Beyond rise over run: Activities to invent and connect slope's five faces

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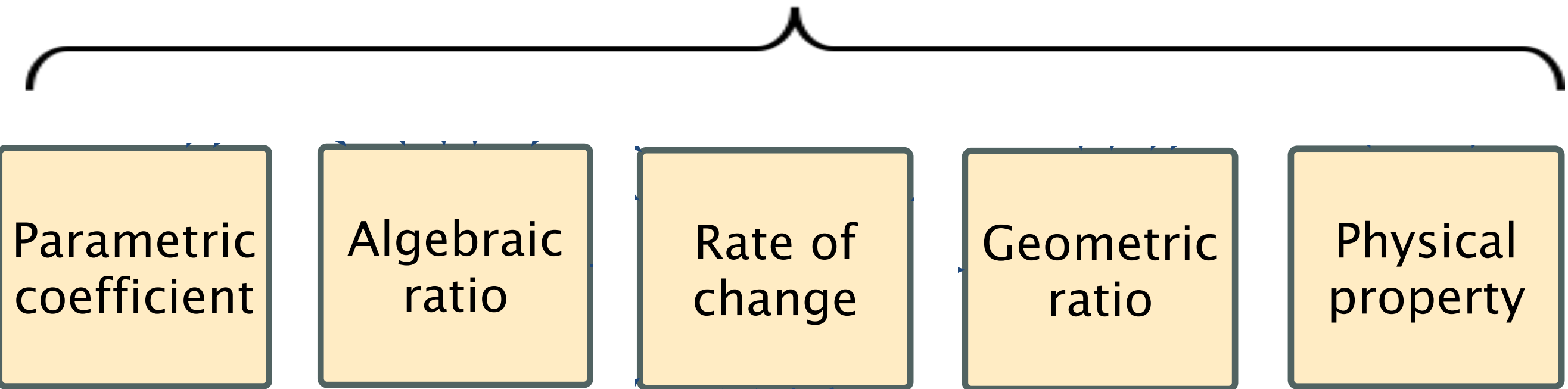
meaningful?

procedural

geometric

meaningful?

How do students make slope meaningful?

slope

Parametric coefficient	Algebraic ratio	Rate of change	Geometric ratio	Physical property

slope

| Parametric coefficient | Algebraic ratio | Rate of change | Geometric ratio | Physical property |

$$y = \textcolor{red}{a}x + b$$

slope

Parametric coefficient | **Algebraic ratio** | Rate of change | Geometric ratio | Physical property

$$\frac{y_2 - y_1}{x_2 - x_1}$$

slope

Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

Physical property

slope

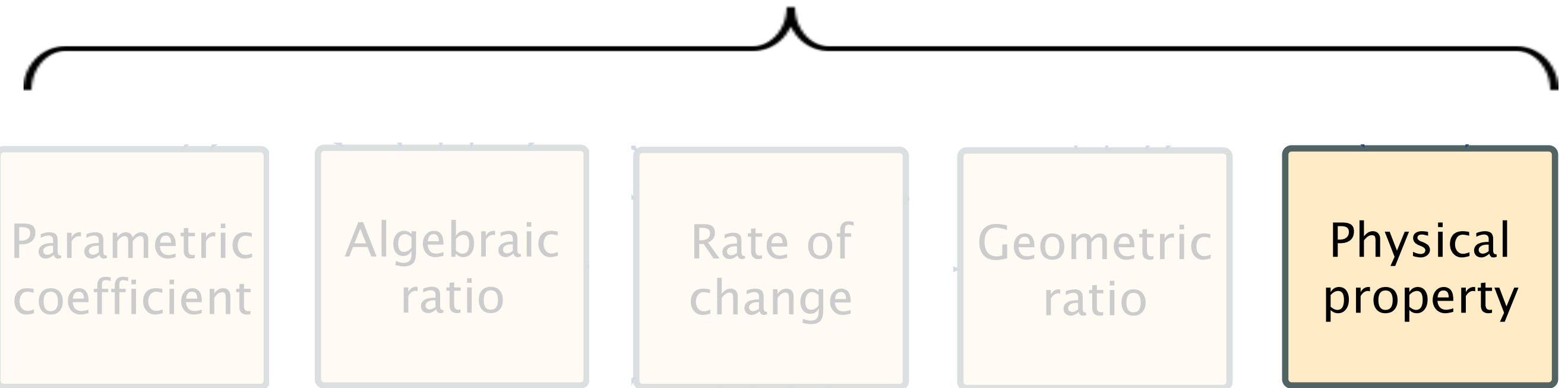Parametric coefficient | Algebraic ratio | Rate of change | **Geometric ratio** | Physical property

$$\frac{\text{rise}}{\text{run}}$$

slope

| Parametric coefficient | Algebraic ratio | Rate of change | Geometric ratio | Physical property |

"steepness"

Artifacts

Parametric coefficient | Algebraic ratio | Rate of change | Geometric ratio | Physical property

"partial solutions to frequently-encountered problems"

Products of history that are resources for the present

artifacts are resources
for current activity

Artifacts

Activity

artifacts are the
residue of historic
activity

artifacts are resources
for current activity

Artifacts   Activity

artifacts are   artifacts become
residue of history   meaningful through
activity   activity

learning
making artifacts
meaningful

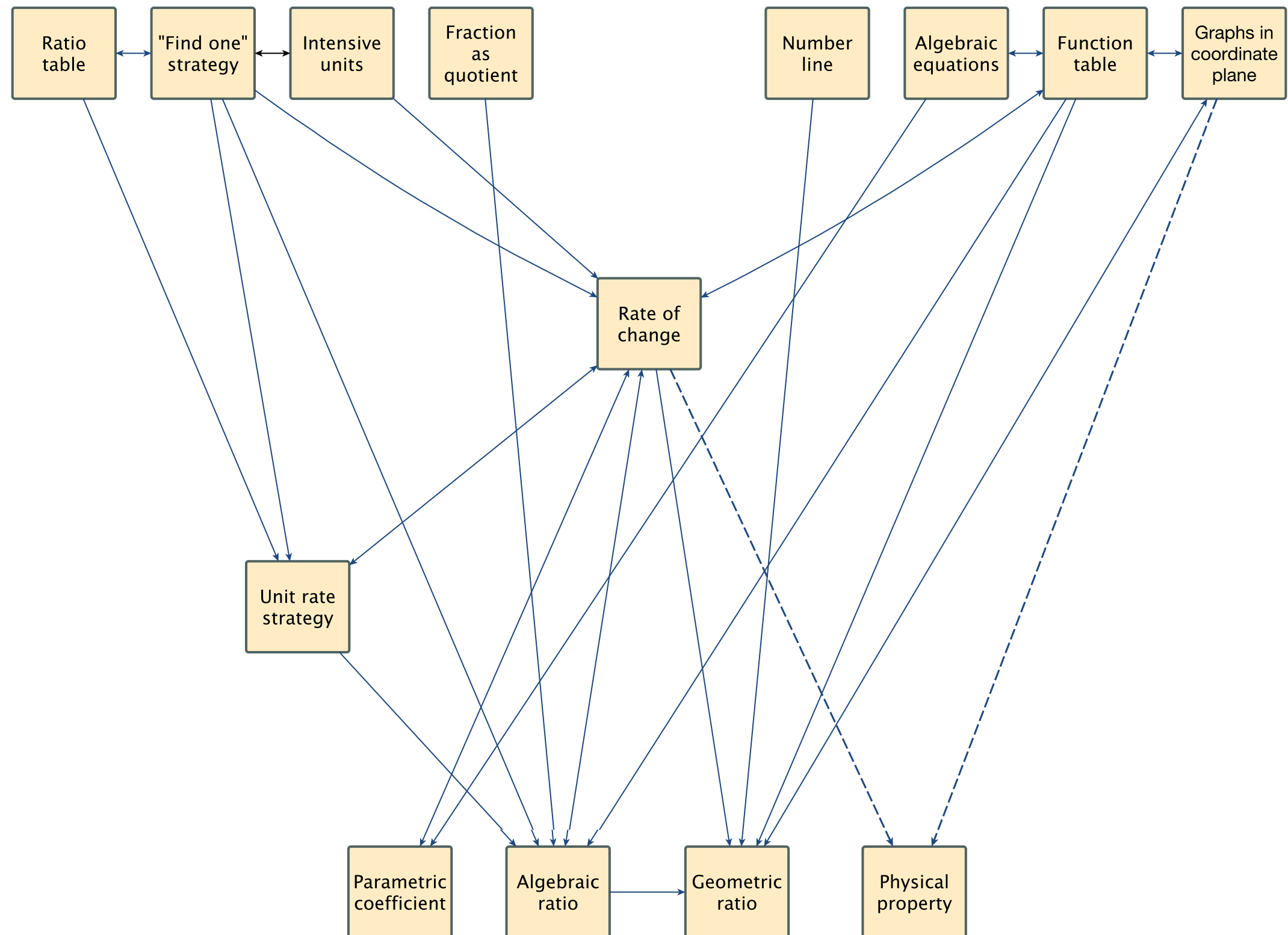| Parametric coefficient	Algebraic ratio	Rate of change	Geometric ratio	Physical property

