' estimates (1, 2, 3) and early reports that suggested Apple's initial iPhone 3G orders spanned only 25 million units through the expected lifespan of the product.

TechCrunch believes Apple's initial order was actually 40 million units over the course of the first twelve months, but is now hearing that "those numbers are being revised upwards sharply."

Apple said it sold 1 million iPhones in the first 72 hours the new iPhone 3G was put on sale, but has not provided an updated sales tally since. The iPhone is currently on sale in 23 countries, with 20 more expected to be added on August 22nd, and another 30 by the end of the calendar year.

Objectifying rate in a prediction

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Objectifying rate in a prediction

... At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months ...

Thursday, April 10, 14

... 800,000 units per week ...

... At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months ...

... 800,000 units per week ...

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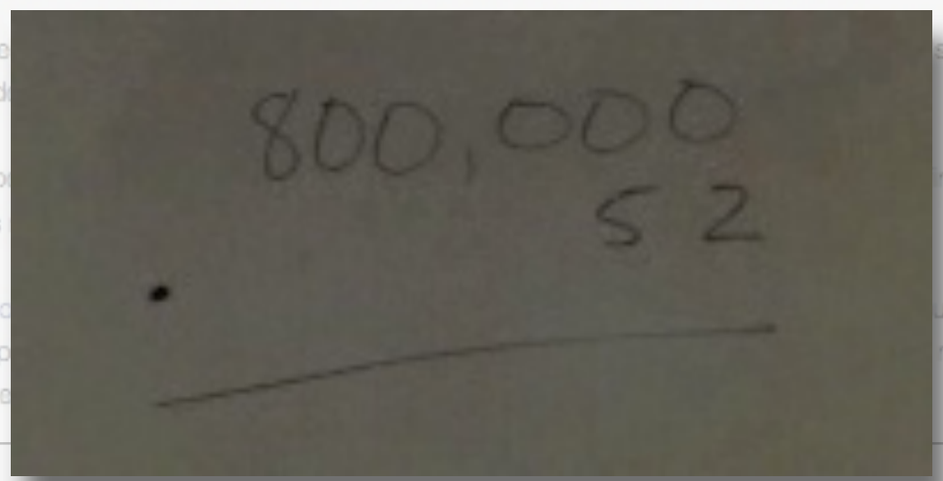

Objectifying rate in a prediction

FAP: Randy why is that [multiplication] going to get us a prediction for the number of iPhones in a year? How does weeks turn into iPhones?

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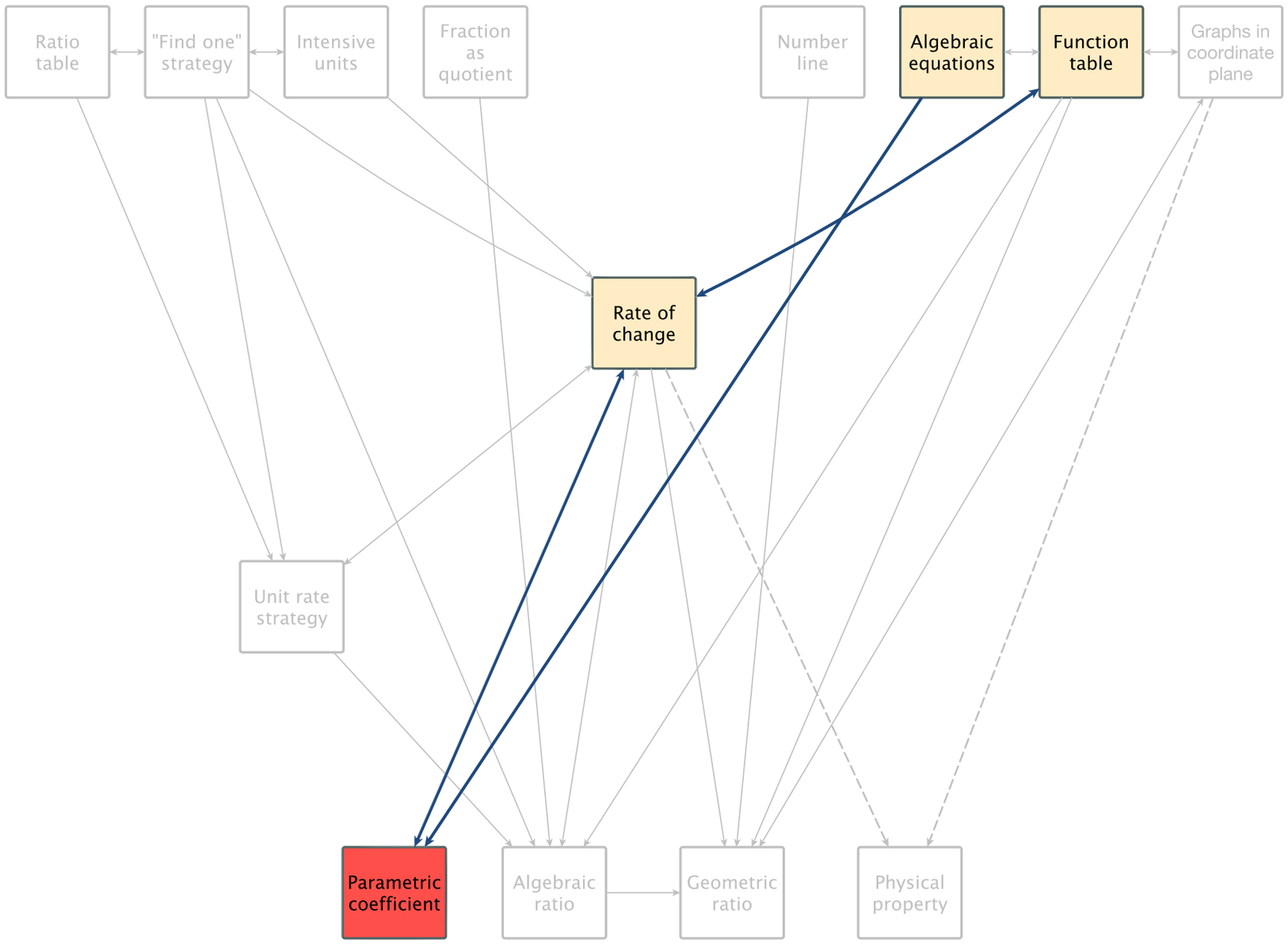
Up and down in the cascade



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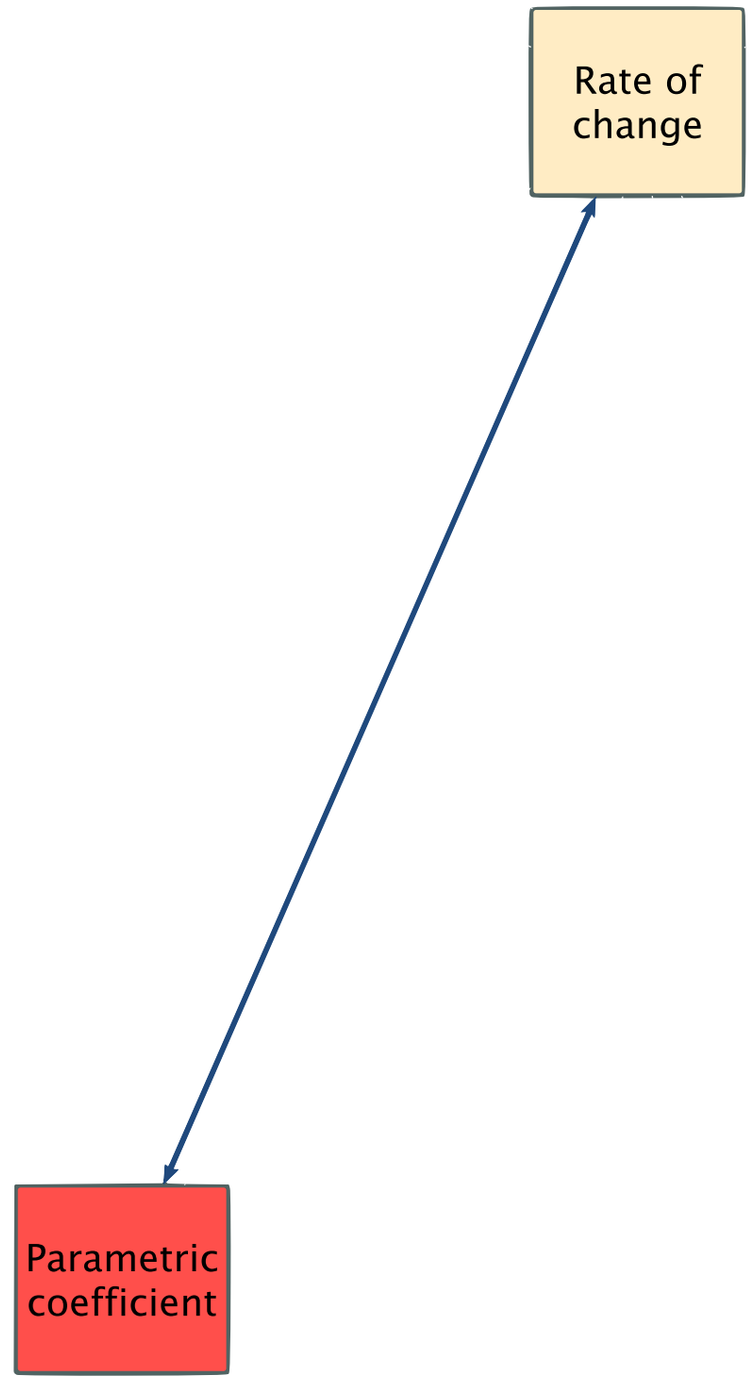

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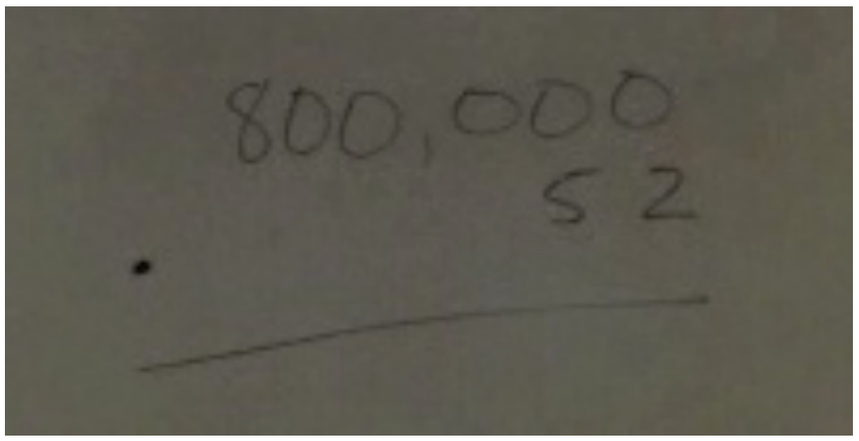
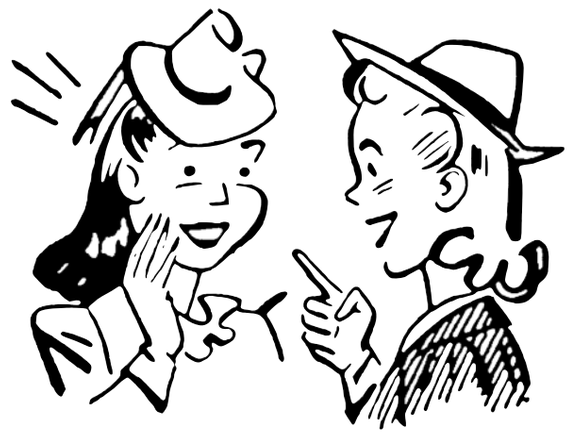
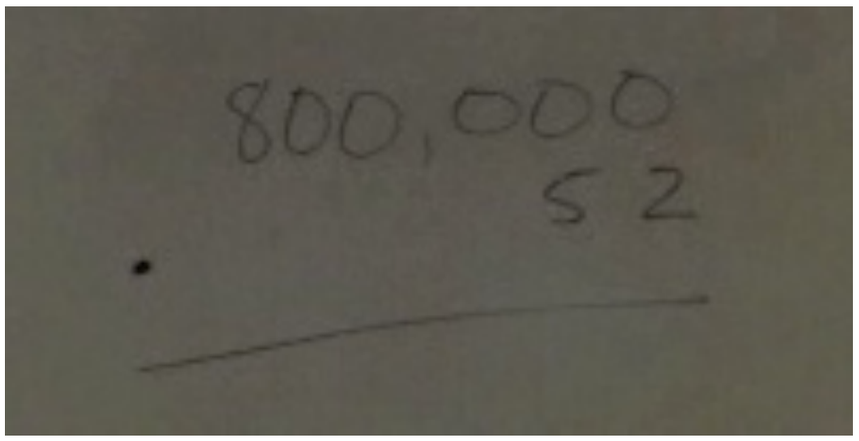


Rate of change



Parametric coefficient

Up and down in the cascade



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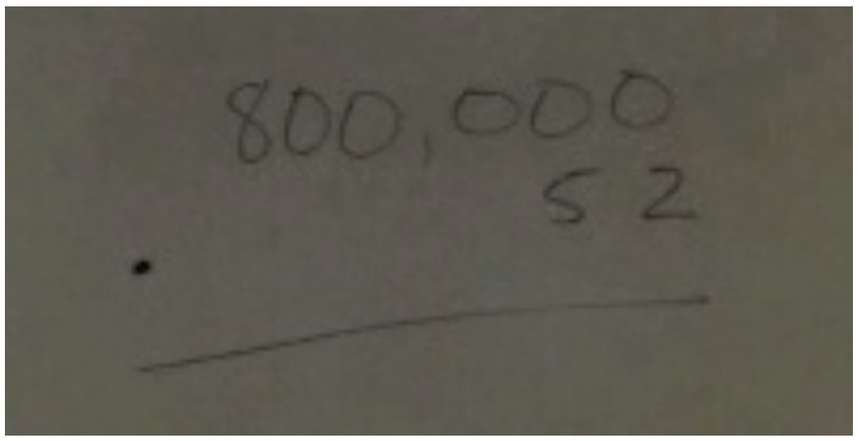
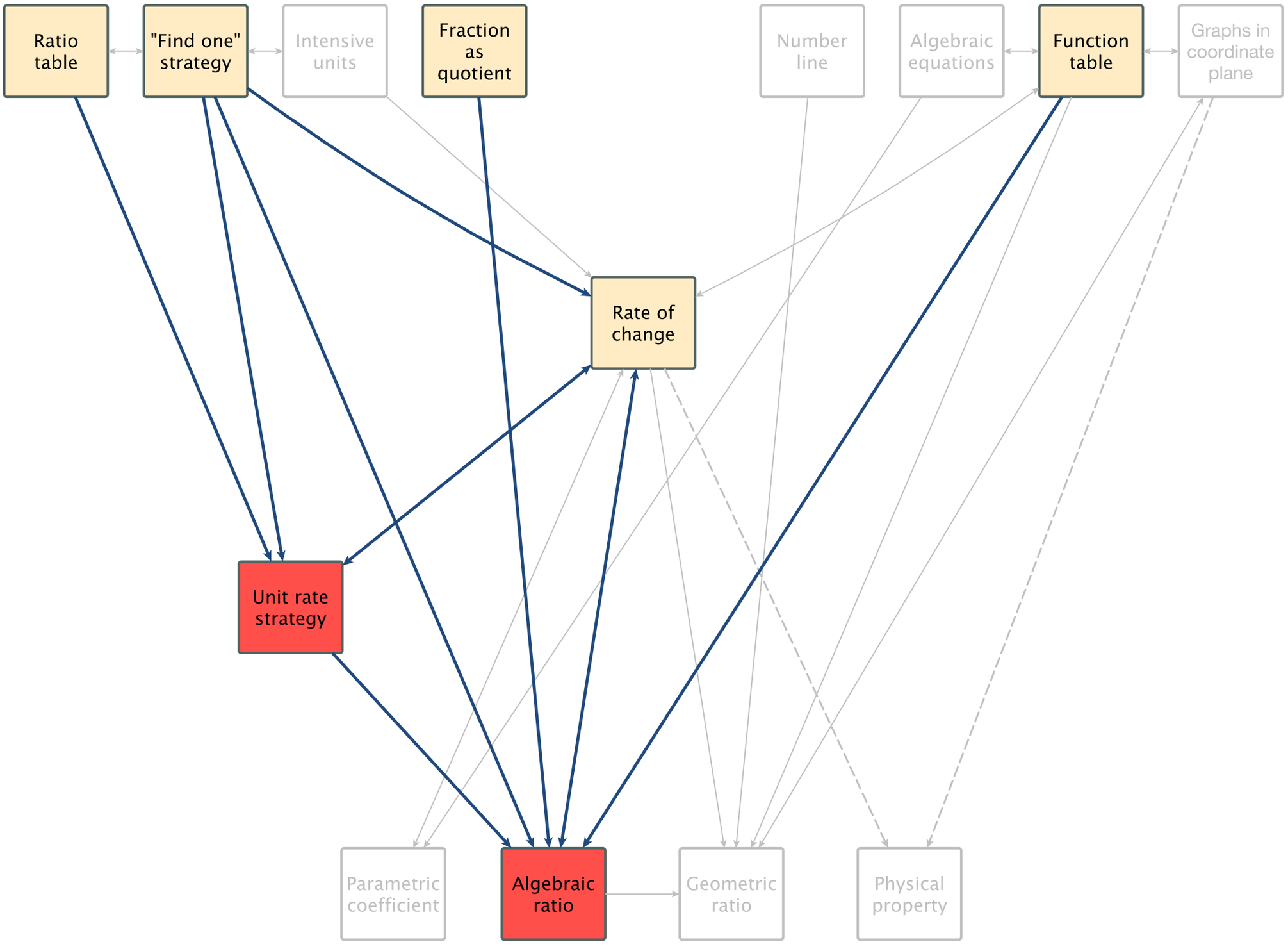