cascade of artifacts
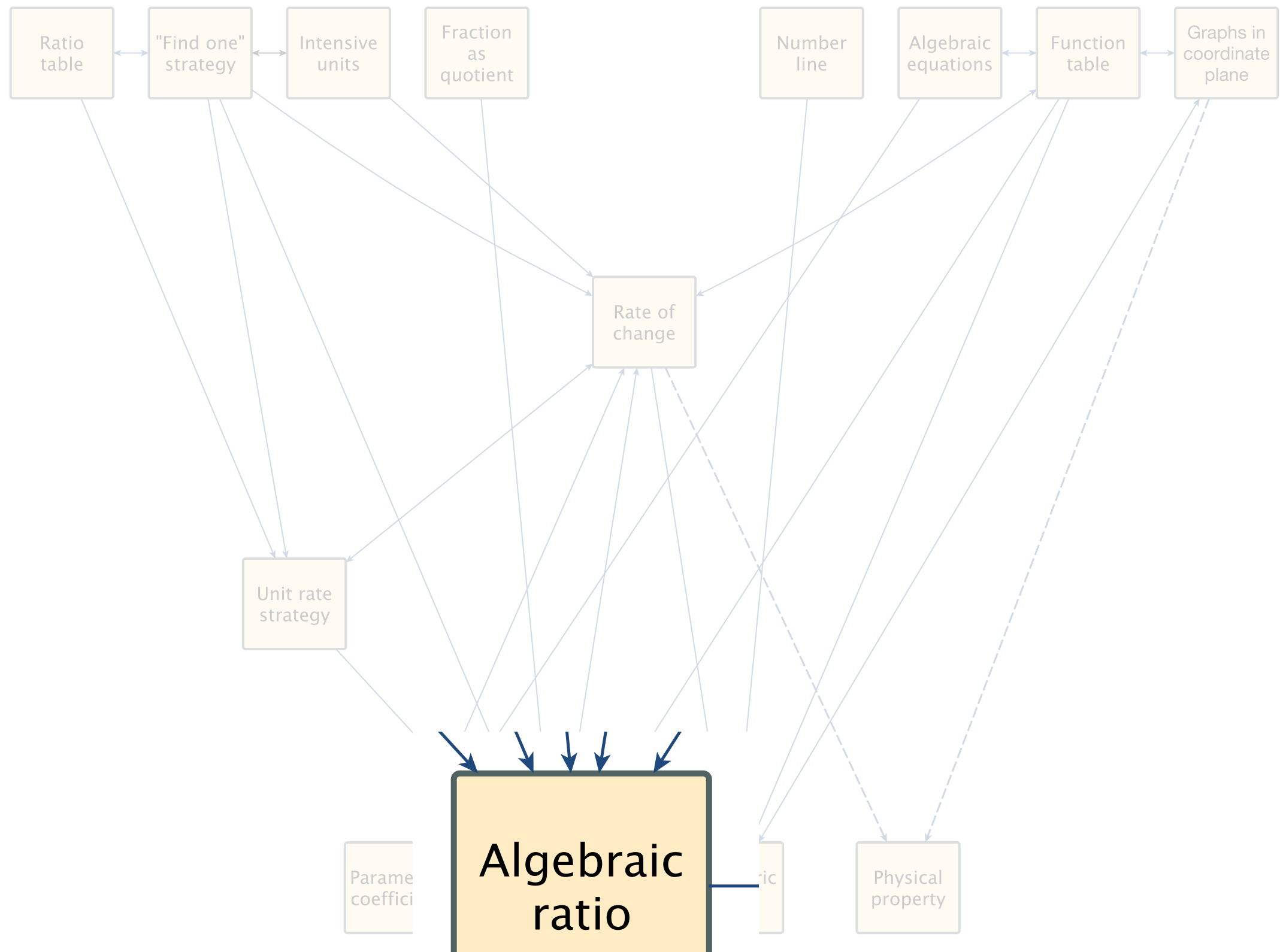
Rate of change

Parametric coefficient

Algebraic ratio
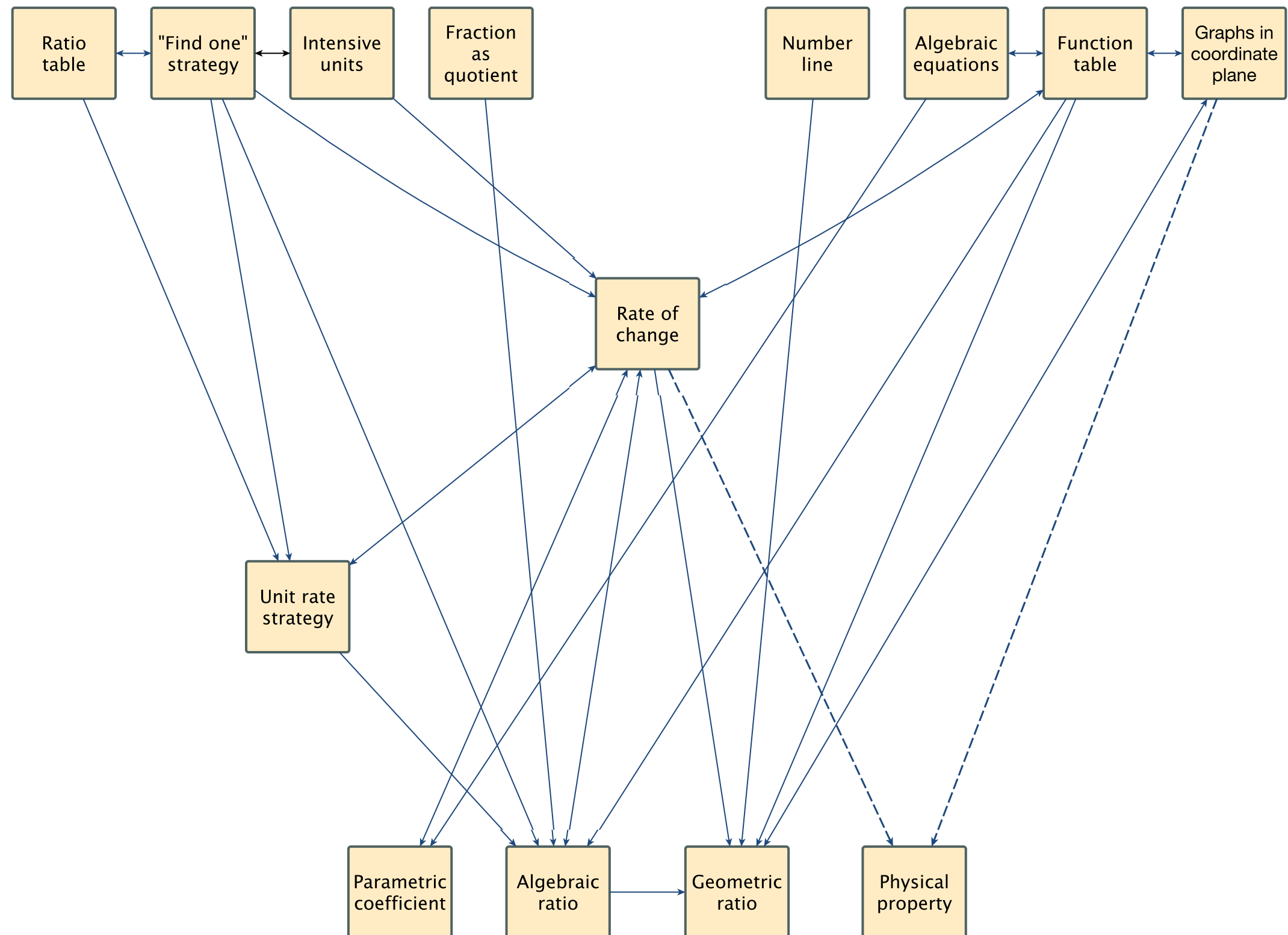
Geometric ratio

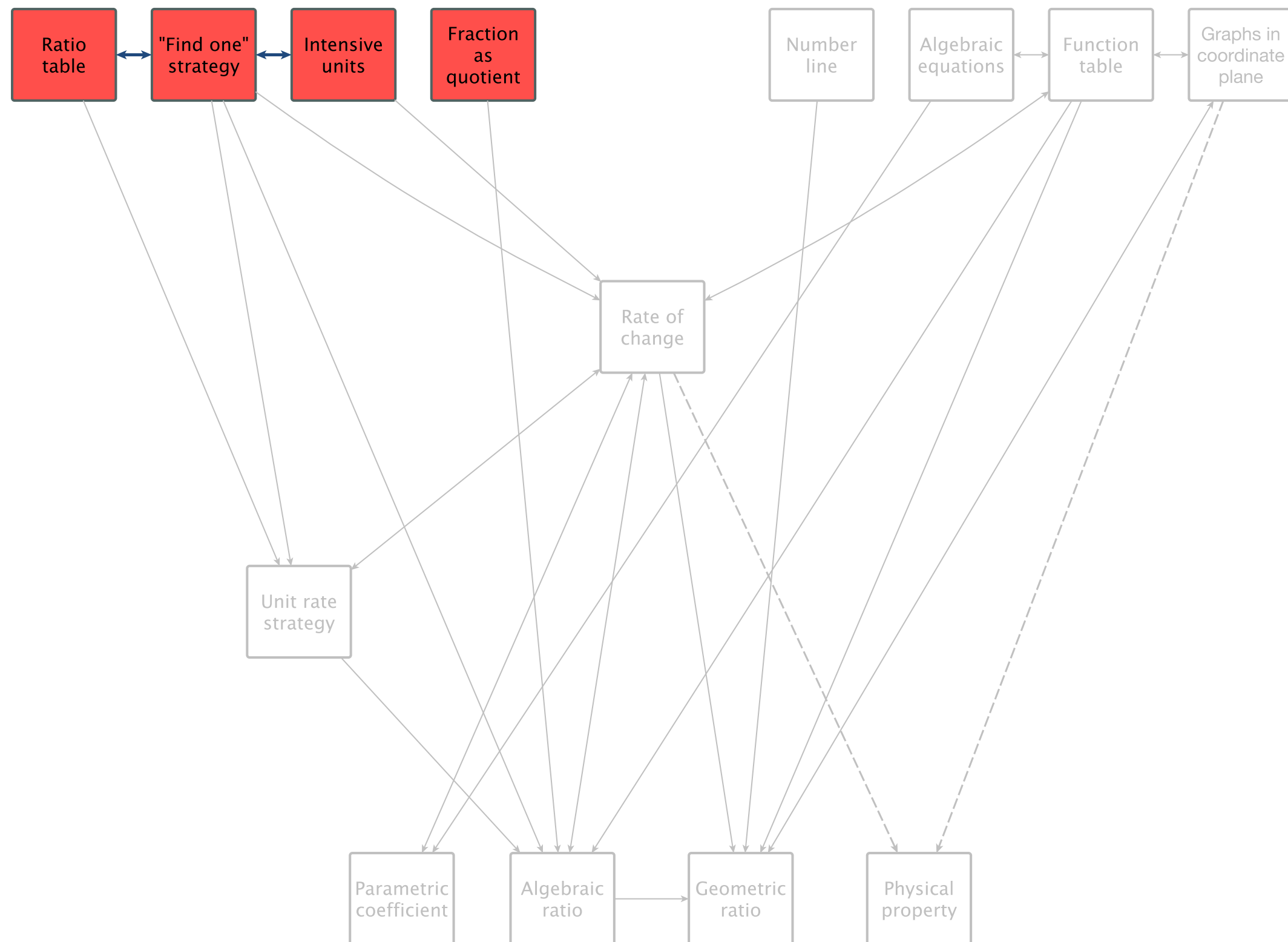Physical property
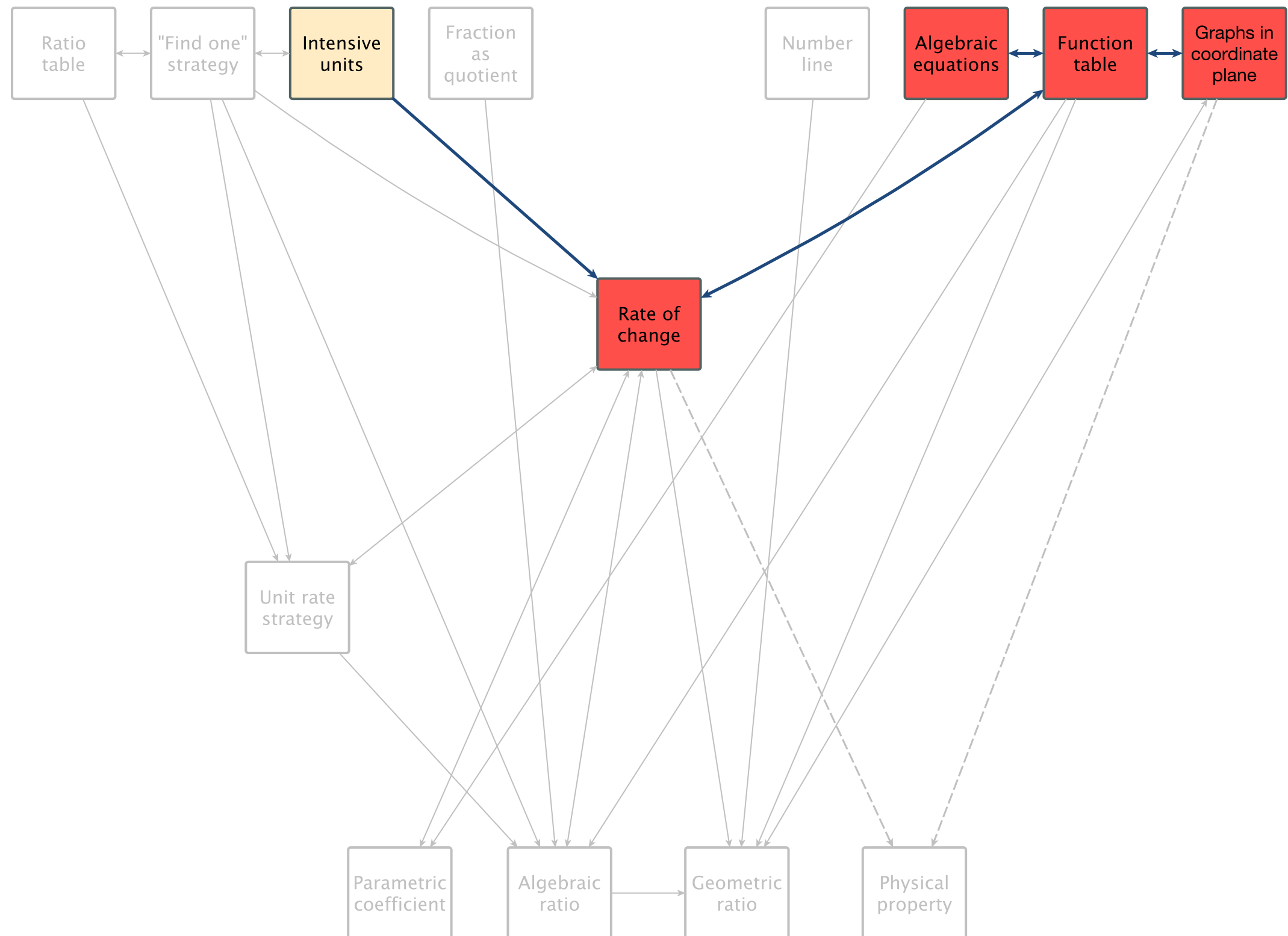
cascade of artifacts

cascade of artifacts

Ratio table

"Find one" strategy

Intensive units

Fraction as quotient

Number line

Algebraic equations

Function table

Graphs in coordinate plane

Rate of change

Unit rate strategy

Parame coeffici

Algebraic ratio

ric

Physical property

cascade of artifacts

Ratio table — "Find one" strategy — Intensive units — Fraction as quotient

Number line — Algebraic equations — Function table — Graphs in coordinate plane

Rate of change

Unit rate strategy

Parametric coefficient — Algebraic ratio — Geometric ratio — Physical property

cascade of artifacts

stage 1

Ratio table — "Find one" strategy — Intensive units — Fraction as quotient

Number line — Algebraic equations — Function table — Graphs in coordinate plane

Rate of change

Unit rate strategy

Parametric coefficient — Algebraic ratio — Geometric ratio — Physical property

stage 2

Ratio table

"Find one" strategy

Intensive units

Fraction as quotient

Number line

Algebraic equations

Function table

Graphs in coordinate plane

Rate of change

Unit rate strategy

Parametric coefficient

Algebraic ratio

Geometric ratio

Physical property

stage 3

Ratio table ←→ "Find one" strategy ←→ Intensive units

Fraction as quotient

Number line

Algebraic equations ←→ **Function table** ←→ Graphs in coordinate plane

Rate of change

Unit rate strategy

Parametric coefficient

Algebraic ratio → Geometric ratio

Physical property

stage 4

Ratio table

"Find one" strategy

Intensive units

Fraction as quotient

Number line

Algebraic equations

Function table

Graphs in coordinate plane

Rate of change

Unit rate strategy

Parametric coefficient

Algebraic ratio

Geometric ratio

Physical property

stage 5

Ratio table

"Find one" strategy

Intensive units

Fraction as quotient

Number line

Algebraic equations

Function table

Graphs in coordinate plane

Rate of change

Unit rate strategy

Parametric coefficient

Algebraic ratio

Geometric ratio

Physical property

stage 6

Ratio table — "Find one" strategy — Intensive units — Fraction as quotient — Number line — Algebraic equations — Function table — Graphs in coordinate plane — Rate of change — Unit rate strategy — Parametric coefficient — Algebraic ratio — Geometric ratio — Physical property

progression of learning

stage 1

Ratio table ↔ "Find one" strategy ↔ Intensive units   Fraction as quotient

Number line   Algebraic equations ↔ Function table ↔ Graphs in coordinate plane

Rate of change

Unit rate strategy
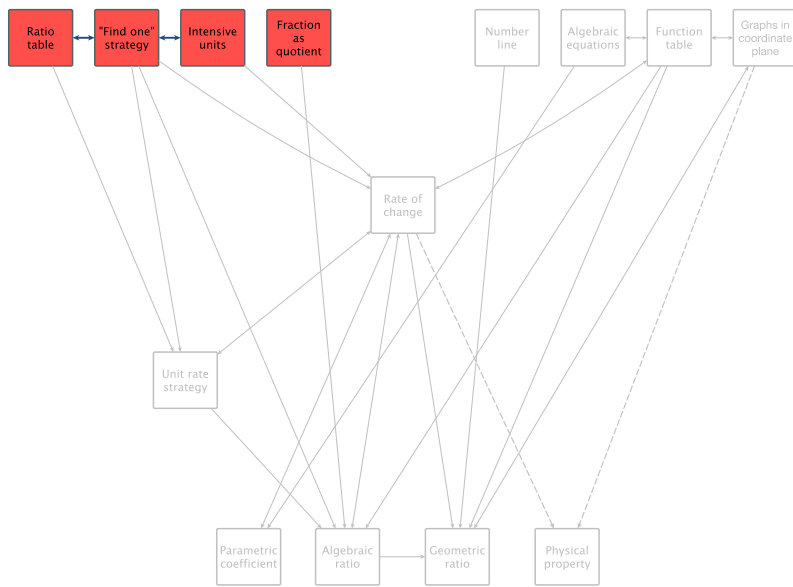
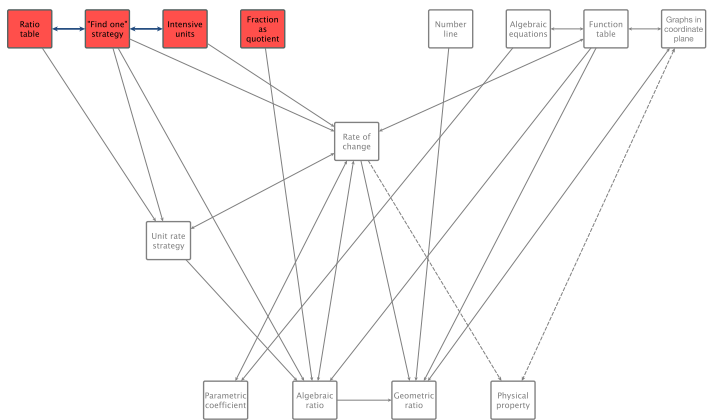Parametric coefficient   Algebraic ratio → Geometric ratio   Physical property

stage 1

Reinvented & objectified
- ratio table
- "find one" strategy
- intensive units (many-to-one)
- fraction-as-quotient

Activities
- "partitive division" situations
  - fair sharing
  - find unit values given many-to-many

Stage		Artifacts	Characteristics of tasks
1		Reinvented & objectified: • Ratio table • "find one" strategy • Intensive units • Fraction-as-quotient	Tasks that involve the activity of partitive division, including: • finding fair shares • finding unit values

Example activities:

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1. Three children shared five chocolate bars equally. How much chocolate did each child receive?

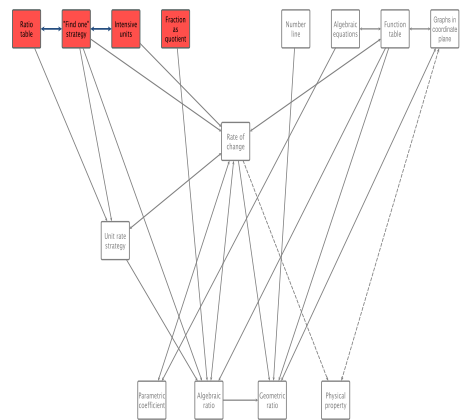State your final answer using units: _____   _____ *per* _____ .

2.

3 pizzas cost 12 dollars

1 pizza costs _____ dollars

_____   _____ *per* _____ .

3 pizzas cost 12   dollars

_____ pizzas costs 1 dollar

_____   _____ *per* _____ .

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Stage		Artifacts	Characteristics of tasks
1		Reinvented & objectified: • Ratio table • "find one" strategy • Intensive units • Fraction-as-quotient	Tasks that involve the activity of partitive division, including: • finding fair shares • finding unit values

Example activities:

1. Three children shared five chocolate bars equally. How much chocolate did each child receive?

   State your final answer using units: _____   _____ *per* _____ .

Fraction-as-quotient

Intensive units

2.

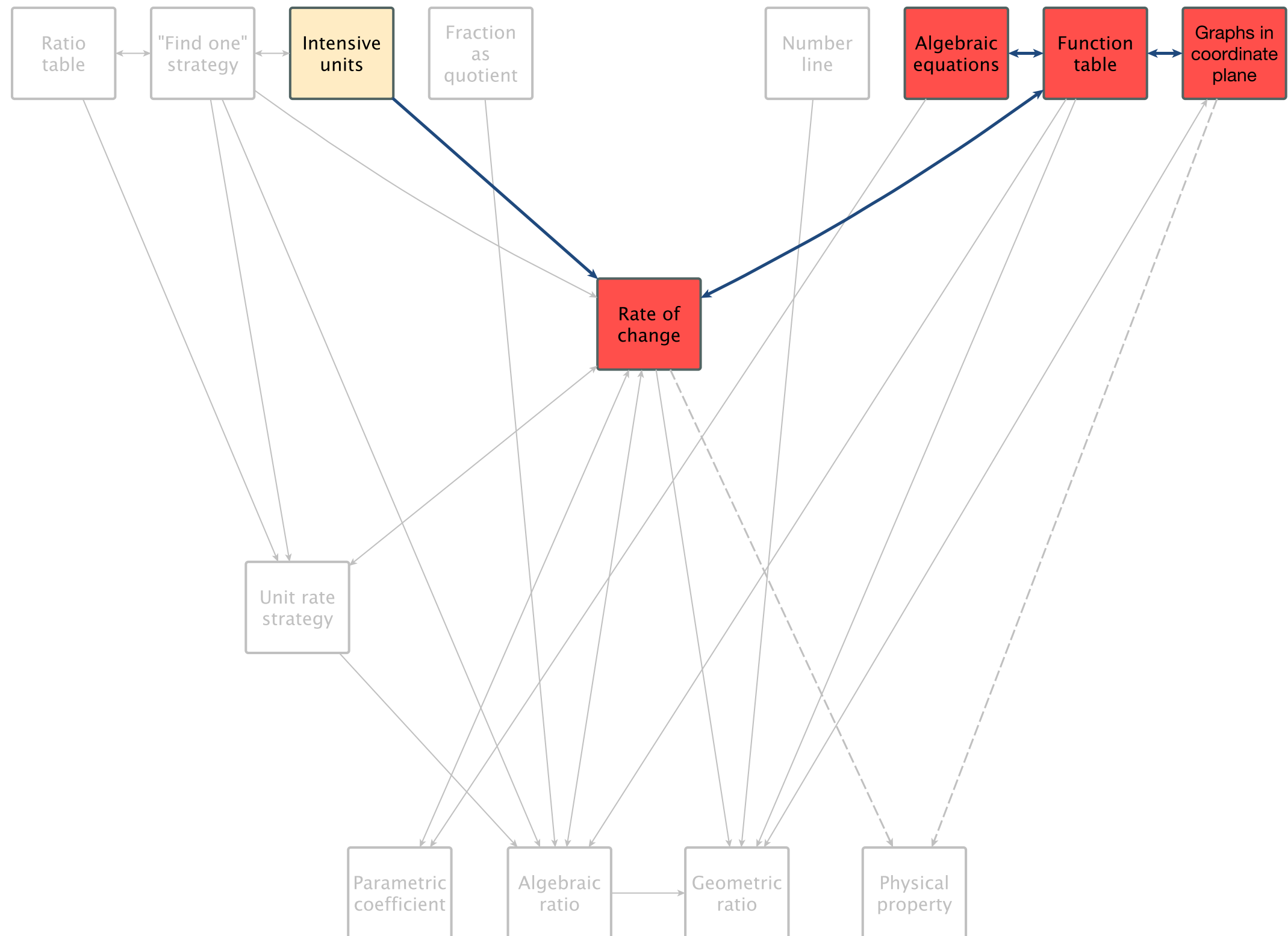   3 pizzas cost 12 dollars   3 pizzas cost 12 dollars
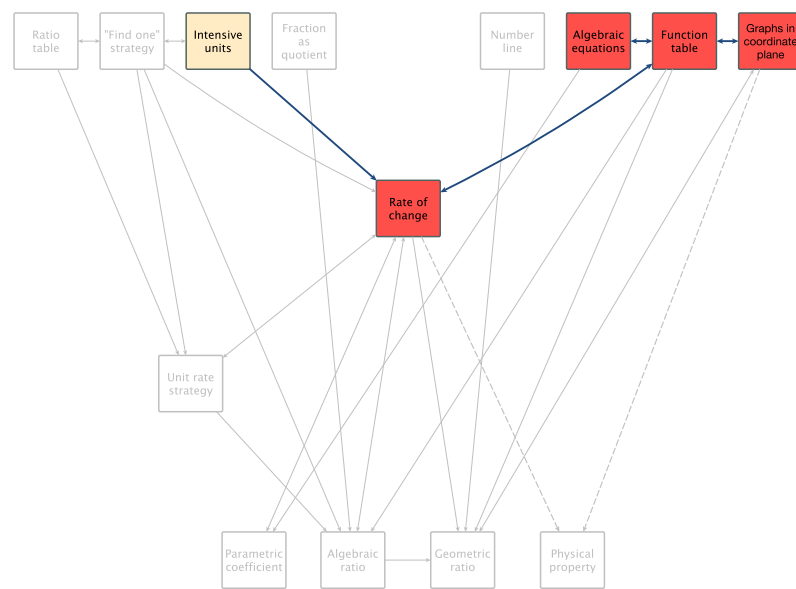
   1 pizza costs ___ dollars   ___ pizzas costs 1 dollar