Rate of change

Parametric coefficient

stage 4


Ratio table
"Find one" strategy
Intensive units
Fraction as quotient
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Rate of change
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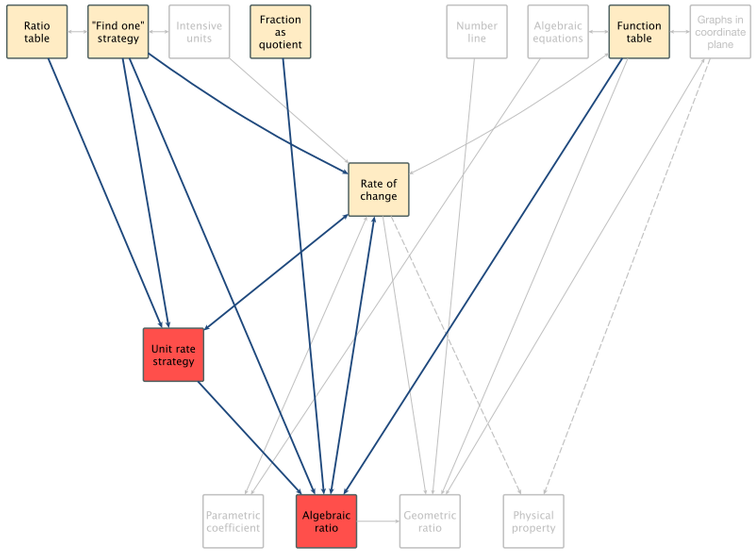
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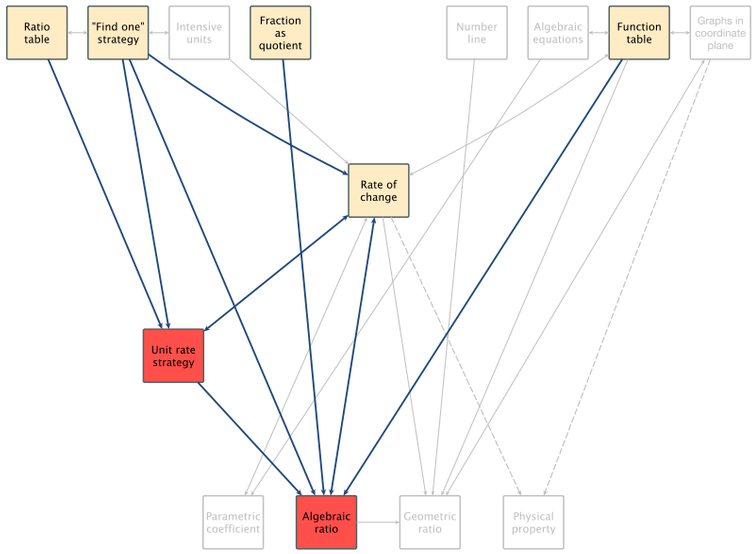
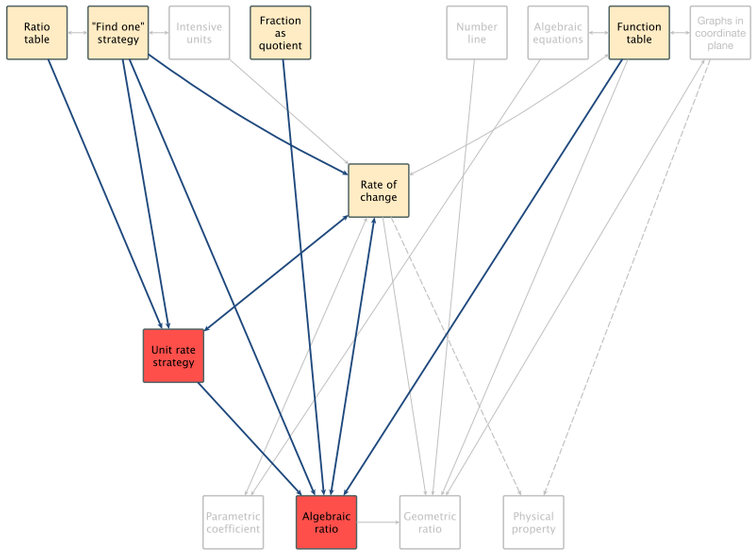


stage 4

Reinvented & objectified

- unit rate strategy
- algebraic ratio



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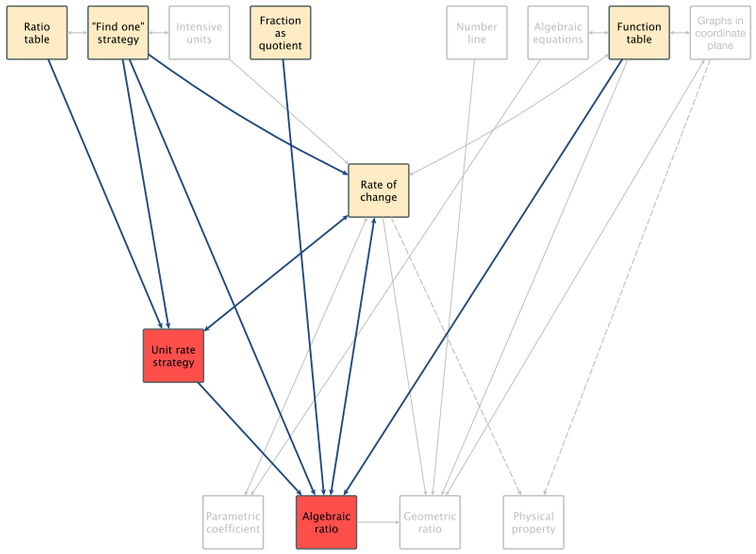
Reinvented & objectified

- unit rate strategy
- algebraic ratio

Assembled & coordinated

- ratio table
- “find one” strategy
- fraction as quotient
- rate of change
- function tables



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Reinvented & objectified

- unit rate strategy
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- ratio table
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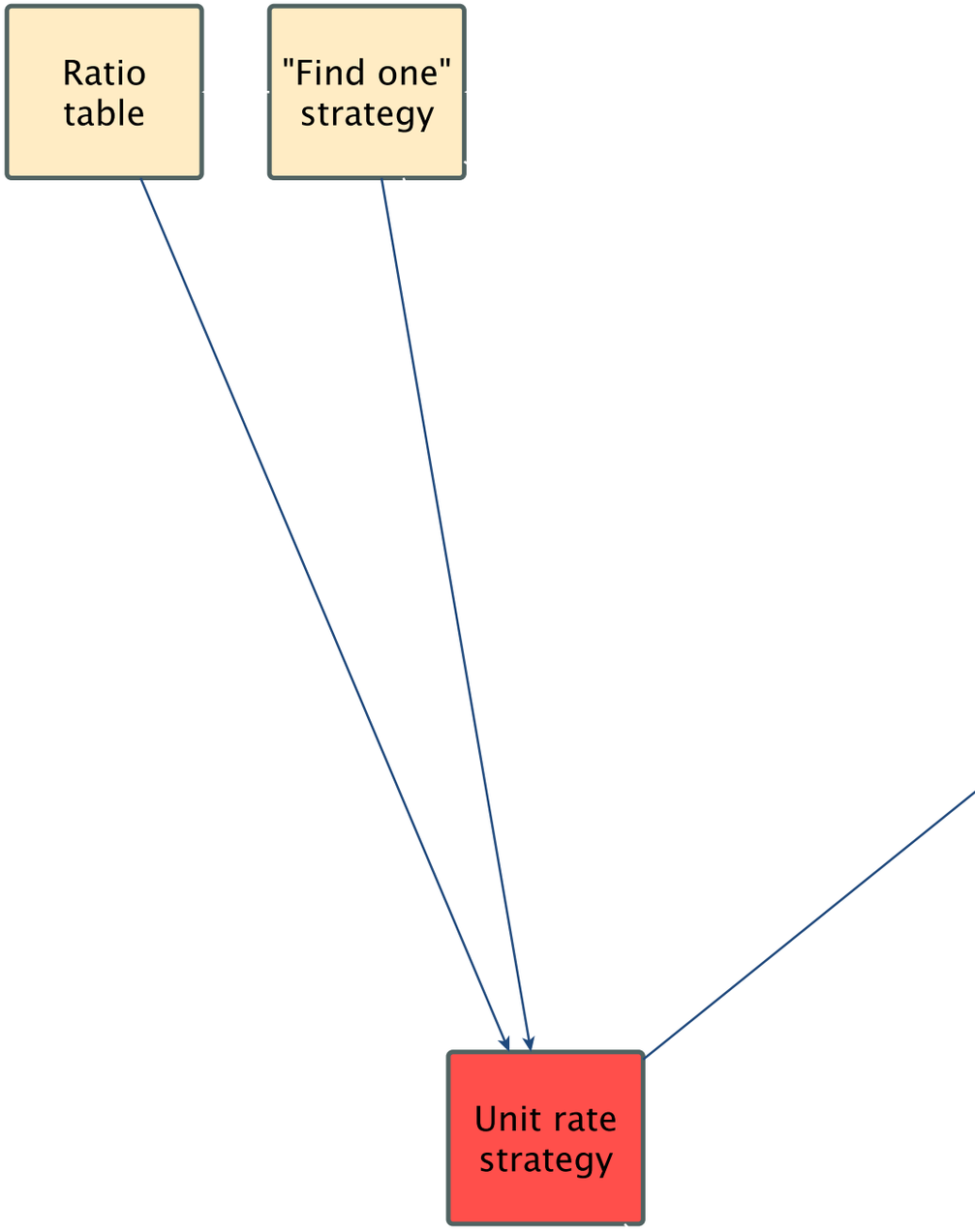
Activities

make predictions given:

- one value in proportional situation
- two data points with $\Delta x \neq 1$

stage 4


Ratio table
"Find one" strategy
Rate of change
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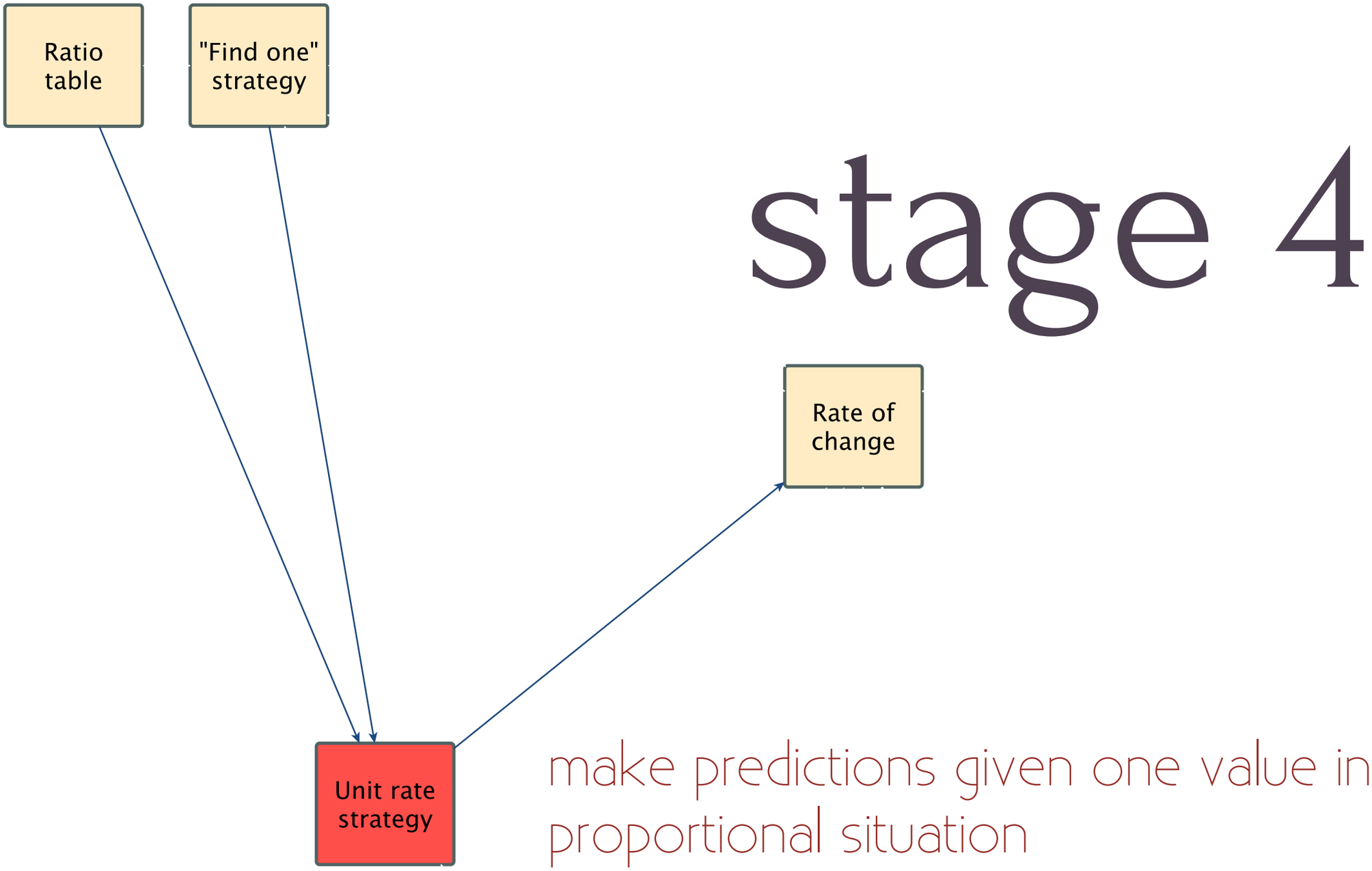
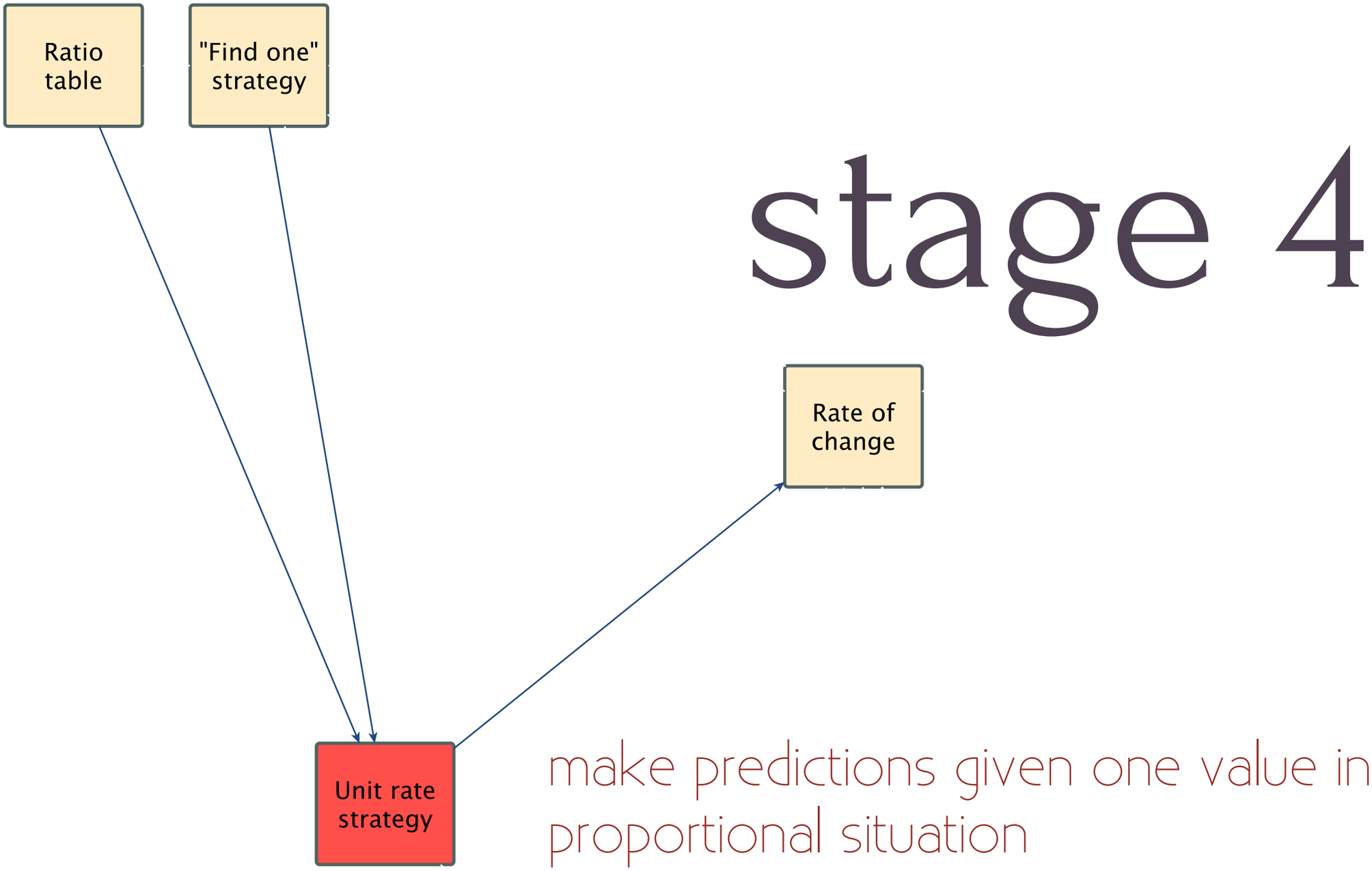
stage 4