Ratio table

"find one" strategy intensive units

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stage 2

Ratio table

"Find one" strategy

Intensive units

Fraction as quotient

Number line

Algebraic equations

Function table

Graphs in coordinate plane

Rate of change

Unit rate strategy

Parametric coefficient

Algebraic ratio

Geometric ratio

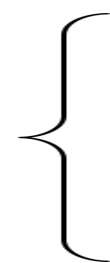Physical property

stage 2

Reinvented
& objectified {
- function tables
- algebraic equations
- graphs in coord. plane
- rate of change

Assembled
& coordinated {
- intensive units

Activities {
- find and continue patterns
- convert between multiple representations of functions

Stage	Artifacts	Characteristics of tasks
2	Assembled and coordinated: • Intensive units Reinvented & objectified: • Algebraic equations • Function tables • Graphs in coord. plane • Rate of change	Tasks that involve: • Finding and continuing patterns in geometric figures and tables of values, where there is a "starting value" and the independent variable increases by 1 • Converting between multiple representations of functions (focusing on table rows and points in the plane as *solutions* to two-variable equations)
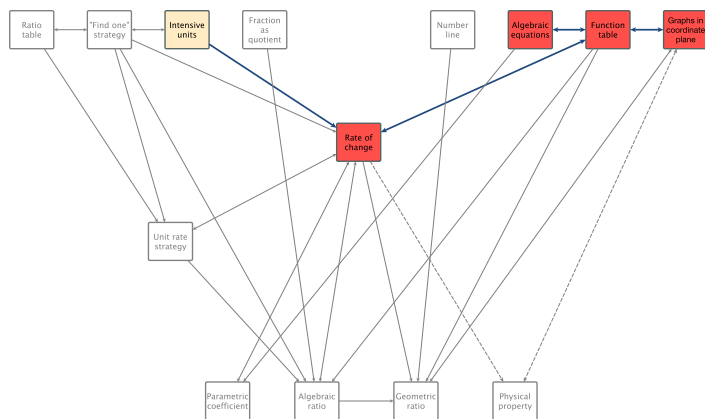
Example activities:

Plain cheese (no toppings) - - - - - - - - - - - - - - - - - $6.95

1 topping - $8.20

2 toppings - $9.45

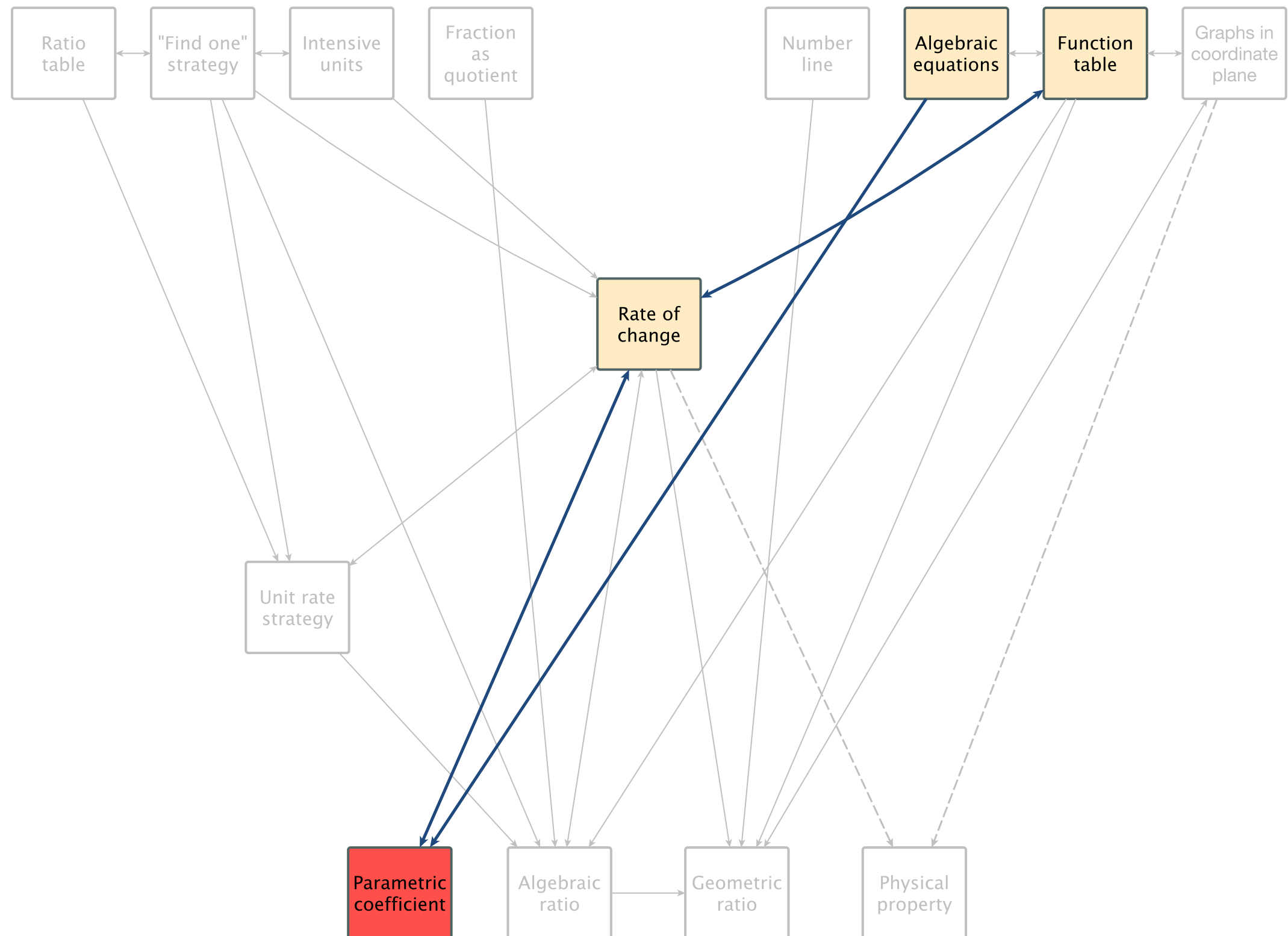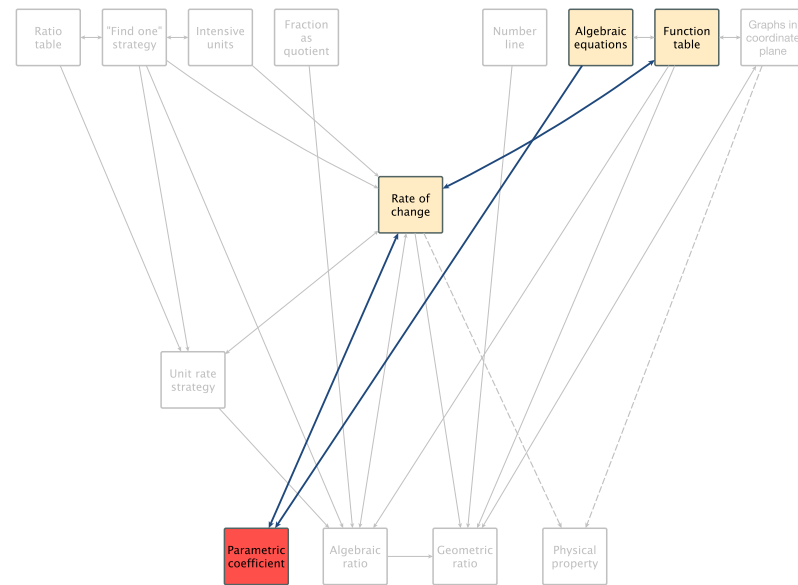3 toppings - $10.70

Find patterns: What patterns do you notice in the prices?

Find a rule: Write a rule that you can use to predict the total cost of a pizza if you know the number of toppings.

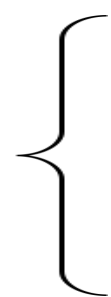| *Write the rule as an equation*	*Write the rule with an arrow chain*

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stage 3

Ratio table ←→ "Find one" strategy ←→ Intensive units

Fraction as quotient

Number line

Algebraic equations ←→ **Function table** ←→ Graphs in coordinate plane

Rate of change

Unit rate strategy

Parametric coefficient

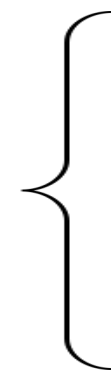Algebraic ratio → Geometric ratio

Physical property
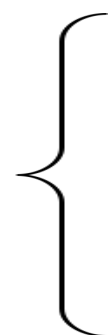
stage 3

Reinvented & objectified

- parametric coefficient

Assembled & coordinated

- algebraic equations
- function tables
- rate of change

Activities

make predictions given:
- rate and start
- well-ordered function table ($\triangle x = 1$)

Stage	Artifacts	Characteristics of tasks
3	Assembled and coordinated: • Algebraic equations • Rate of change Reinvented & objectified: • Parametric coefficient Objectified • Rate of change • Function tables	Making predictions in linear situations, given: • The rate of change and starting value • Multiple data points (e.g., in a table), where the independent variable increases by one.

Example activities

1.

(just search Google for "at the current rate" (in quotes))

Class discussion:
- "What prediction does the author make?"
- "How does the author make this prediction?"
- "Why does multiplication make a prediction?"

Goal is to discipline perception to the role of rates and multiplication to make predictions.

2. The table below shows the cost of shipping used X-box games from CHEEP GAMZ ONLINE. Some of the data is missing. Based on the data in the table, how much would it cost to have 12 games shipped?

Number of games	Total cost
0	
1	
2	20.00
3	26.00
4	32.00
5	38.00
6	44.00

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Apple already building iPhones at rate of 40 million a year?

By Slash Lane

Apple is reportedly testing the limits of its overseas manufacturing facilities in order to keep up with demand for the new iPhone 3G, with production already cranked nearly sevenfold compared to the first-generation model.

Foxconn, the company's Taiwanese handset and iPod manufacturer, has recently ramped production of the new iPhone to 800,000 units per week, says *TechCrunch*, citing a person 'close to Apple with direct knowledge of the numbers.'

The build rate is said to be 'above current full capacity' for the Foxconn facilities alloted to Apple's handset business, which has led to concerns that quality control may suffer. At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months.

That paces well ahead of analysts' estimates (1, 2, 3) and early reports that suggested Apple's initial iPhone 3G orders spanned only 25 million units through the expected lifespan of the product.

TechCrunch believes Apple's initial order was actually 40 million units over the course of the first twelve months, but is now hearing that 'those numbers are being revised upwards sharply.'

Apple said it sold 1 million iPhones in the first 72 hours the new iPhone 3G was put on sale, but has not provided an updated sales tally since. The iPhone is currently on sale in 23 countries, with 20 more expected to be added on August 22nd, and another 30 by the end of the calendar year.

... 800,000 units per week ...

... At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months ...

800,000
52

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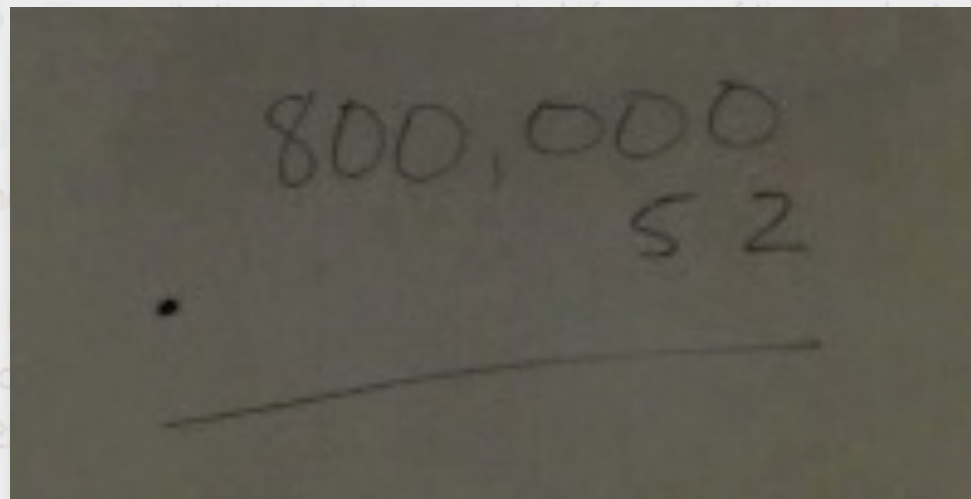
FAP: Randy why is that [multiplication] going to get us a prediction for the number of iPhones in a year? How does weeks turn into iPhones?

Randy: Because for every week you have, you produce a certain amount of iPhones, so if you multiply it by a certain amount of weeks, the amount of iPhones will go up. [The reason-

FAP: [For every-

Randy: -that might be important is for (investors to know)

Rate as many-per-one

FAP: Randy why [... on] going to get us a pre ... r of iPhones in a ye ... rn into iPhones?

Randy: Because **for every week you have, you produce a certain amount of iPhones,** so if you multiply it by a certain amount of weeks, the amount of iPhones will go up. [The reason-

FAP: [For every-

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Randy: Because **for every week you have, you produce a certain amount of iPhones,** so **if you multiply it by a certain amount of weeks, the amount of iPhones will go up.** [The reason-

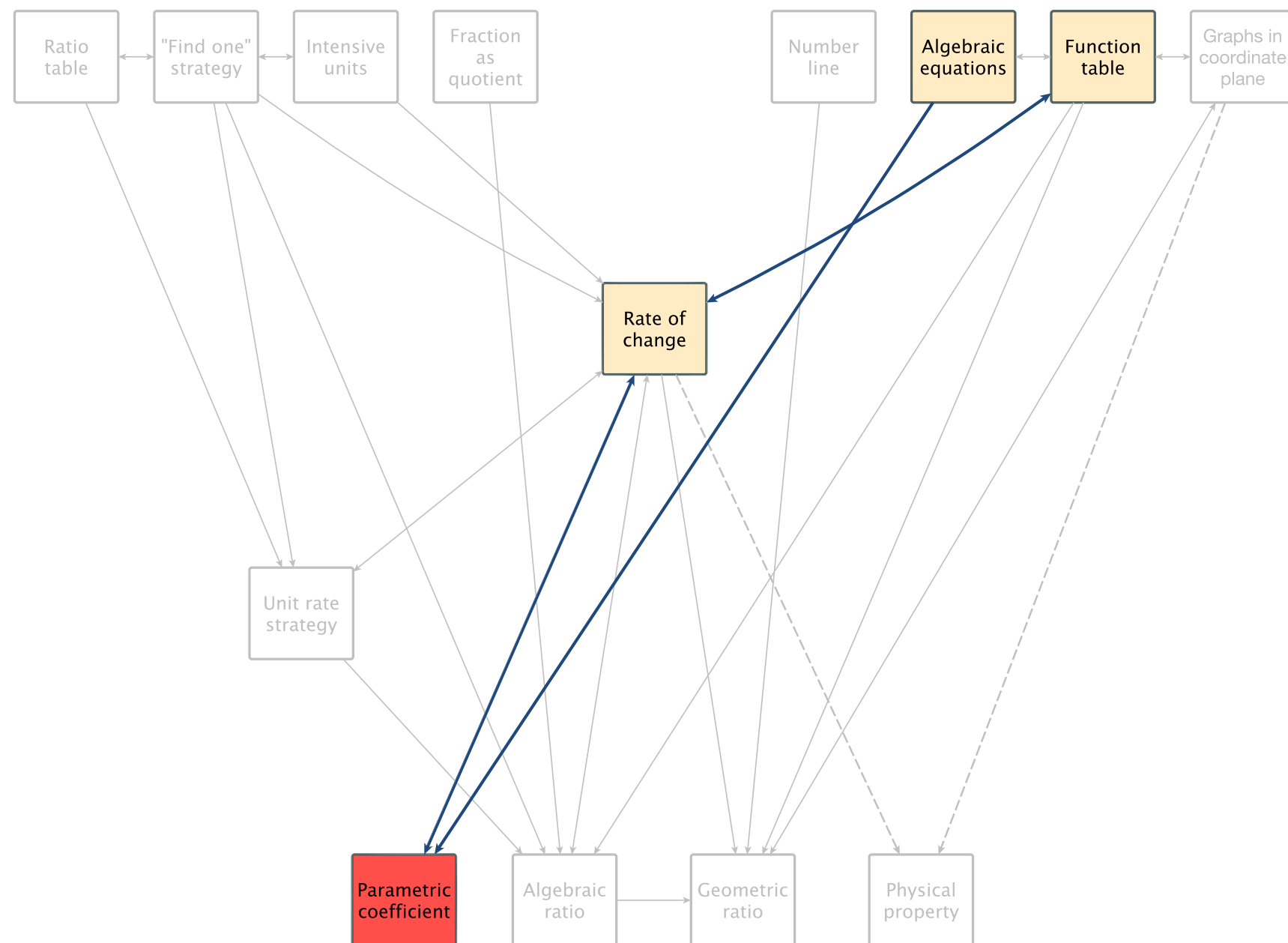FAP: [For every-
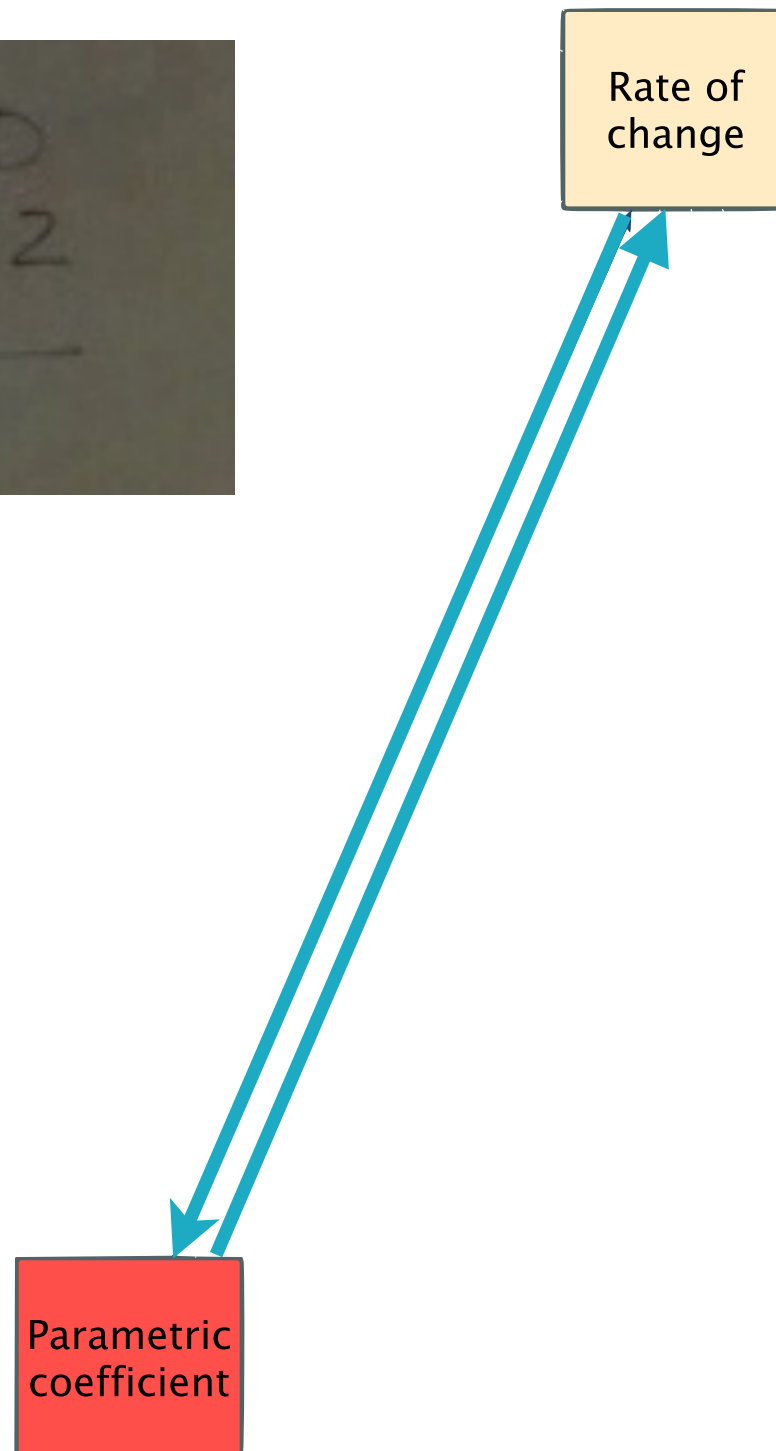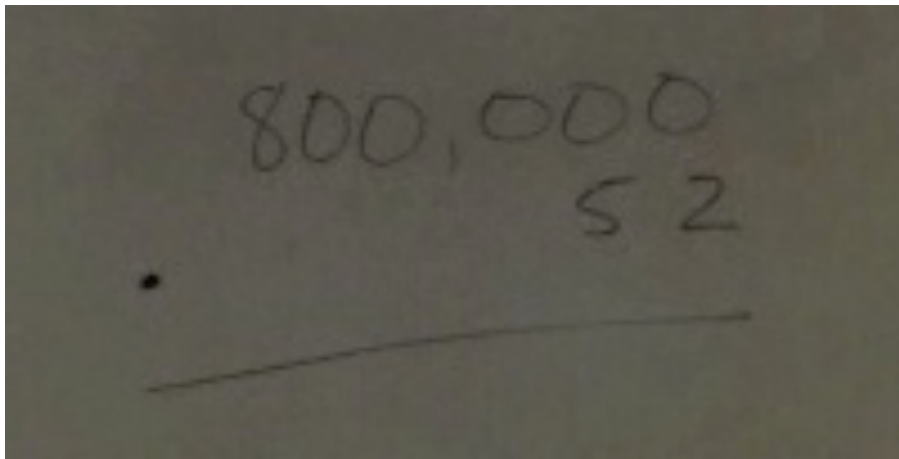
Randy: -that m[   ]r (investors to

Rate as many-per-one

Rate as number that can be accumulated

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Up and down in the cascade

Up and down in the cascade
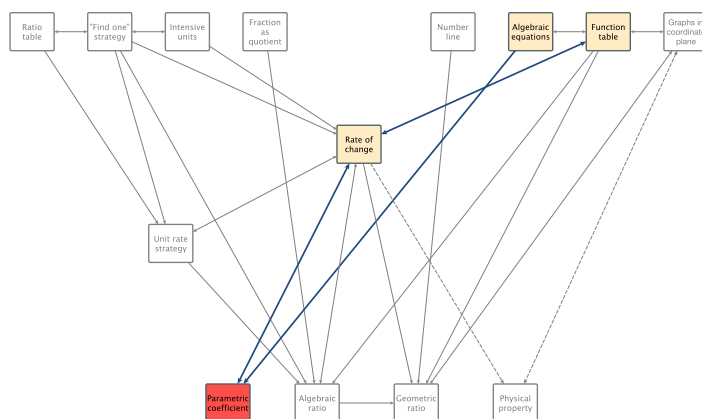
800,000
52

Rate of change

Parametric coefficient

Stage	Artifacts	Characteristics of tasks

3

Assembled and coordinated:

- Algebraic equations
- Rate of change

Reinvented & objectified:

- Parametric coefficient

Objectified

- Rate of change
- Function tables

Making predictions in linear situations, given:

- The rate of change and starting value
- Multiple data points (e.g., in a table), where the independent variable increases by one.