make predictions given one value in proportional situation

stage 4



make predictions given one value in proportional situation

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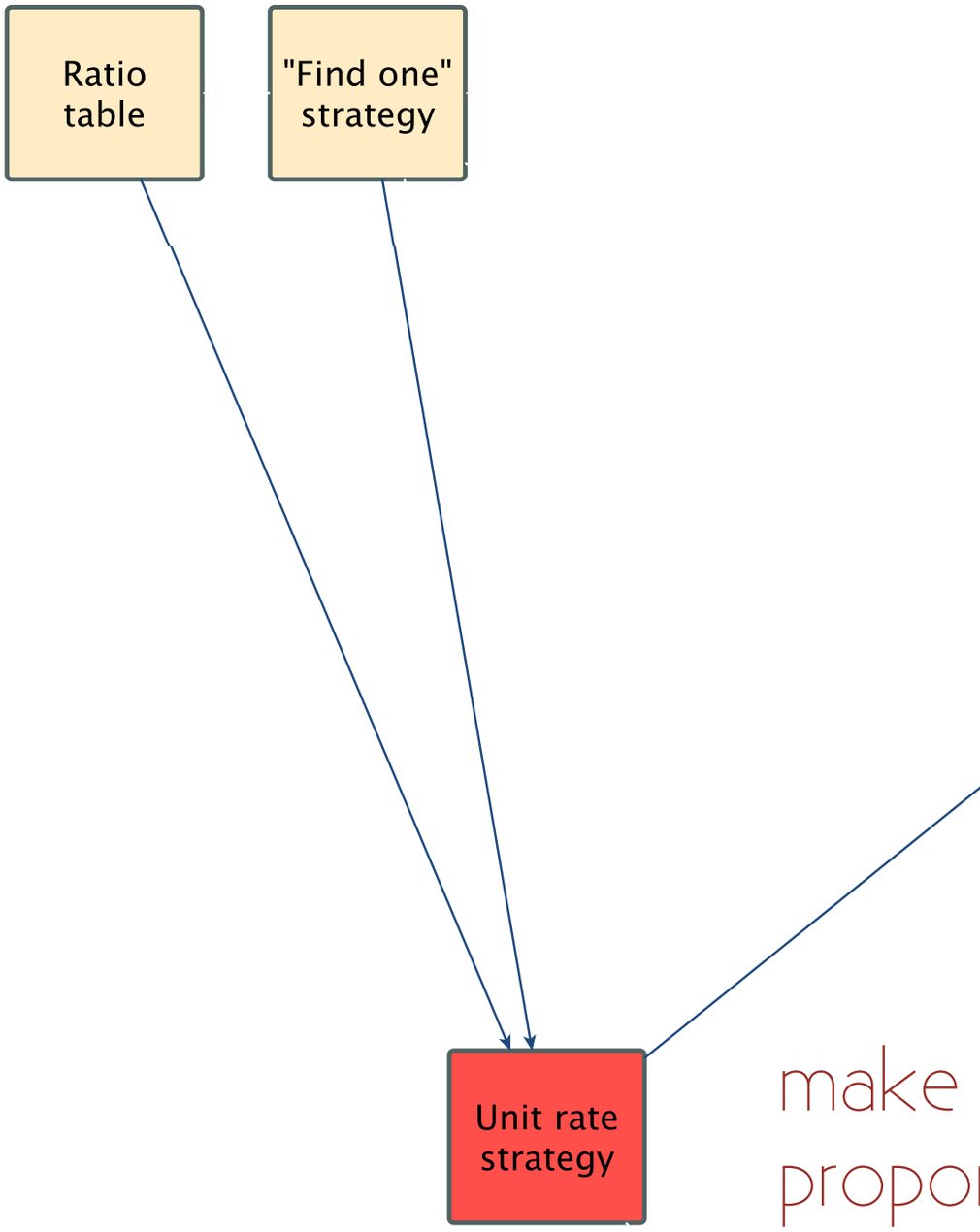

stage 4

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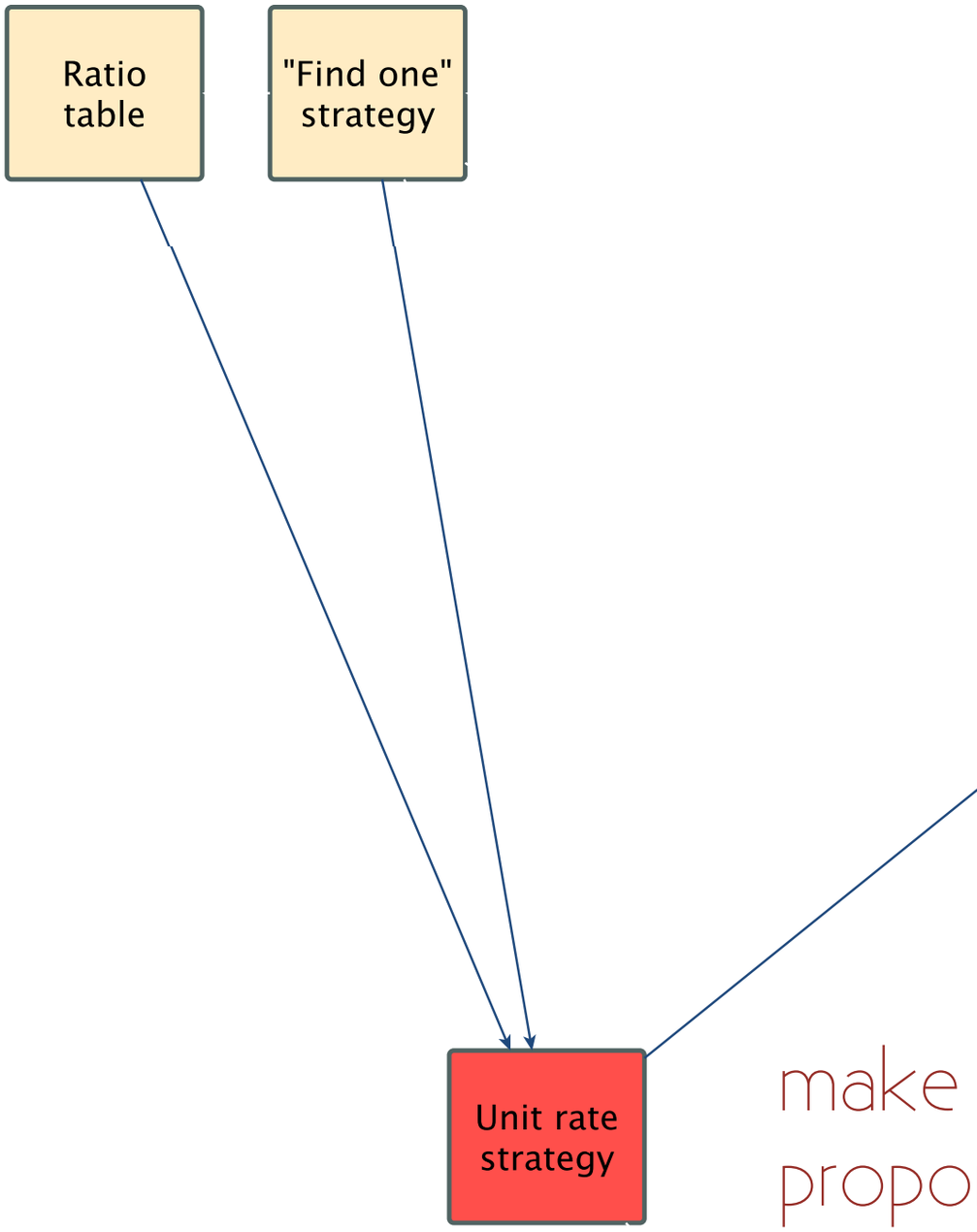
6 ÷ (6 mile > 54 min
1 mile 9 min
1 ÷ 6

9 x 11 = 99.

Takes 99 minutes



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"Find one" strategy

stage 4

Rate of change

Unit rate strategy

make predictions given one value in proportional situation

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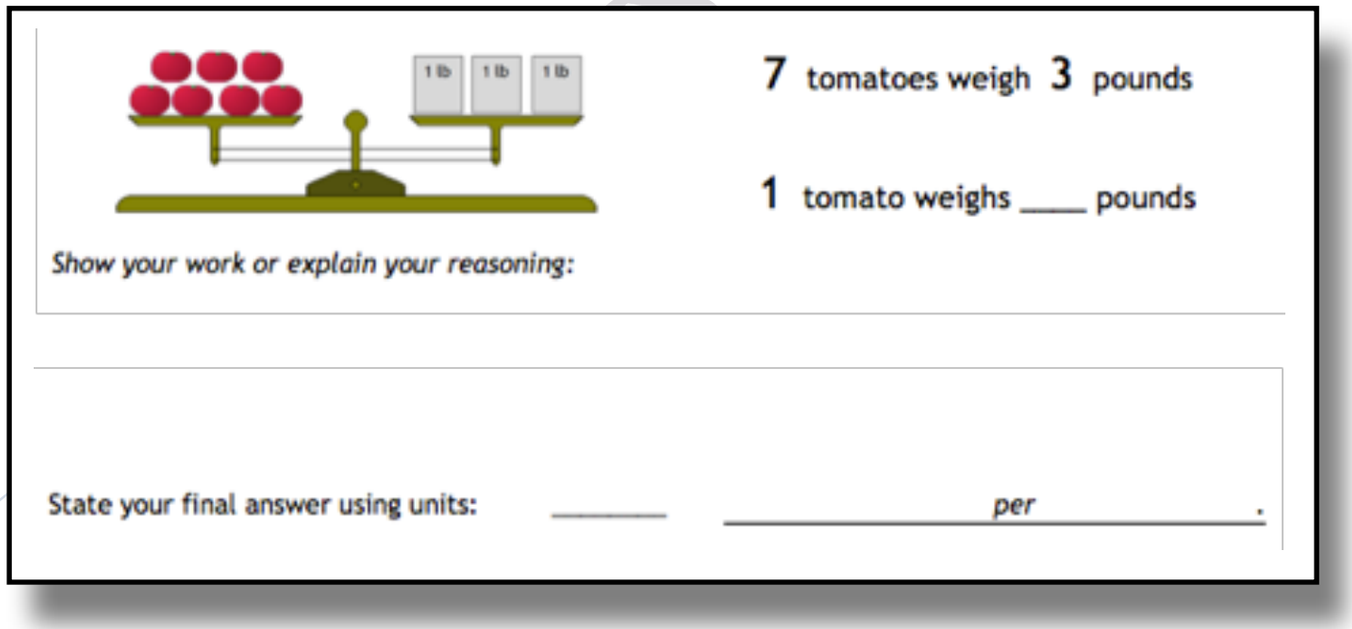
"Find one" strategy

Unit rate strategy

stage 4



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54 ÷ 6 = 9

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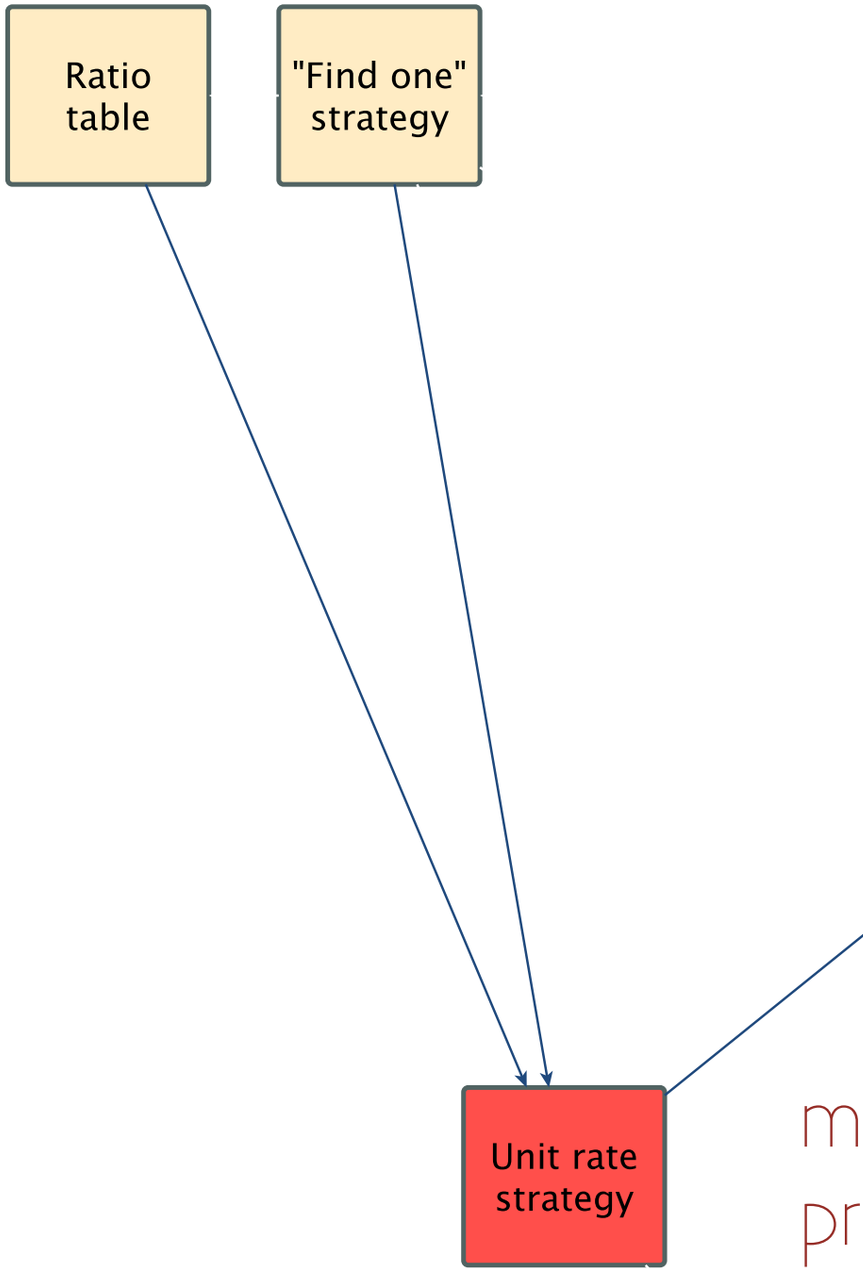
Takes 99 minutes