Example activities

1.

Monday, August 04, 2008, 07:00 am PT (10:00 am ET)

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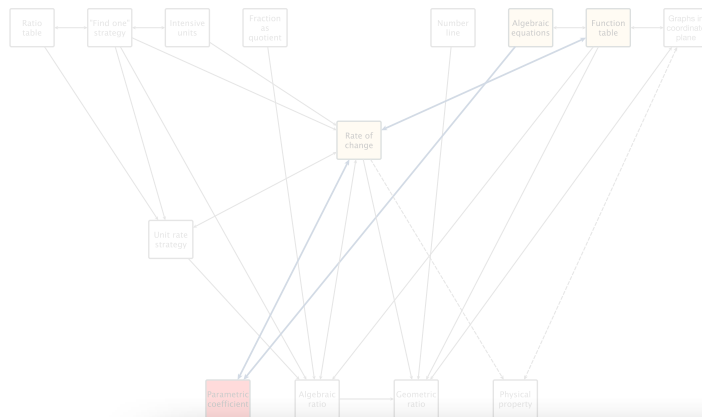| Stage	Artifacts	Characteristics of tasks
3	Assembled and coordinated: • Algebraic equations • Rate of change Reinvented & objectified: • Parametric coefficient Objectified • Rate of change • Function tables	Making predictions in linear situations, given: • The rate of change and starting value • Multiple data points (e.g., in a table), where the independent variable increases by one.

Example acti

1.

(ju

Class disc

Goal is to discipline perception to the role of rates and multiplication to make predictions.

The table shows the cost of shipping XBox games

Rate of change:

2 dollars per game

Number of games	Total cost
0	
1	
2	8.00
3	10.00
4	12.00
5	14.00
6	16.00

+1

+2

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stage 4

Ratio table

"Find one" strategy

Intensive units

Fraction as quotient

Number line

Algebraic equations

Function table

Graphs in coordinate plane

Rate of change

Unit rate strategy

Parametric coefficient

Algebraic ratio

Geometric ratio

Physical property
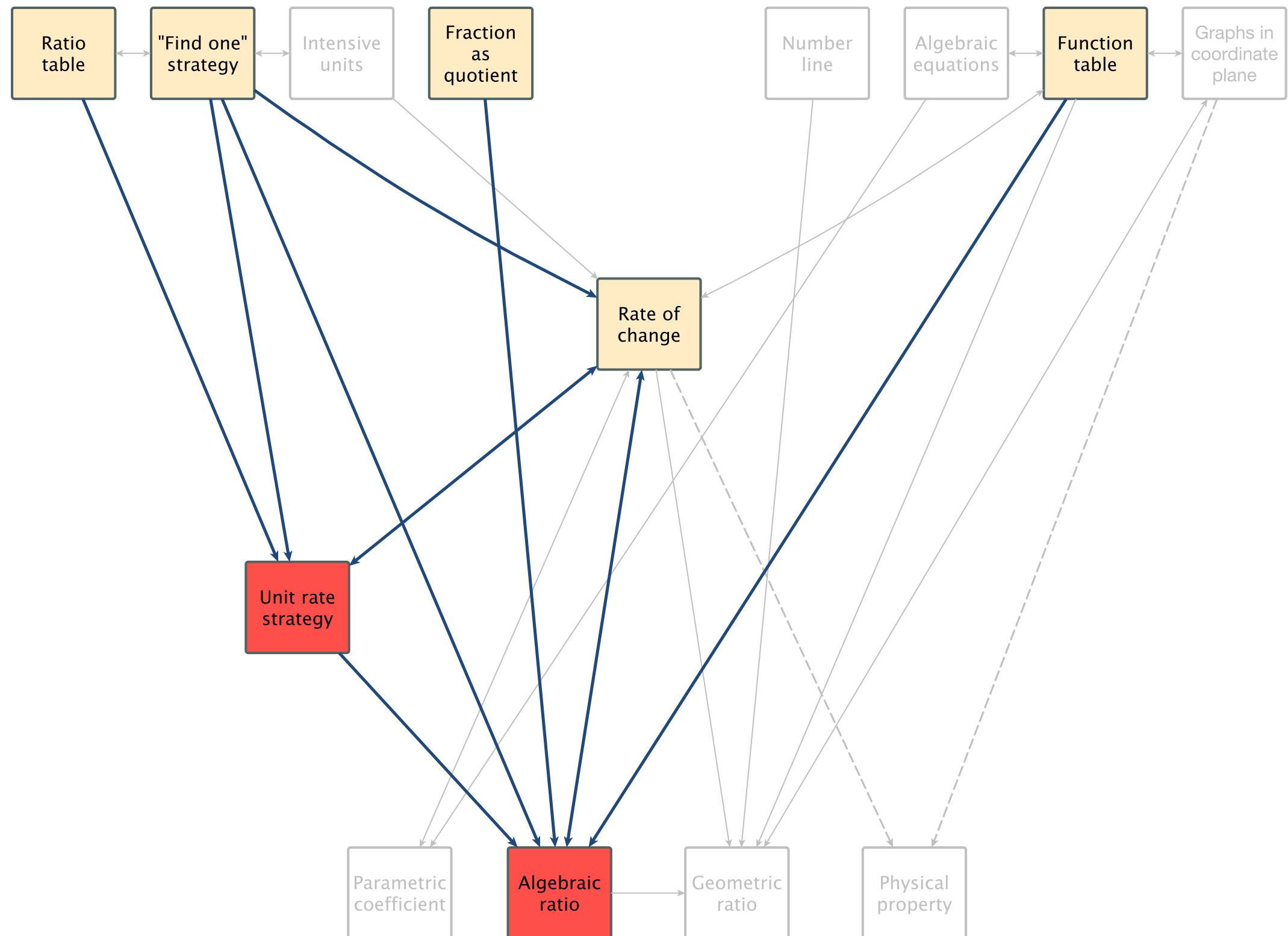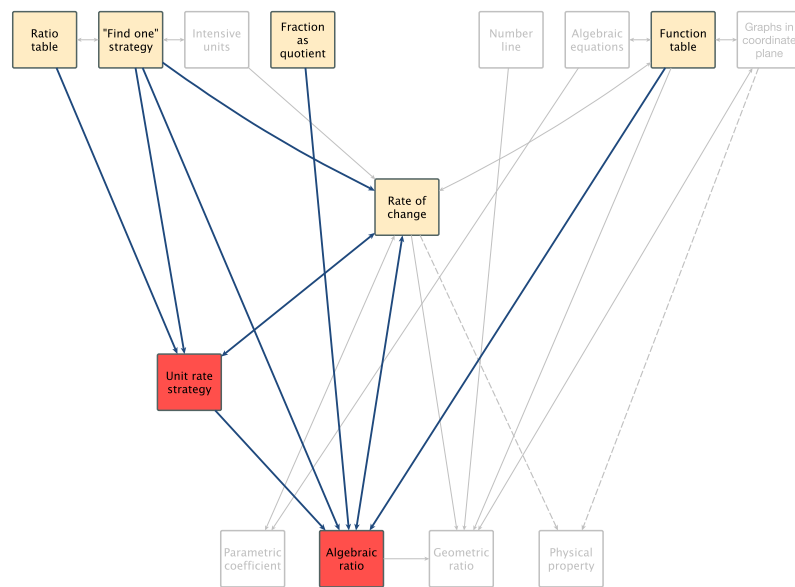
stage 4

Reinvented
& objectified

- unit rate strategy
- algebraic ratio

Assembled
& coordinated

- ratio table
- "find one" strategy
- fraction as quotient
- rate of change
- function tables

Activities

make predictions given:
- one value in proportional situation
- two data points with $\triangle x \neq 1$

Stage	Artifacts	Characteristics of tasks
4	**Assembled and coordinated:** • "Find one" strategy • Ratio table • Fraction as quotient • Function tables • Rate of change **Reinvented & objectified:** • Unit rate strategy • Algebraic ratio **Objectified** • Rate of change	**Make predictions in linear situations given:** • A single data point, for situations where the values of the variables are proportional • Two data points, for situations where there is a starting value. Problem contexts should be chosen to make clear the distinction between changes and values.

Example activities:

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1. Unit rate strategy

Ms. Magro runs six miles every day. On average it takes her 54 minutes to run six miles. At this rate, how long will it take Ms. Magro to run an 11-mile race?

---

2. Algebraic ratio

At the end of the summer, the YMCA drains their swimming pool. Raif and Julie are in charge of measuring the height of the pool as it drains.

Raif says: I checked the pool two hours after we started draining it. When I checked, the height of the water was 517 mm.

Julie says: I checked the pool seven hours after we started draining it. When I checked, the height of the water was 420 mm.

Imagine you checked the height of the pool five hours after the YMCA started draining it. What would the height have been?

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NCTM 2014

stage 4

Ratio table

"Find one" strategy

Rate of change

Unit rate strategy

make predictions given one value in proportional situation

Ms. Magro runs 6 miles every day. On average, she can run six miles in 54 minutes. At this rate, how long would it take Ms. Magro to run an 11-mile race?

$$6 \div \left(\begin{array}{l} 6 \text{ mile} > \\ 1 \text{ mile} \end{array} \right.$$ $$\begin{array}{l} 54 \text{ min} \\ ) \div 6 \\ 9 \text{ min} \end{array}$$ Takes 99 minutes

$$9 \times 11 = 99.$$

State your final answer using units: _____

stage 4

2.

Rate of change

Unit rate strategy

make predictions given one value in proportional situation

3 pizzas cost 12 dollars

1 pizza costs _____ dollars

Ms. Magro runs 6 miles every day. On average, she can run six miles in 54 minutes. At this rate, how long would it take Ms. Magro to run an 11-mile race?

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$6 \div \left(\begin{matrix} 6 \text{ mile} > \\ 1 \text{ mile} \end{matrix}\right.$ $\begin{matrix} 54 \text{ min} \\ ) \div 6 \\ 9 \text{ min} \end{matrix}$ $9 \times 11 = 99.$ Takes 99 minutes

stage 4

Ratio table

"Find one" strategy

Rate of change

Unit rate strategy

make predictions given one value in proportional situation

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Takes 99 minutes

stage 4

Ratio table

"Find one" strategy

Rate of change

Unit rate strategy

make predictions given one value in proportional situation

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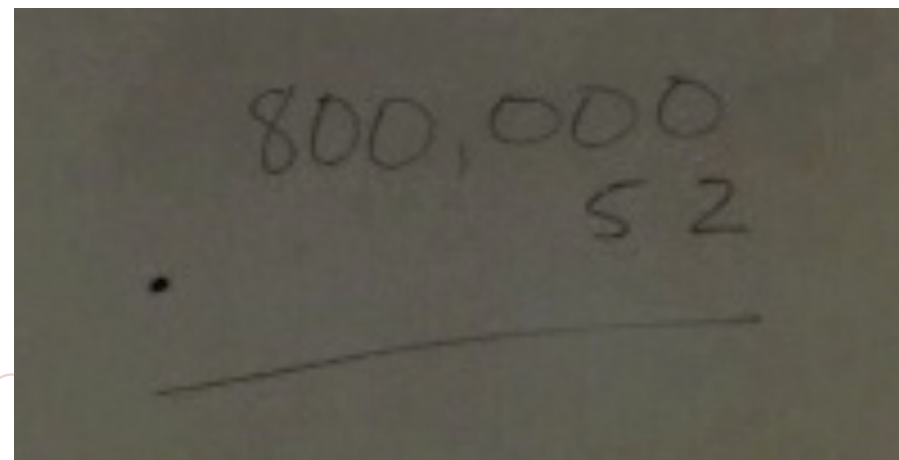
$9 \times 11 = 99.$

Takes 99 minutes

Ratio table

"Find one" strategy

Because **for every week you have, you produce a certain amount of iPhones,** so **if you multiply it by a certain amount of weeks, the amount of iPhones will go up**.

Rate of change

make predictions given proportional situation

Unit rate strategy

800,000
52

Ms. Magro runs 6 miles every day. On average, she can run six miles in 54 minutes. At this rate, how long would it take Ms. Magro to run an 11-mile race?

$6 \div ($ 6 mile >   54 min
1 mile   ) ÷ 6
9 mn

9 × 11 = 99.

Takes 99 minutes

Ratio table

"Find one" strategy

stage 4

Rate of change

Unit rate strategy

make predictions given one value in proportional situation

Ms. Magro runs 6 miles every day. On average, she can run six miles in 54 minutes. At this rate, how long would it take Ms. Magro to run an 11-mile race?

$$6 \div \begin{pmatrix} 6 \text{ mile} > \\ 1 \text{ mile} \end{pmatrix} \quad \begin{matrix} 54 \text{ min} \\ ) \div 6 \\ 9 \text{ min} \end{matrix} \quad 9 \times 11 = 99 . \quad \text{Takes } 99 \text{ minutes}$$

stage 4

Ratio table

"Find one" strategy

Fraction as quotient

Rate of change

Unit rate strategy

Algebraic ratio
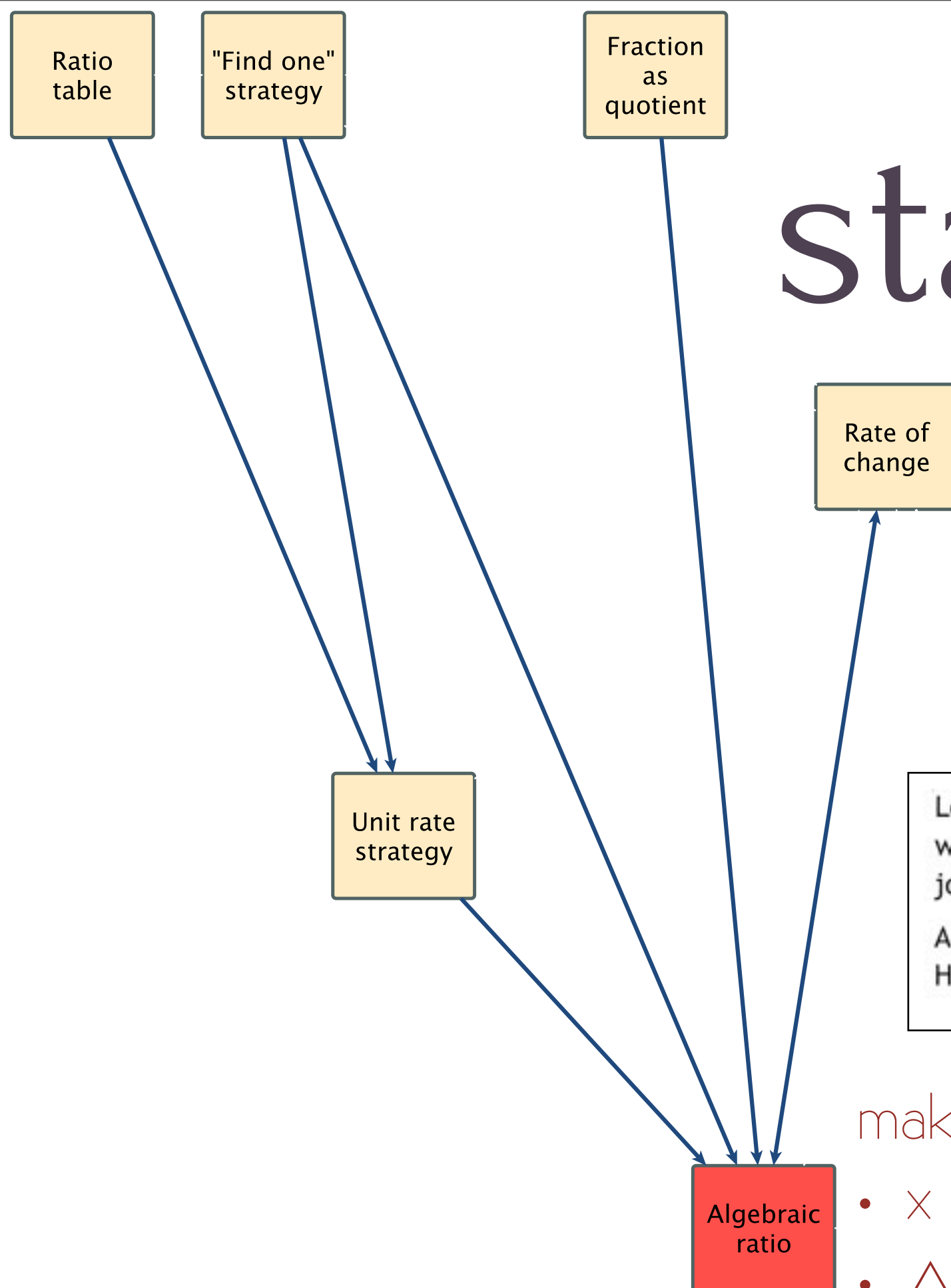
Leslie is a window installer. On Friday, she installed two windows, and charged 402 dollars. Last week, on another job, she charged 517 dollars to install seven windows.

A new customer has asked Leslie to install five windows. How much will this cost?

make predictions given two data points:
- x & y not proportional
- $\triangle x \neq 1$

Leslie is a window installer. On Friday, she installed two windows, and charged 402 dollars. Last week, on another job, she charged 517 dollars to install seven windows.

A new customer has asked Leslie to install five windows. How much will this cost?

2 windows = 402 dollars

7 windows = 517 dollars

$\frac{517}{7}$ = 1 window

$\frac{517}{7}$ •5

364 = 5 windows

$2 = 402$

$\frac{402}{2}$ = 1 window).5

$1,005$ = 5 windows

$7 = 517$

$5 \cdot \frac{517}{7} + \left(\frac{2585}{7}\right) \rightarrow 364$

$\frac{402}{2}$ •5 = $\frac{2010}{2}$ = $\frac{1005}{1}$

23

0	$356
1	$379
2	$402
3	$425
4	$498
5	$471
6	$494
7	$517

5-23

+23

402
23
498
23
471
23
494
23
517

517
-402
115
$\frac{115}{5}$ =23

402
-23
379
-23
356

Rate of change = $23 per window
starting cost = $356
5 windows = $425

Leslie is a window installer. On Friday, she installed two windows, and charged 402 dollars. Last week, on another job, she charged 517 dollars to install seven windows.

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2 windows = 402 dollars

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$\frac{517}{7}$ = 1 window
• 5
$\frac{\$364}{}$ = 5 windows

$\frac{\$402}{2}$ = 1 window) • 5
$\frac{\$1005}{}$ = 5 windows

$\frac{5}{1} \cdot \frac{517}{7} \left(\frac{8585}{7} \right) \rightarrow 364$

$\frac{402}{2}$ $\frac{402}{2} \cdot 5 = \frac{2010}{2} = 1005$

23

486

0	$356
1	$379
2	$402
3	$425
4	$498
5	$471
6	$494
7	$517

5 - 23
+23

402
23
498
23
471
23
494
23
517

517
- 402
115 : 5 = 23

402
- 23
379
- 23
356

Rate of change = $23 per window
starting cost = $356
5 windows = $475

Leslie is a window installer. On Friday, she installed two windows, and charged 402 dollars. Last week, on another job, she charged 517 dollars to install seven windows.

A new customer has asked Leslie to install five windows. How much will this cost?

$$\frac{517}{7} = 1 \text{ window}$$

.5 → $364 = 5 windows

$$\frac{402}{2} = 1 \text{ window} \quad).5$$

$1,005 = 5 windows?

23

0	$356
1	$379
2	$402
3	$425
4	$498
5	$471
6	$494
7	$517

Rate of change = $23 per window
starting cost = $356
5 windows = $475

Leslie is a window installer. On Friday, she installed two windows, and charged 402 dollars. Last week, on another job, she charged 517 dollars to install seven windows.

A new customer has asked Leslie to install five windows. How much will this cost?