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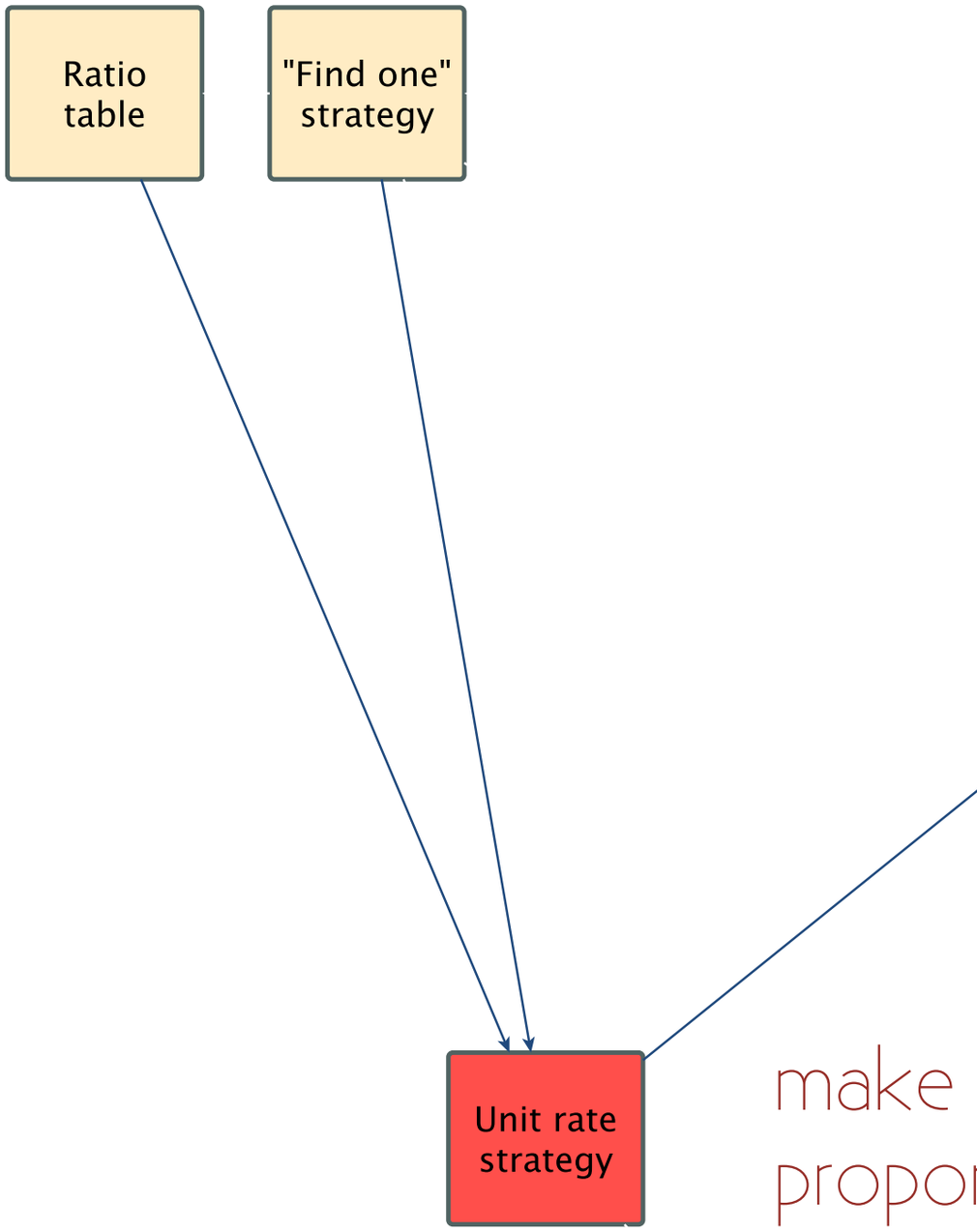

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"Find one" strategy

stage 4

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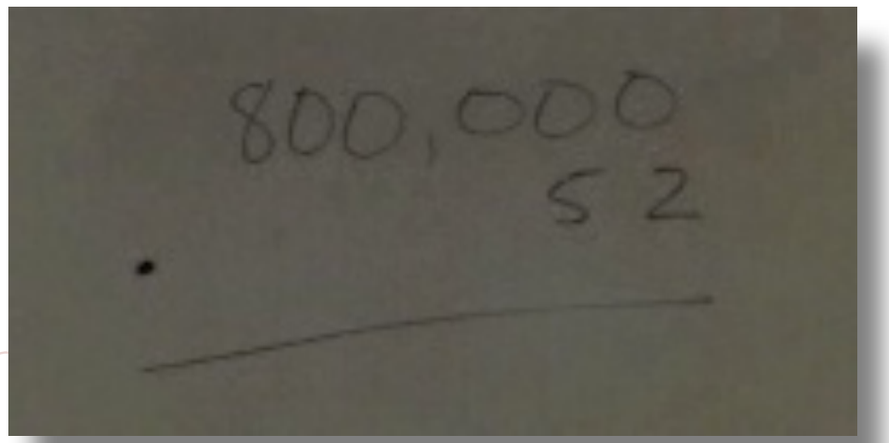

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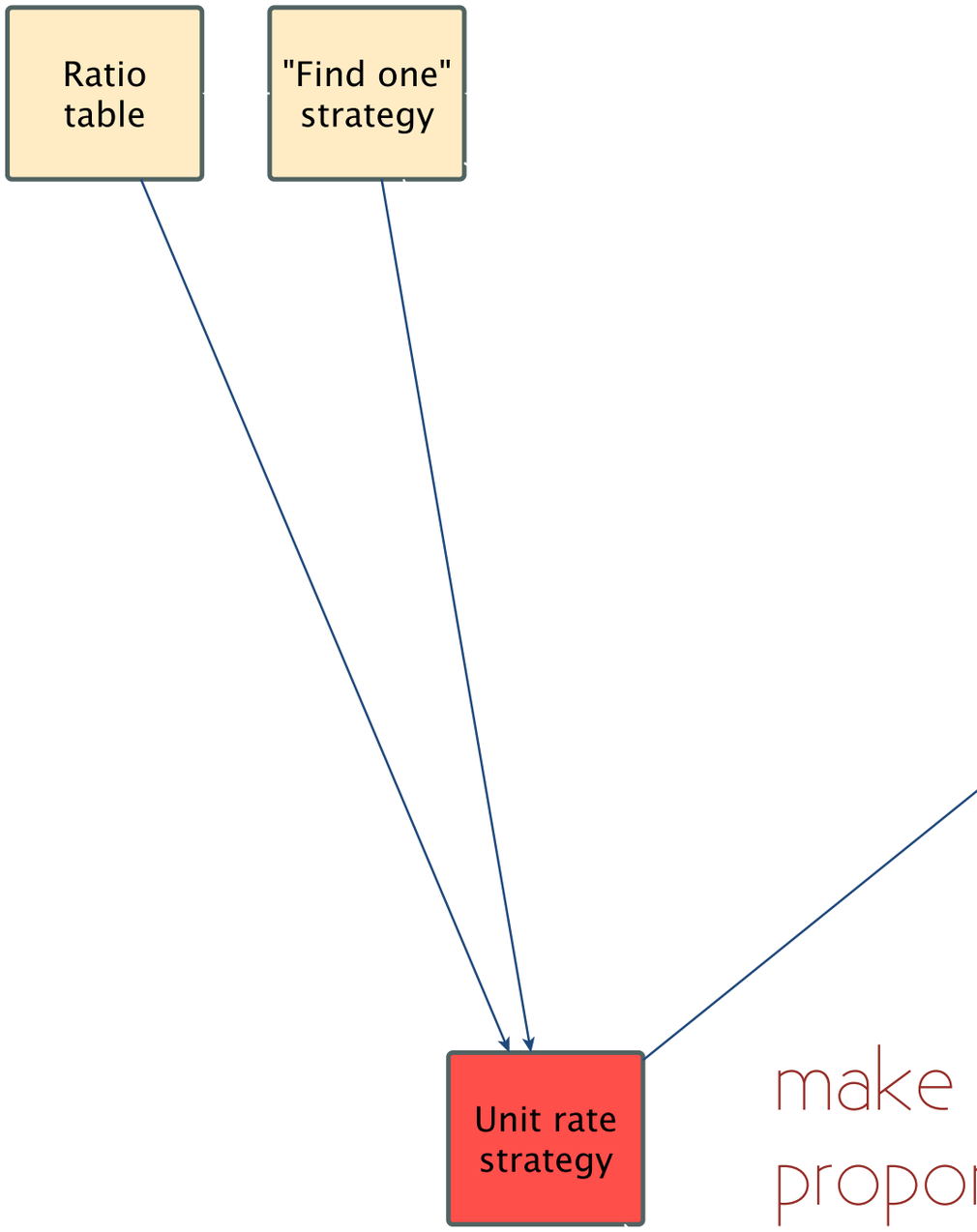

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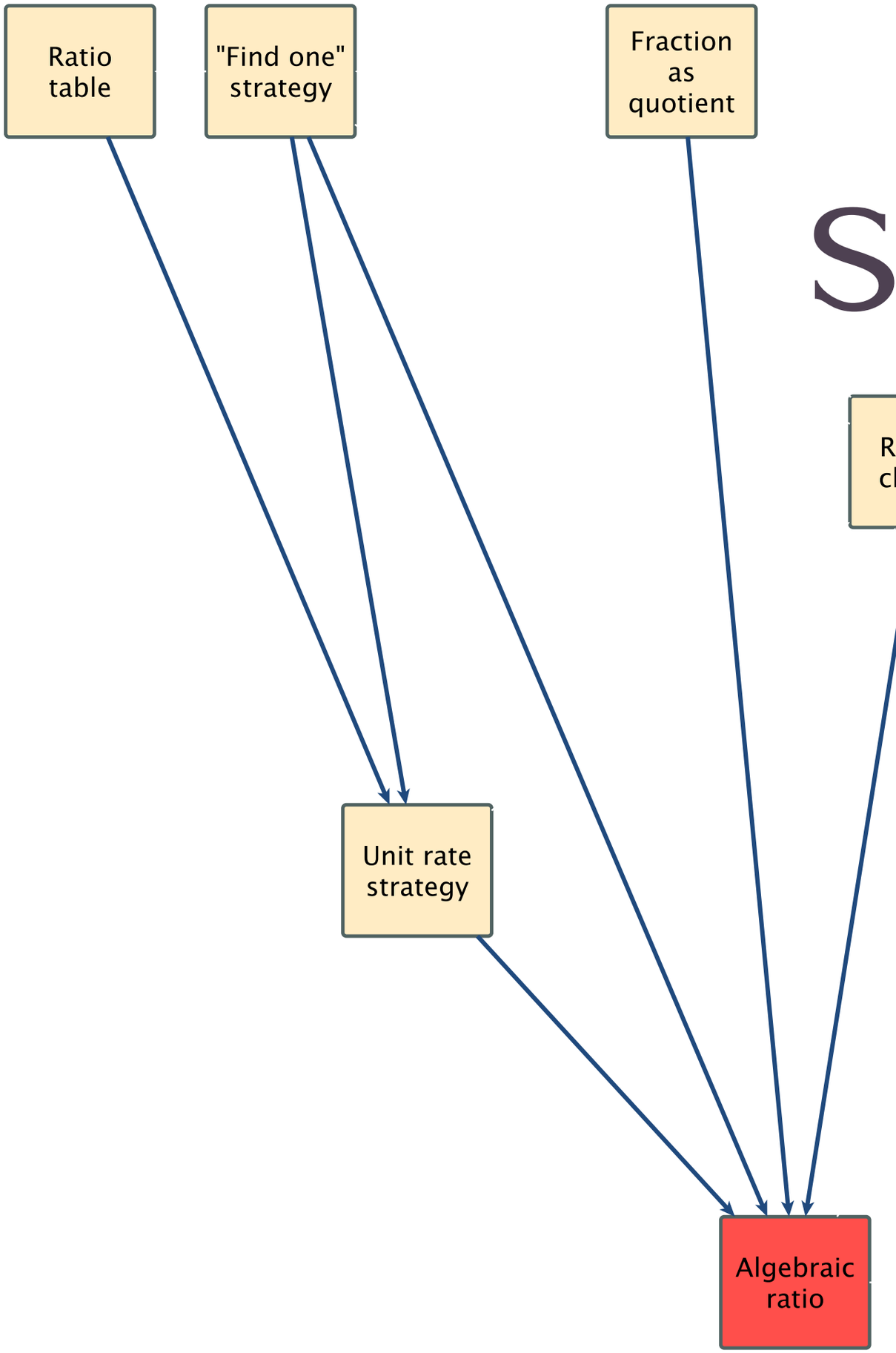
stage 4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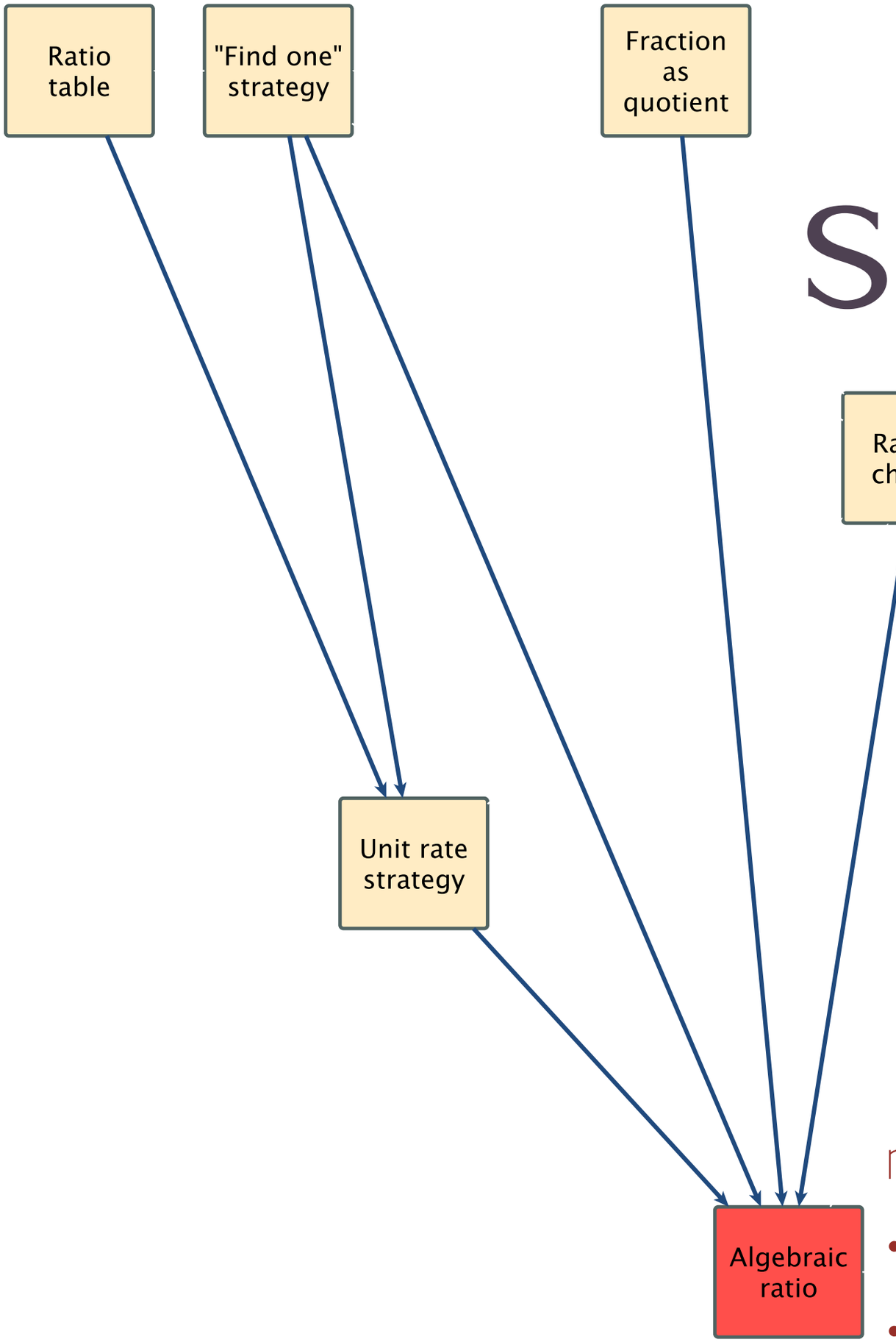
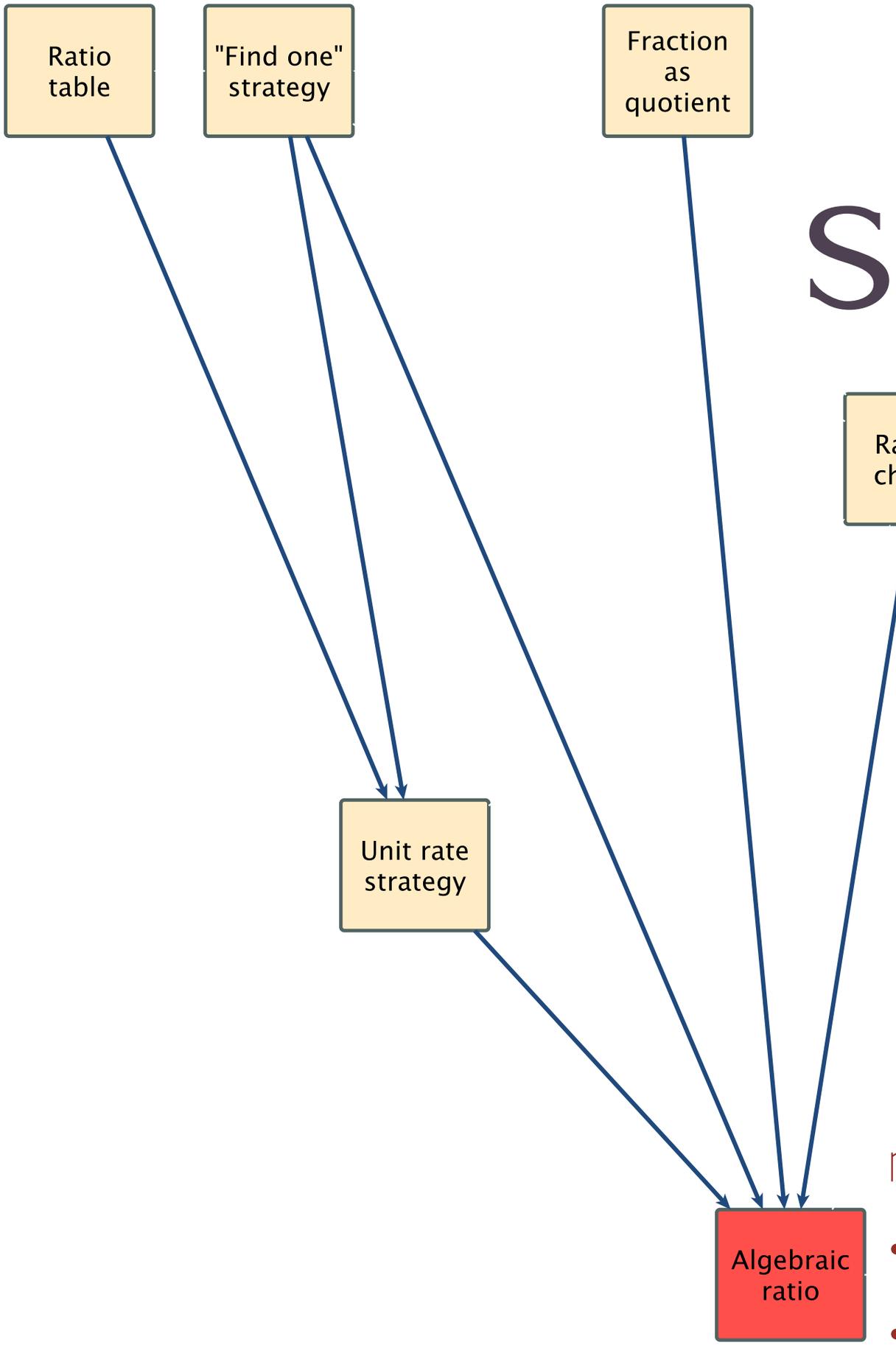


make predictions given two data points:

- x & y not proportional
- $\triangle x \neq 1$

stage 4



Leslie is a window installer. On Friday, she installed two windows, and charged 402 dollars. Last week, on another job, she charged 517 dollars to install seven windows.

A new customer has asked Leslie to install five windows. How much will this cost?

make predictions given two data points:

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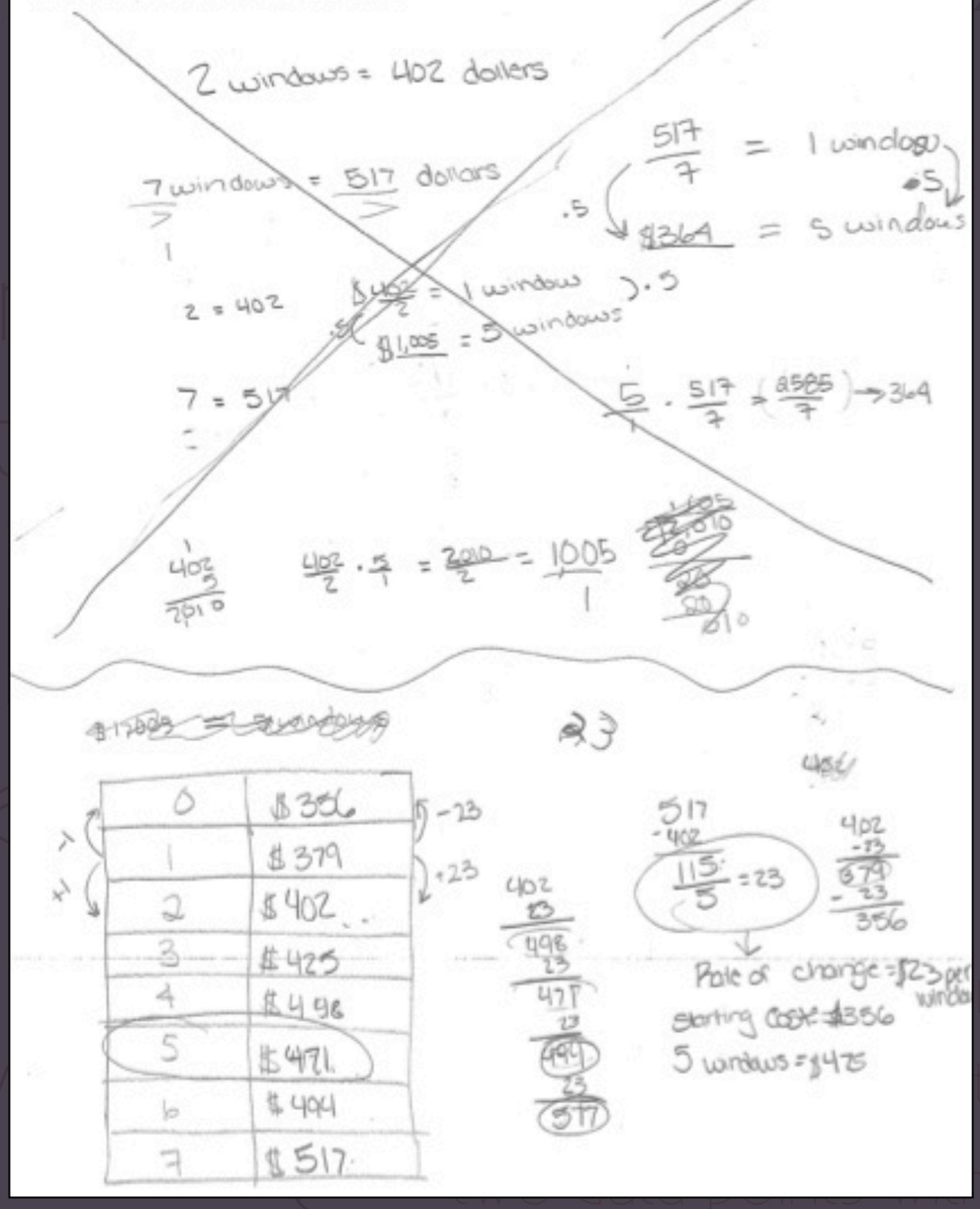

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2 windows = 402 dollars

7 windows = 517 dollars

$\frac{517}{7}$ = 1 window

.5 ↓ $364 = 5 windows ×5

$\frac{\$402}{2}$ = 1 window).5

.5($1,005 = 5 windows

$\frac{5}{1} \cdot \frac{517}{7}$ ($\frac{\$3585}{7}$) → 364

$\frac{405}{3}$ $\frac{402}{2} \cdot 5 = \frac{2010}{2} = \frac{1005}{1}$

23

0	$356
1	$379
2	$402
3	$425
4	$498
5	$471
6	$494
7	$517

-23 +23

402 -23 379 -23 356

517 - 402 = 115 ÷ 5 = 23

402 +23 498 +23 471 +23 494 +23 517

Rate of change = $23 per window

Starting cost: $356

5 windows = $471

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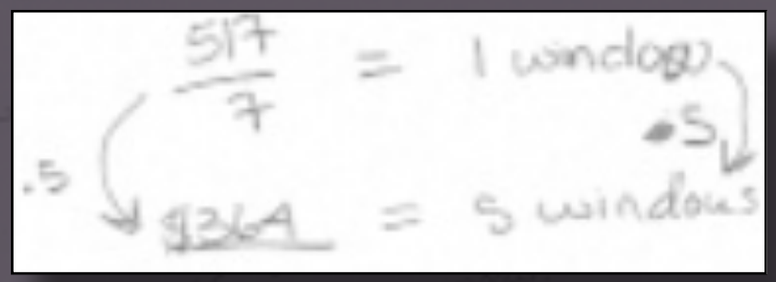

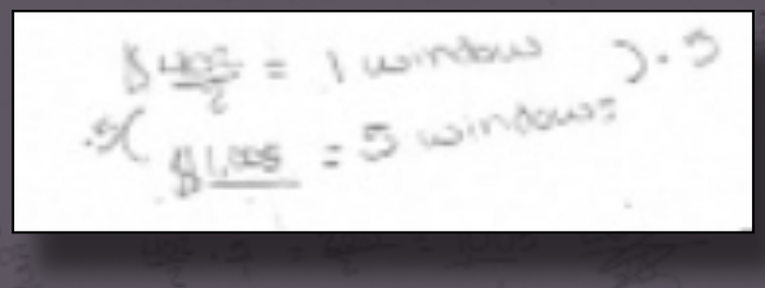

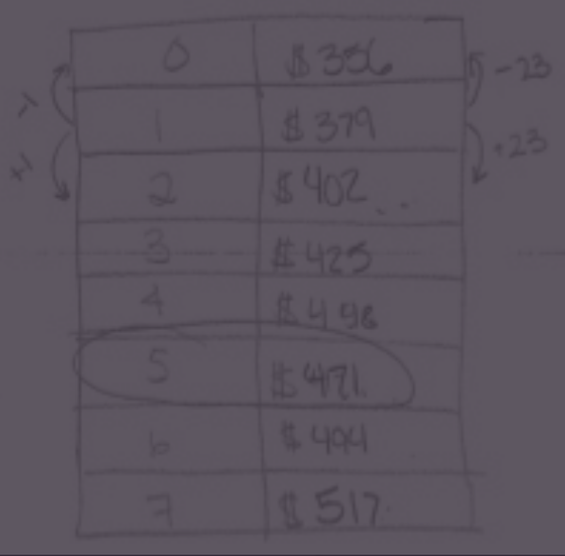

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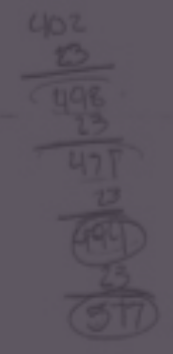

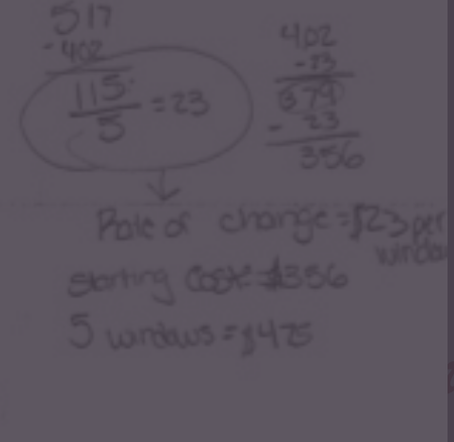

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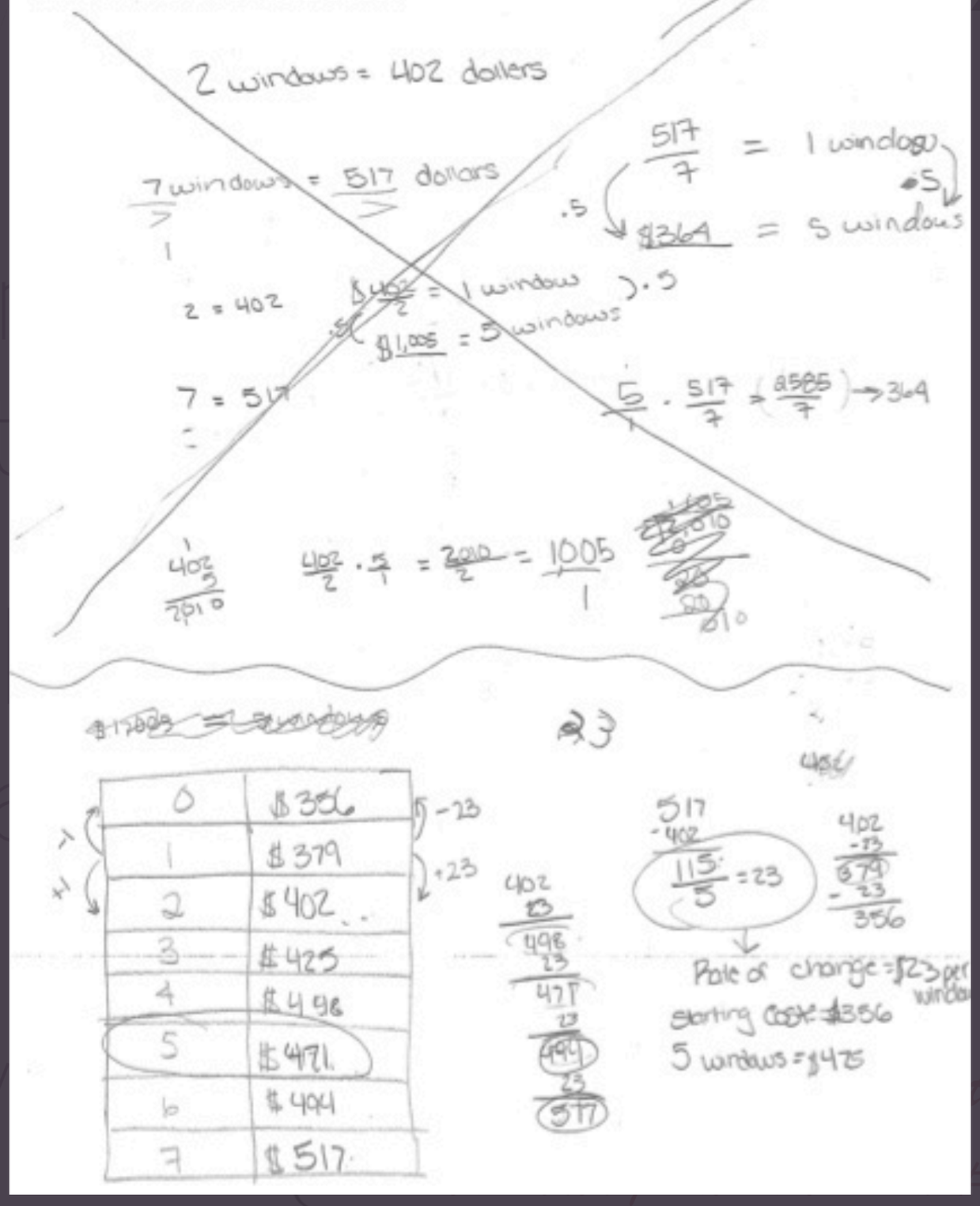

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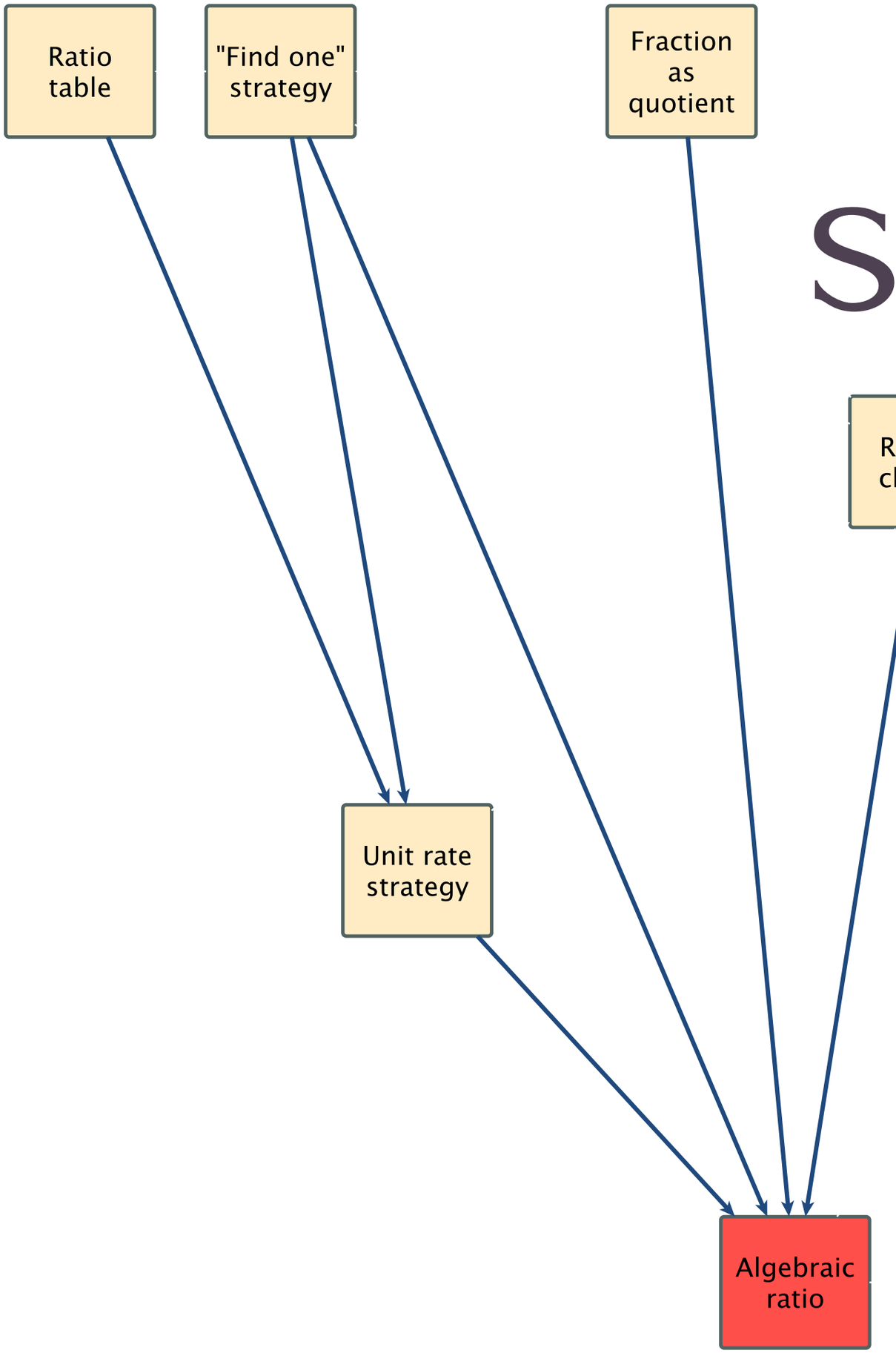
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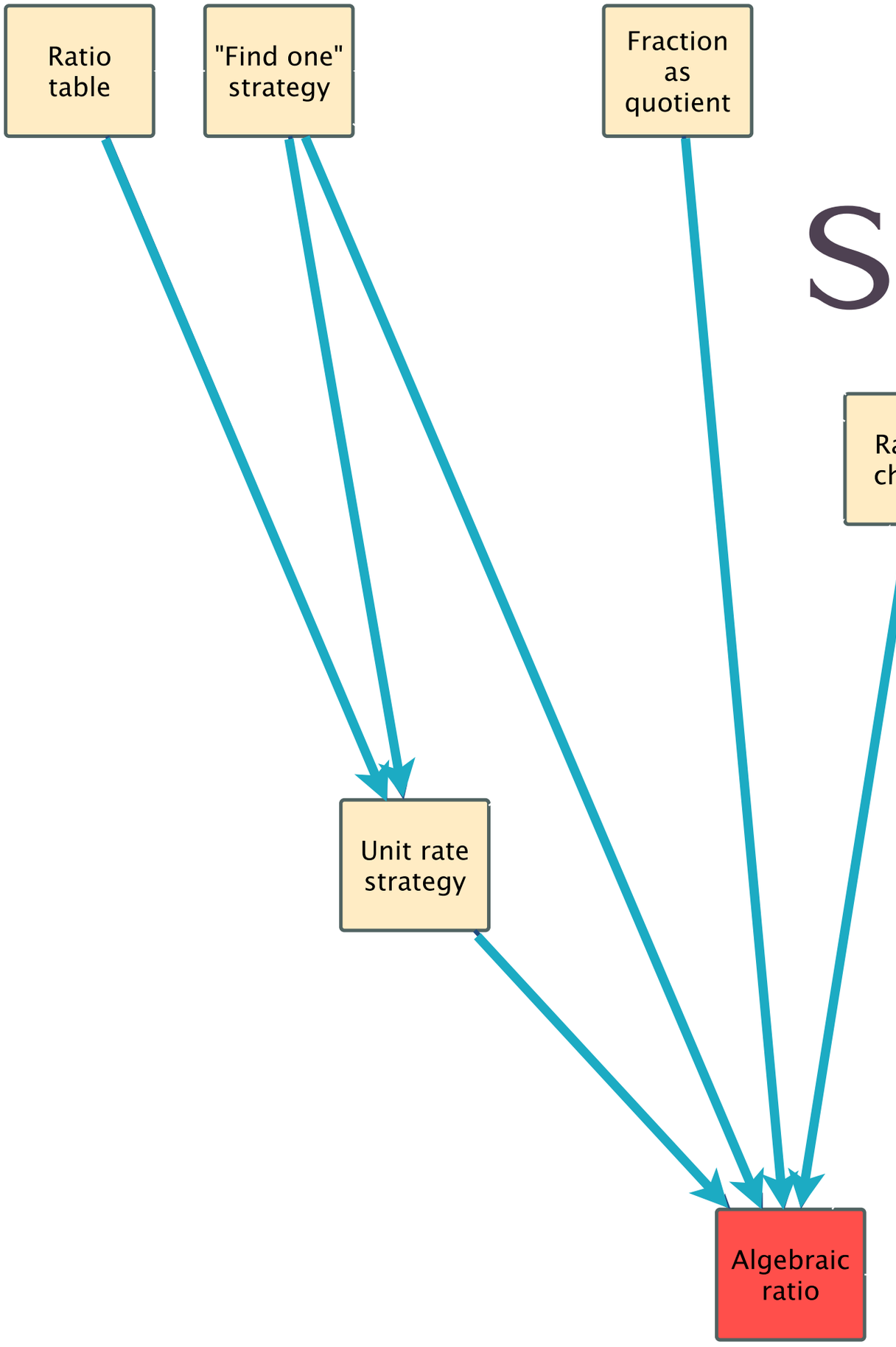
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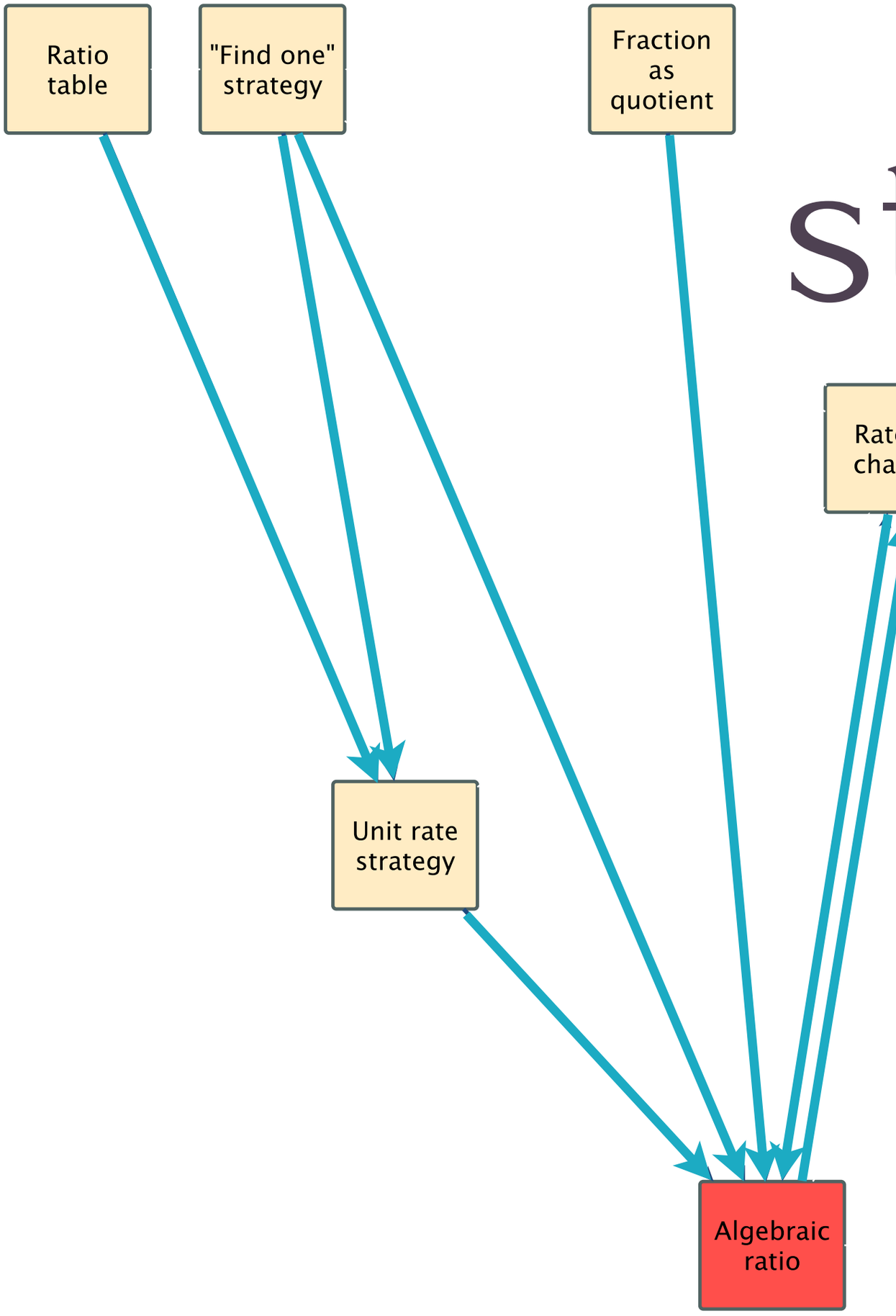
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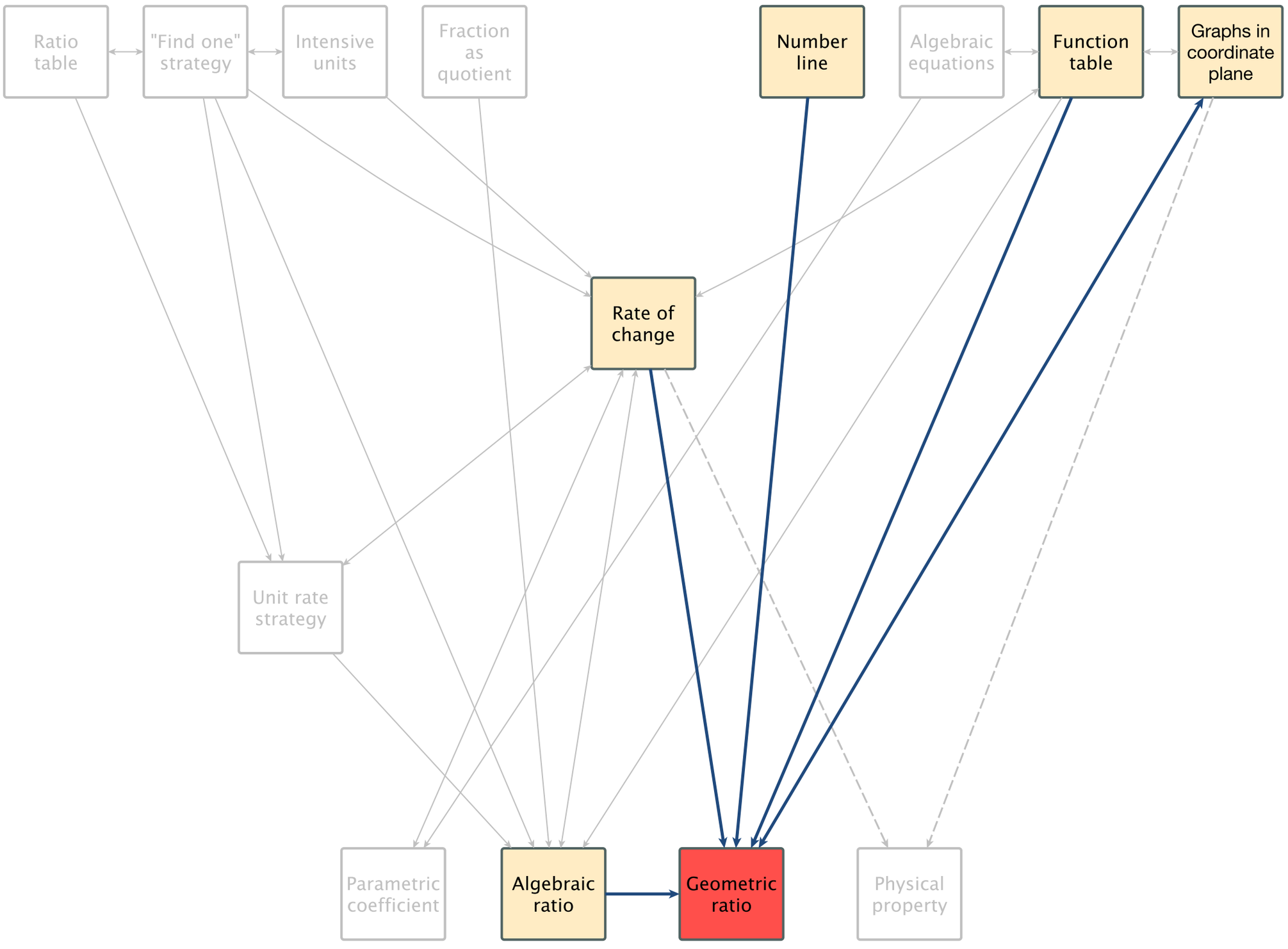
stage 5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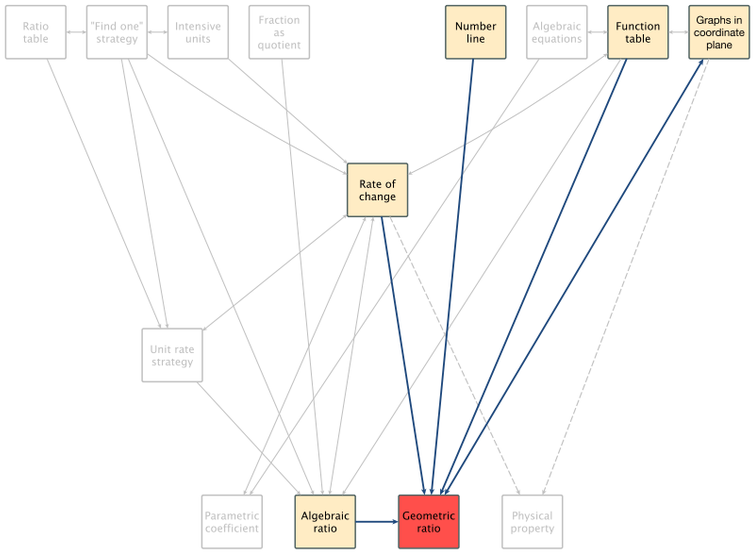
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Intensive units
Fraction as quotient
Number line
Algebraic equations
Function table
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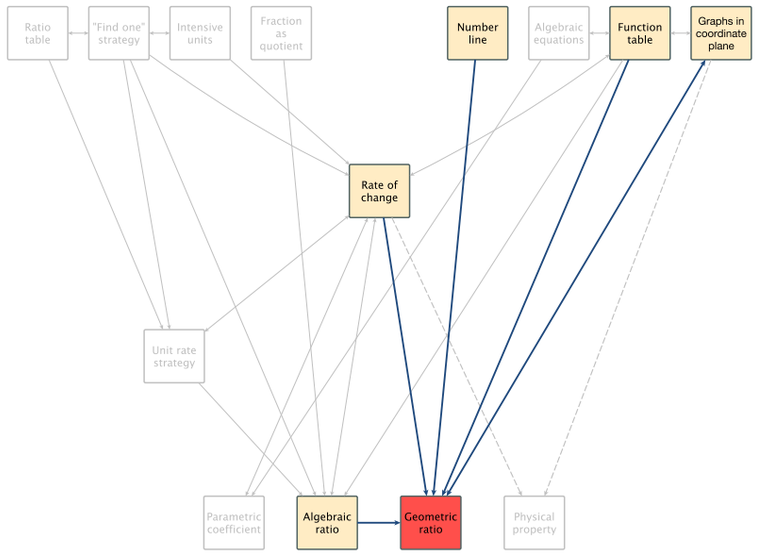
Rate of change
Unit rate strategy
Parametric coefficient
Algebraic ratio
Geometric ratio
Physical property

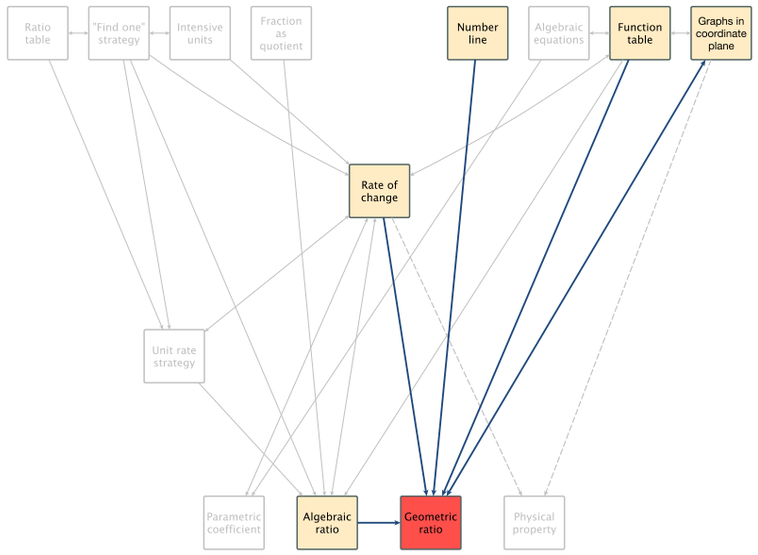
stage 5



stage 5

Reinvented & objectified

- geometric ratio



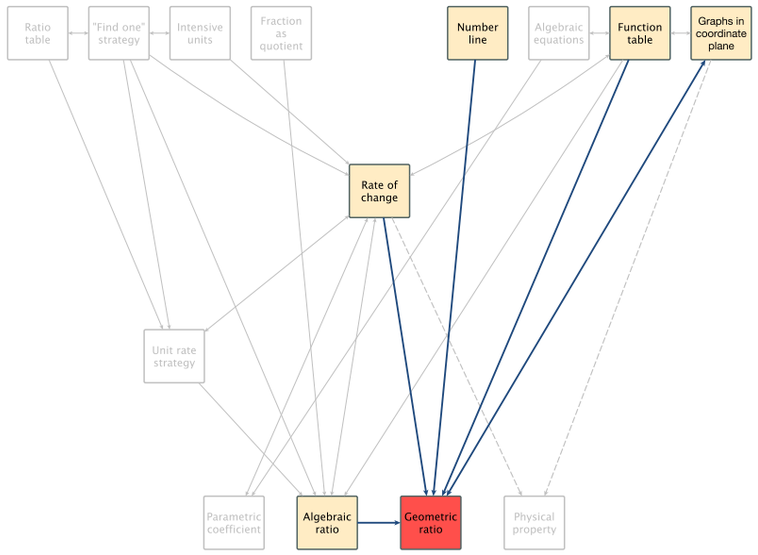
stage 5

Reinvented & objectified

- geometric ratio

Assembled & coordinated

- algebraic ratio
- rate of change
- number line
- function tables
- graphs in coordinate plane



stage 5

Reinvented & objectified

- geometric ratio

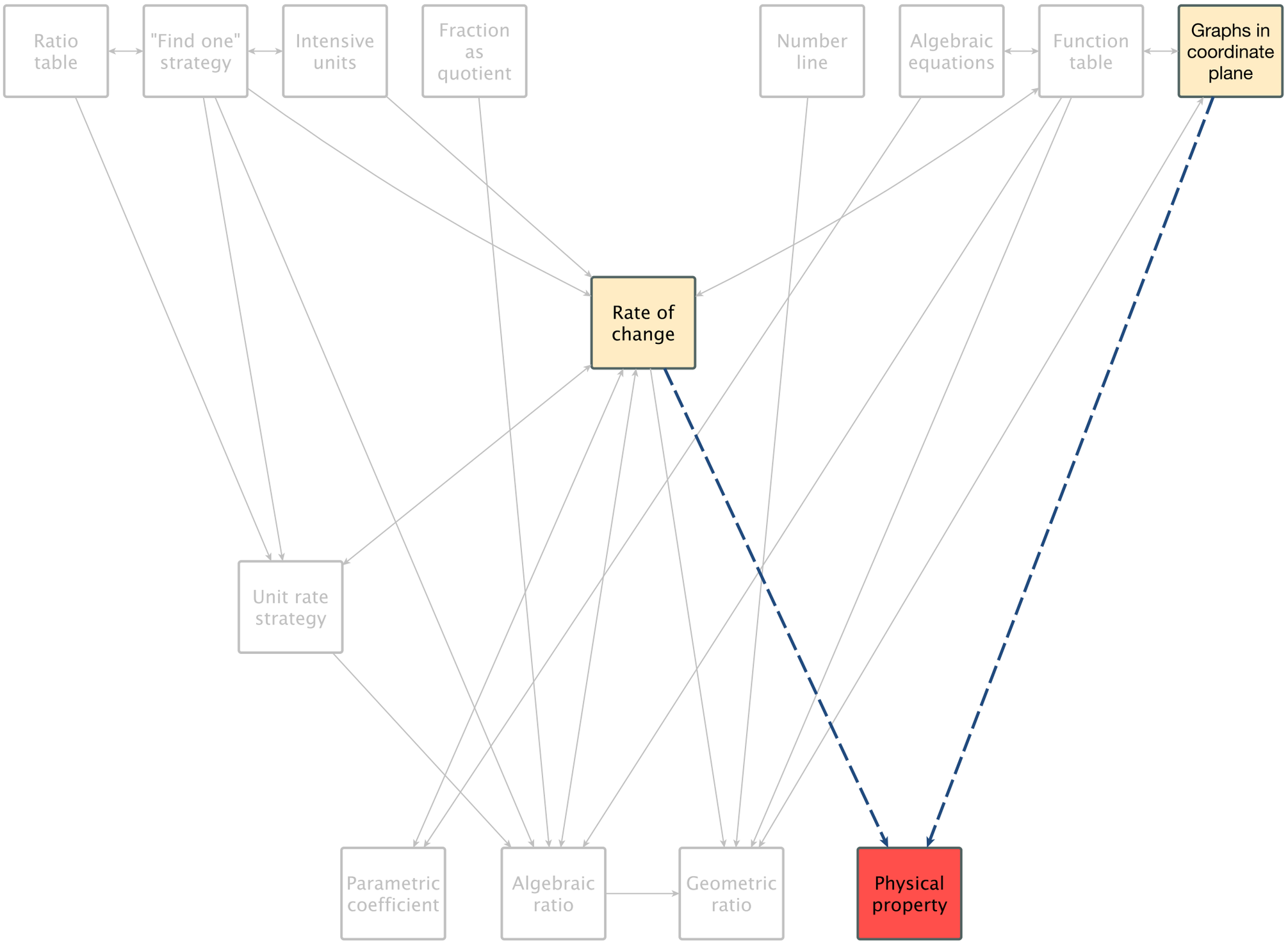
Assembled & coordinated

- algebraic ratio
- rate of change
- number line
- function tables
- graphs in coordinate plane

Activities

- show change on number line diagrams
- make predictions given graph

stage 6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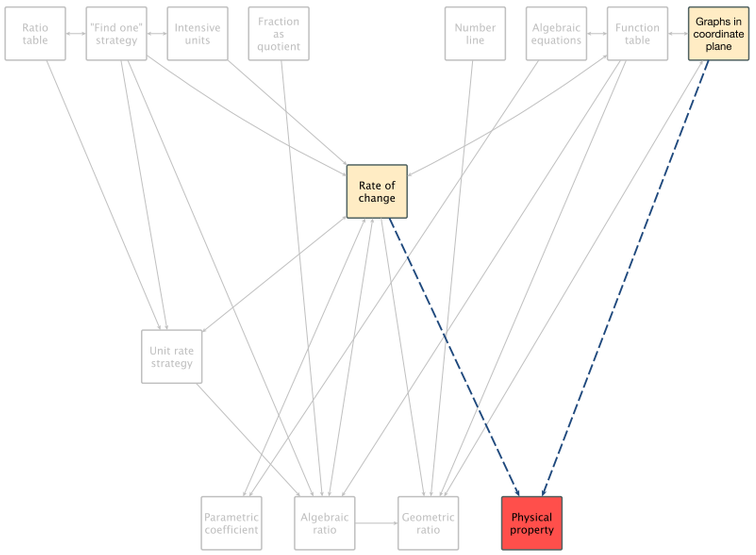
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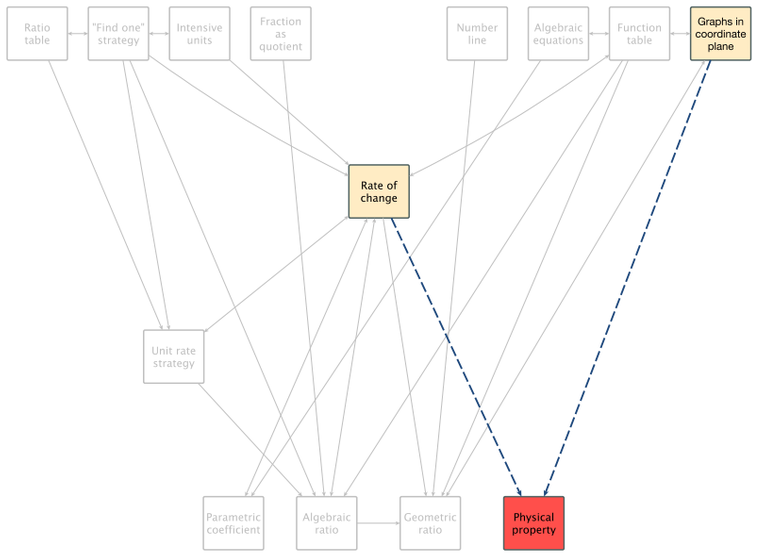
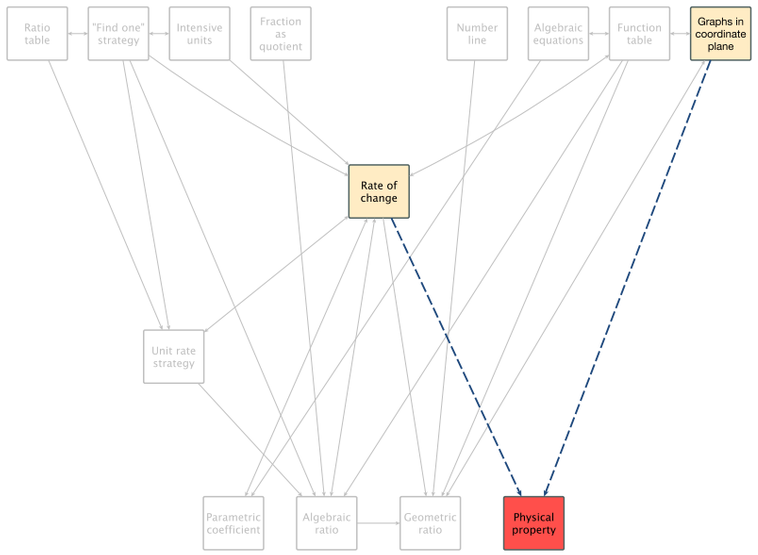
stage 6



stage 6

Reinvented & objectified {

- physical property



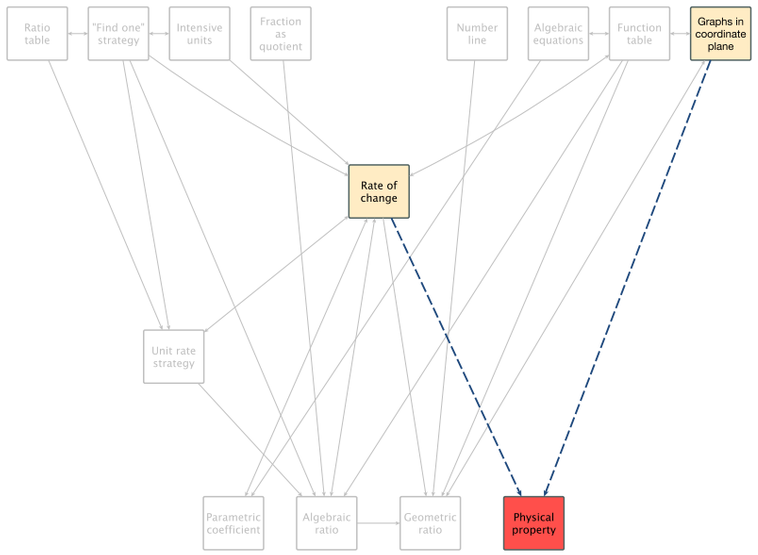
stage 6

Reinvented & objectified

- physical property

Assembled & coordinated

- rate of change
- graphs in coordinate plane



stage 6

Reinvented & objectified

- physical property

Assembled & coordinated

- rate of change
- graphs in coordinate plane

Activities

- compare rates given graph of two intersecting linear functions
- measure steepness of objects

summary

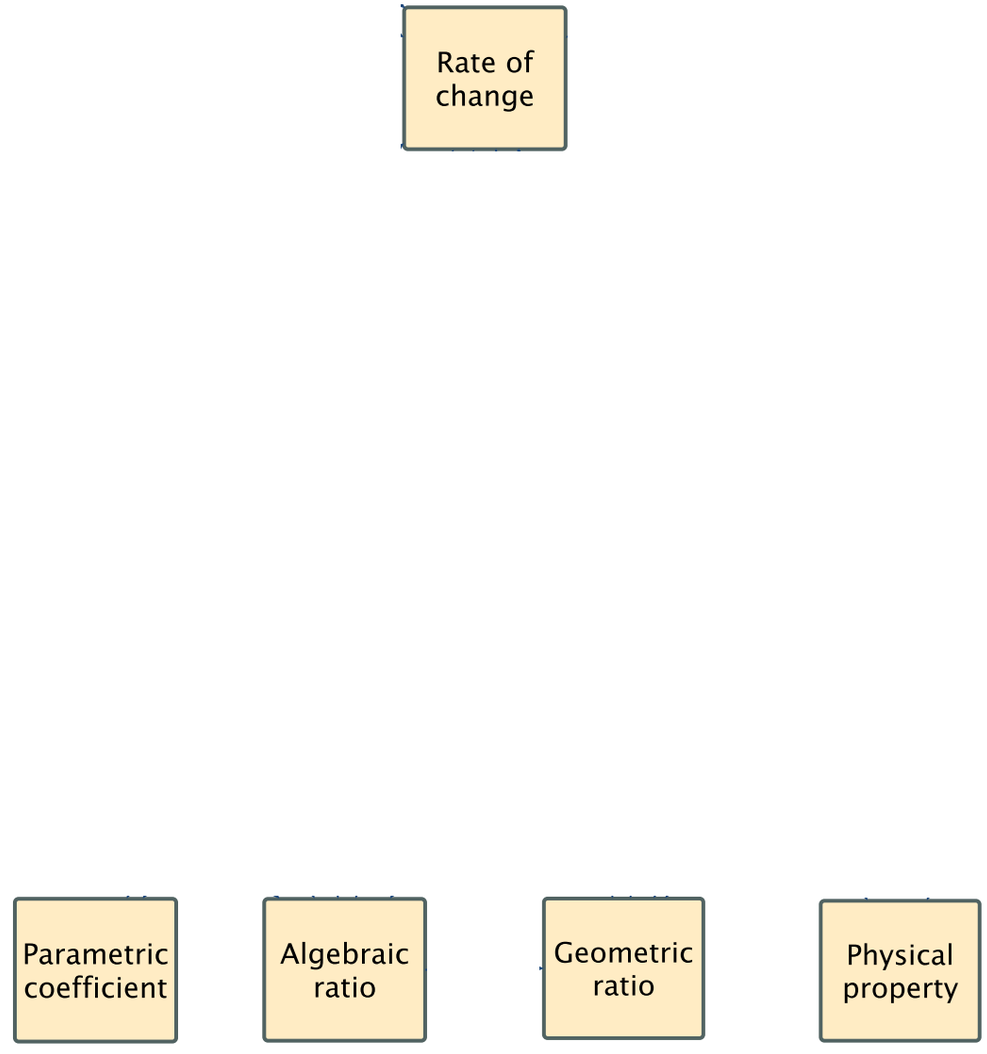
How do students make slope meaningful?

slope

slope

- Parametric coefficient
- Algebraic ratio
- Rate of change
- Geometric ratio
- Physical property

cascade of artifacts



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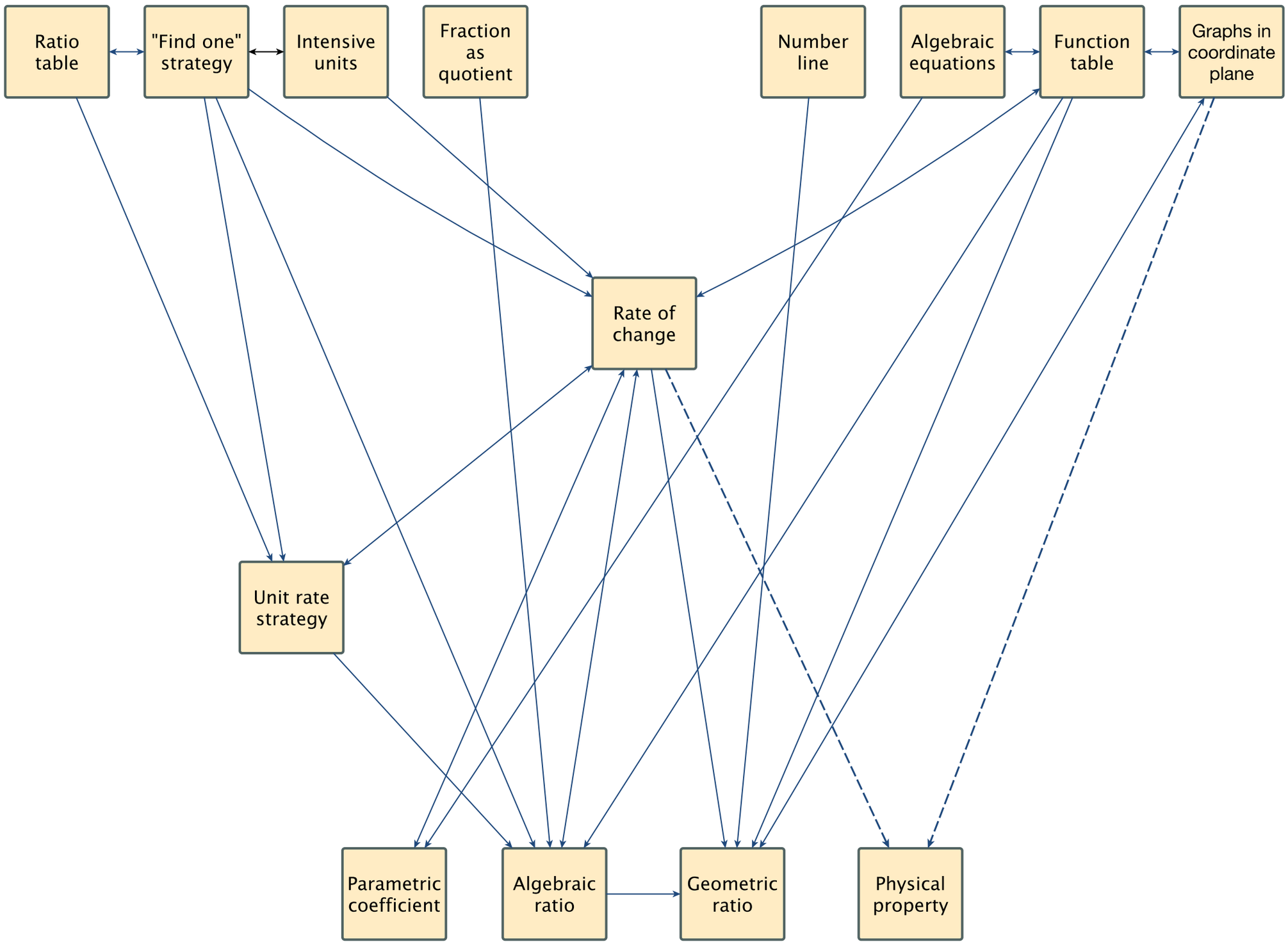

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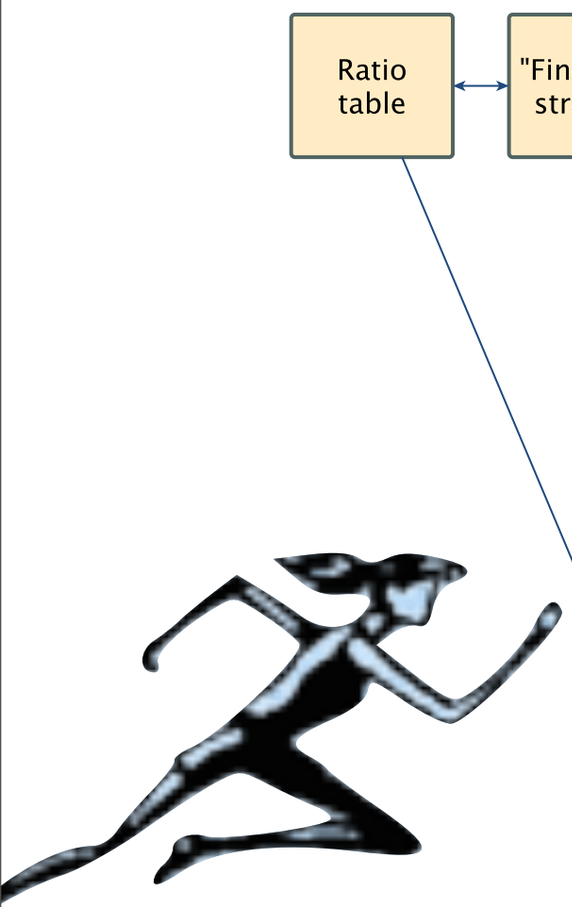

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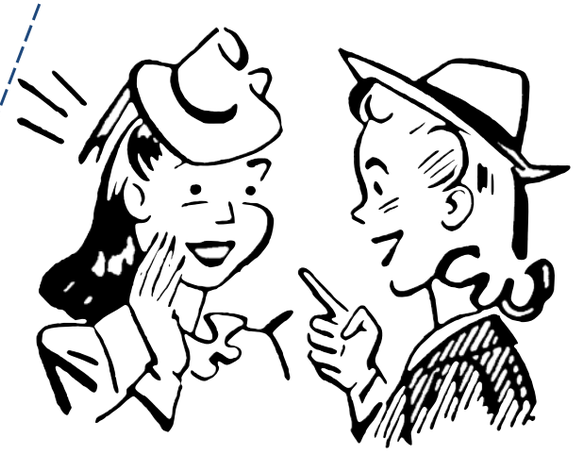
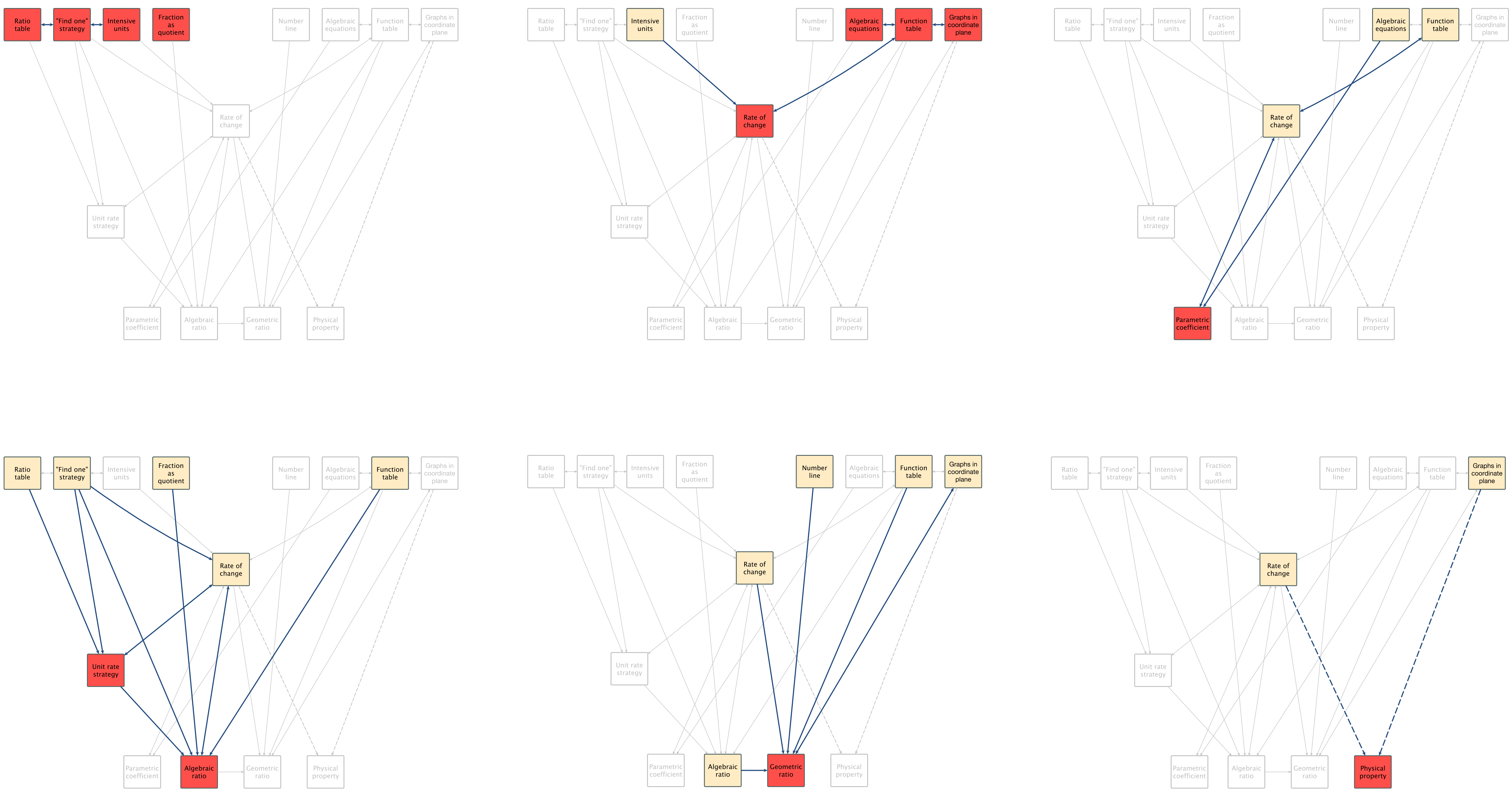


local instructional theory



Questions and discussion

Questions and discussion

Frederick Peck

Freudenthal Institute US

University of Colorado, USA

Frederick.Peck@Colorado.edu

www.RMEInTheClassroom.com

Method

Method

- Design experiment in HS algebra I classroom

Method

- Design experiment in HS algebra I classroom
  - Before: Thought experiment; conjectured local instructional theory

Method

- Design experiment in HS algebra I classroom
  - Before: Thought experiment; conjectured local instructional theory
  - During: Daily micro-cycles of design and analysis

Method

- Design experiment in HS algebra I classroom
  - Before: Thought experiment; conjectured local instructional theory
  - During: Daily micro-cycles of design and analysis
  - After: Retrospective analysis

Method

- Design experiment in HS algebra I classroom
  - Before: Thought experiment; conjectured local instructional theory
  - During: Daily micro-cycles of design and analysis
  - After: Retrospective analysis
  - RME as a design theory

Method

- Design experiment in HS algebra I classroom
- Outcome: Local instructional theory

Method

- Design experiment in HS algebra I classroom
- Outcome: Local instructional theory
- Data:

Method

- Design experiment in HS algebra I classroom
- Outcome: Local instructional theory
- Data:
  - Student work

Method

- Design experiment in HS algebra I classroom
- Outcome: Local instructional theory
- Data:
  - Student work
  - Observer field notes

Method

- Design experiment in HS algebra I classroom
- Outcome: Local instructional theory
- Data:
  - Student work
  - Observer field notes
  - Video and audio recordings of individual, group, and full-class work, student interviews, and research team meetings

Method

- Design experiment in HS algebra I classroom
- Outcome: Local instructional theory
- Data: Student work, field notes, video & audio
- 15 days; 19 students; I was the teacher

learning as

reinvention & objectification

learning as reinvention & objectification

- assembling and coordinating other artifacts

learning as reinvention & objectification

- assembling and coordinating other artifacts
- disciplining perception to particular affordances of artifacts

learning

{ Process
{ Product

learning

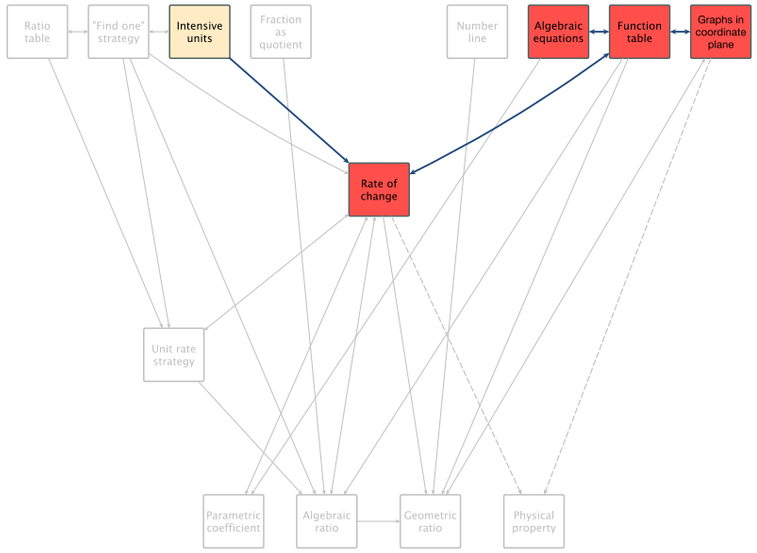
- Culture { Process / Product

learning

- Culture { Process / Product
- Mediation

learning

- Culture { Process / Product
- Mediation
- Objectification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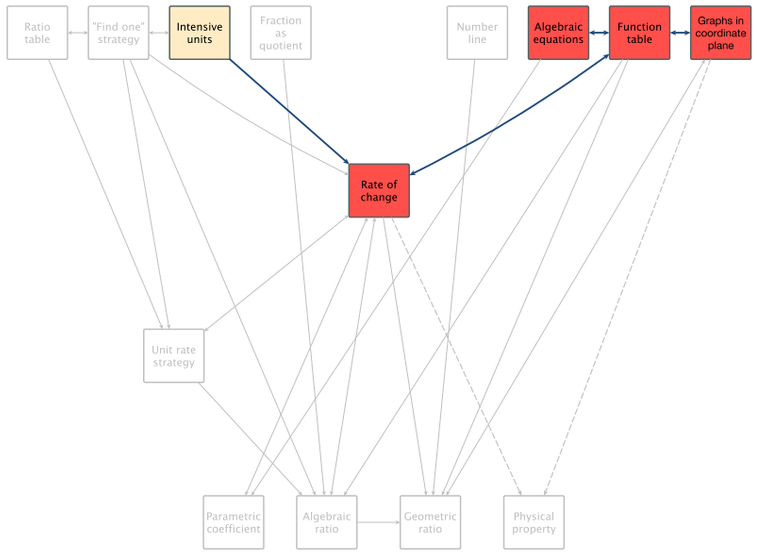
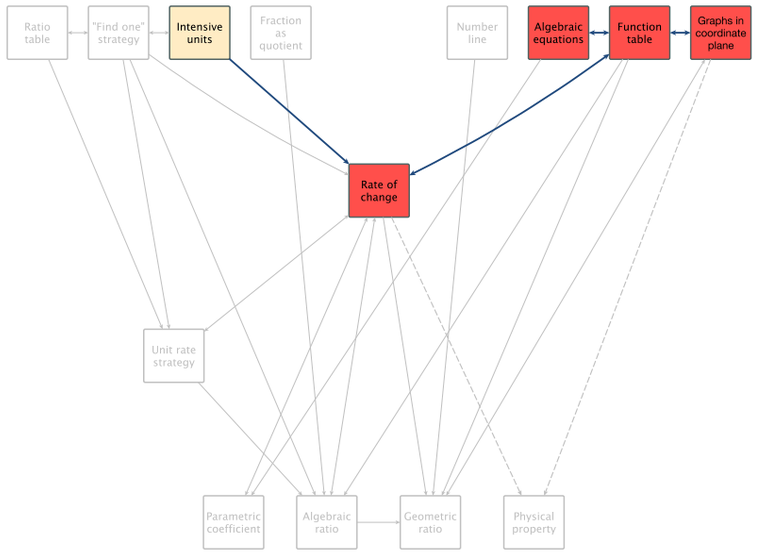
stage 2



stage 2

Reinvented & objectified

- function tables
- algebraic equations
- graphs in coord. plane
- rate of change



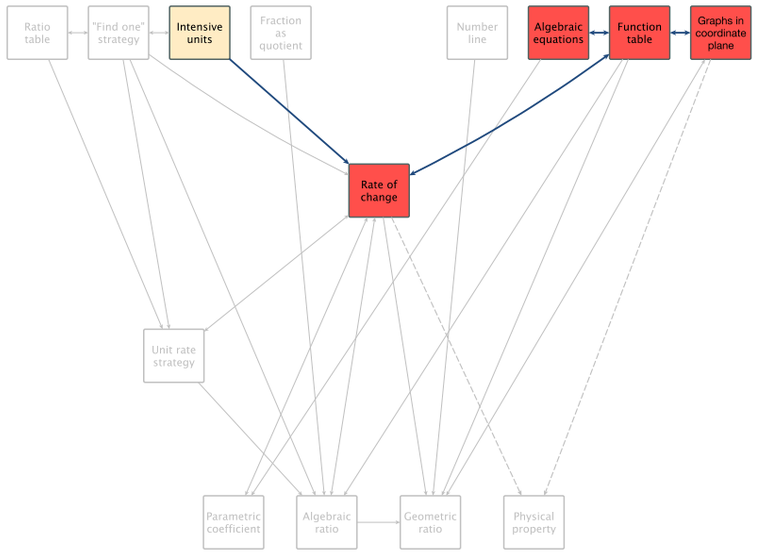
stage 2

Reinvented & objectified

- function tables
- algebraic equations
- graphs in coord. plane
- rate of change

Assembled & coordinated

- intensive units



stage 2

Reinvented & objectified

- function tables
- algebraic equations
- graphs in coord. plane
- rate of change

Assembled & coordinated

- intensive units

Activities

- find and continue patterns
- convert between multiple representations of functions