2 windows = 402 dollars

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$\frac{517}{7}$ = 1 window

$364 = 5 windows

2 = 402

$\frac{402}{2}$ = 1 window

$1,005 = 5 windows

7 = 517

$5 \cdot \frac{517}{7} + (\frac{2585}{7}) \rightarrow 364$

$\frac{402}{2}$

$\frac{402}{2} \cdot \frac{5}{1} = \frac{2010}{2} = \frac{1005}{1}$

23

0	$356
1	$379
2	$402
3	$425
4	$498
5	$471
6	$404
7	$517

5 - 23

+23

402
23
498
23
471
23
494
23
517

517
- 402
115 : 5 = 23

402
- 23
379
- 23
356

Rate of change = $23 per window
starting cost = $356
5 windows = $425

stage 4

Ratio table

"Find one" strategy

Fraction as quotient

Rate of change

Unit rate strategy

Algebraic ratio

2. Algebraic ratio

At the end of the summer, the YMCA drains their swimming pool. Raif and Julie are in charge of measuring the height of the pool as it drains.
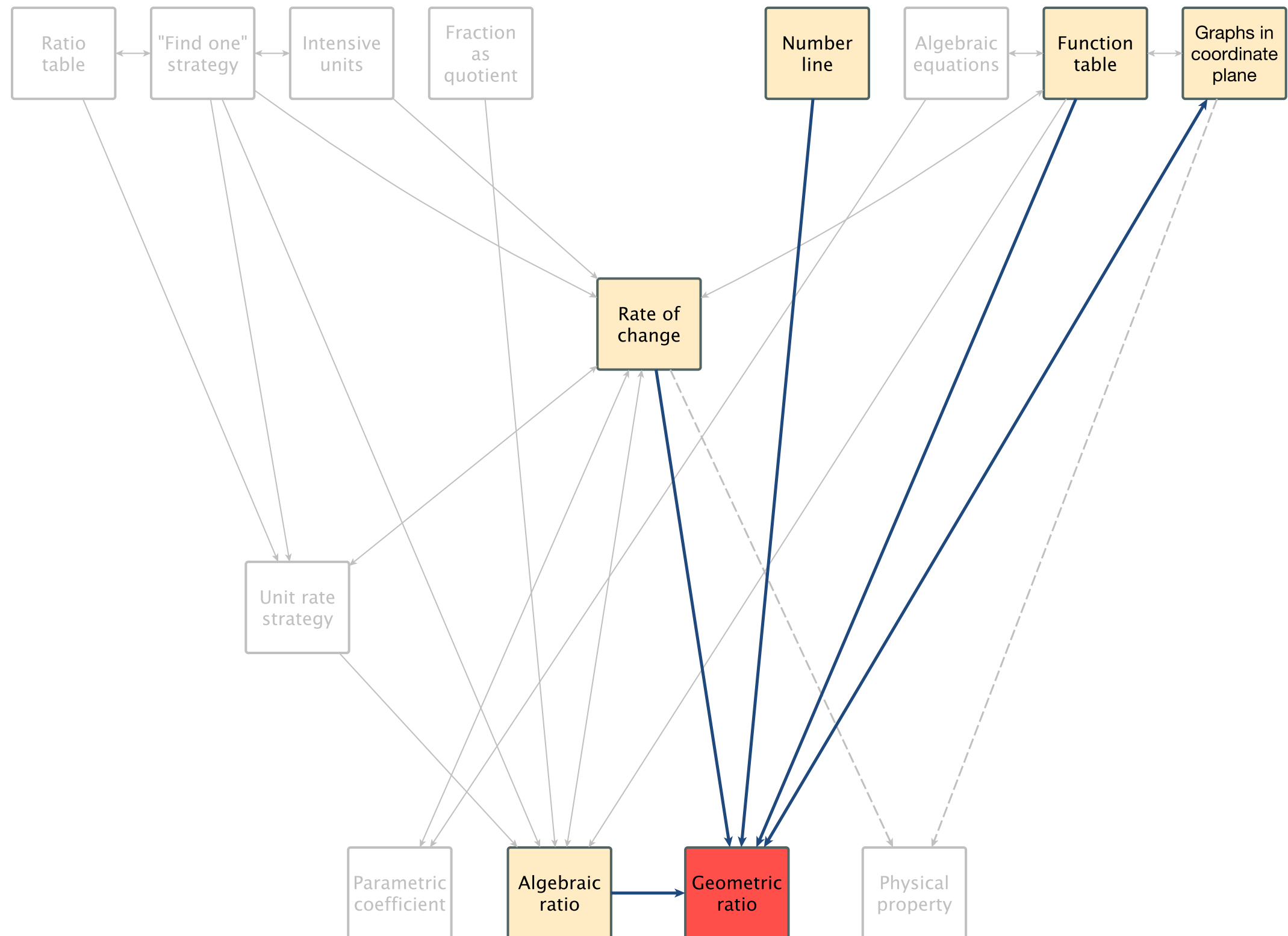
Raif says: I checked the pool two hours after we started draining it. When I checked, the height of the water was 517 mm.
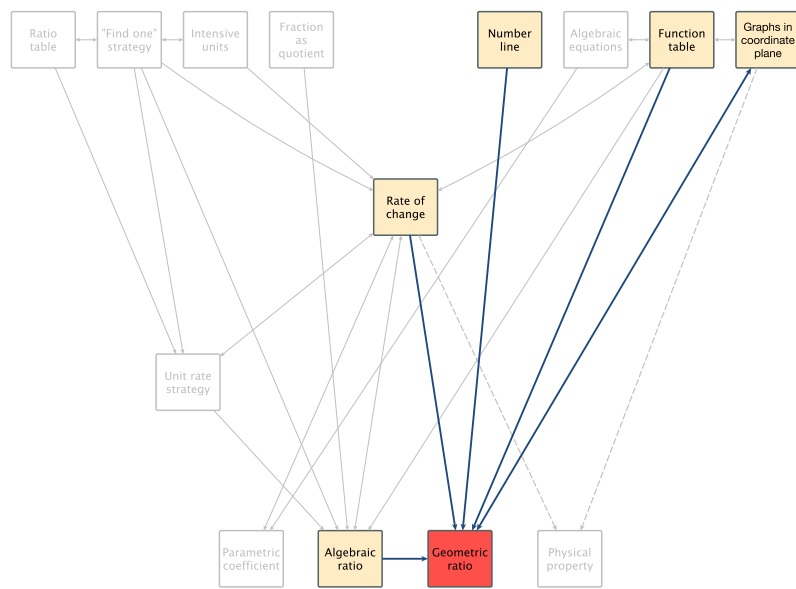
Julie says: I checked the pool seven hours after we started draining it. When I checked, the height of the water was 420 mm.

Imagine you checked the height of the pool five hours after the YMCA started draining it. What would the height have been?

stage

Ratio table

"Find one" strategy

Fraction as quotient

Rate of change

Dynamic

Negative change where negative value doesn't make sense

Algebraic ratio

2. Algebraic ratio

At the end of the summer, the YMCA drains their swimming pool. Raif and Julie are in charge of measuring the height of the pool as it drains.

Raif says: I checked the pool two hours after we started draining it. When I checked, the height of the water was 517 mm.

Julie says: I checked the pool seven hours after we started draining it. When I checked, the height of the water was 420 mm.

Imagine you checked the height of the pool five hours after the YMCA started draining it. What would the height have been?

stage 5

Ratio table

"Find one" strategy

Intensive units

Fraction as quotient

Number line

Algebraic equations

Function table

Graphs in coordinate plane

Rate of change

Unit rate strategy

Parametric coefficient
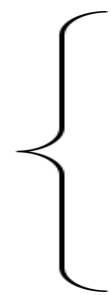
Algebraic ratio

Geometric ratio

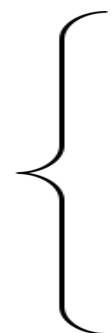Physical property

stage 5

Reinvented & objectified {
- geometric ratio

Assembled & coordinated {
- algebraic ratio
- rate of change
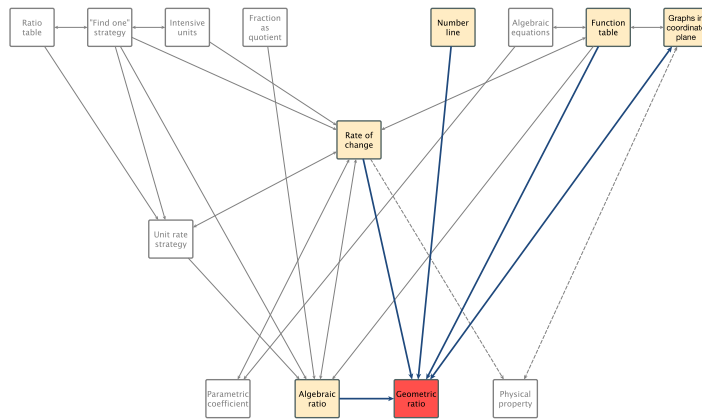- number line
- function tables
- graphs in coordinate plane

Activities {
- show change on number line diagrams
- make predictions given graph

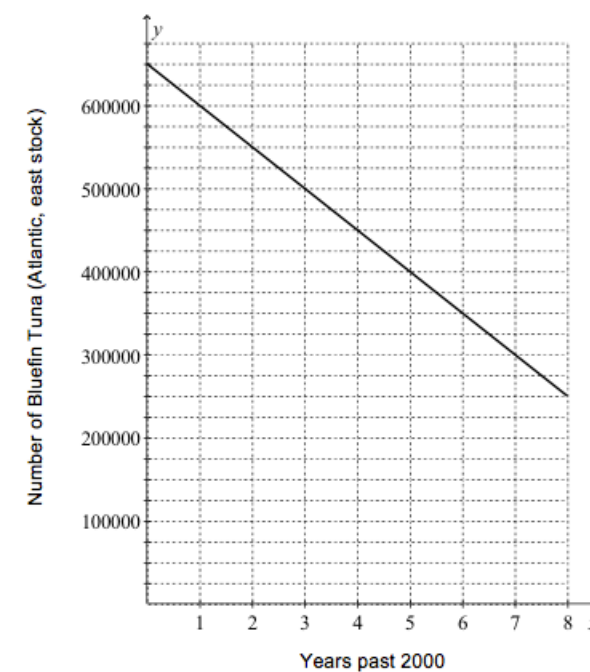| Stage	Artifacts	Characteristics of tasks
5	Assembled and coordinated: • Number line • Graphs in coord. plane • Rate of change • Algebraic ratio Reinvented & objectified: • Geometric ratio Objectified: • Graphs in coord. plane	• Show change on number-line diagrams. • Make predictions in linear situations where there is a starting value, given a graph of a function in a coordinate plane.

Example activities:

1. The temperature in Alamosa, Colorado rose from -12° to 24°.

   Draw an arrow on the number line below to show this change.
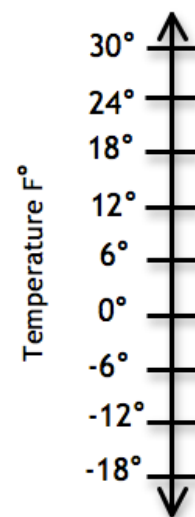
2.

   • What are two variables that are changing in the graph?
   • Write a sentence that describes how these two variables are changing
   • Find the rate of change in the graph
   • At this rate, when will the stock of Bluefin tuna be depleted?

Frederick Peck

NCTM 2014

Stage	Artifacts	Characteristics of tasks
5	Assembled and coordinated:	• Show change on number-line diagrams.
	• Number line	• Make predictions in linear situations where

Explain or show how you found the rate of change in the graph.

X	Y
2	550,000
4	450,000

$$450,000$$
$$-\ 550,000$$
$$-\ 100,000$$
$$\div\ \ \ \ \ \ \ 2$$
$$\boxed{-\ 50,000}$$

We picked 2 points of the graph, subtracted output₁ from output₂, and then ÷ that by 2 which was Input₂ – Input₁.

1. The temperature in Alamosa, Colorado rose from -12° to 24°.

   Draw an arrow on the number line below to show this change.

2.

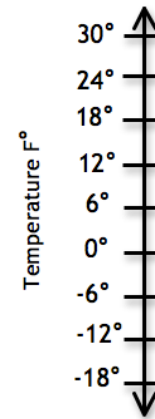$$2\left(\begin{array}{c|c} 1 & 600000 \\ \hline 3 & 500000 \end{array} \right) 100000$$

Thursday, April 10, 14

Stage	Artifacts	Characteristics of tasks

5

Explain or show how you

X	Y
2	550,000
4	450,000

1. The temperature in Alamosa, Colorado rose from -12° to 24°.

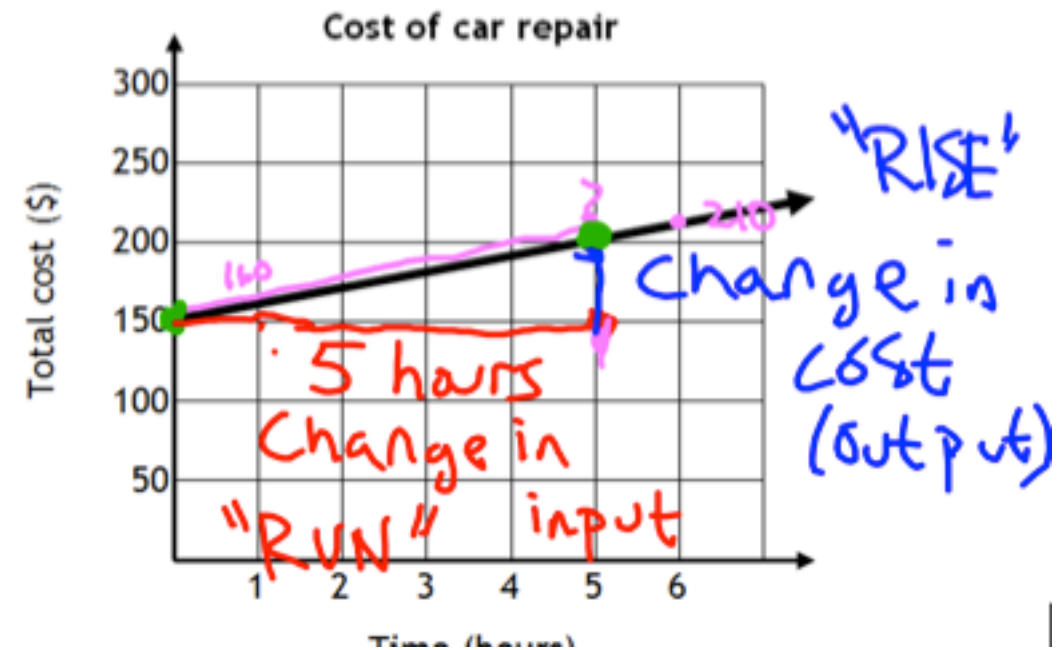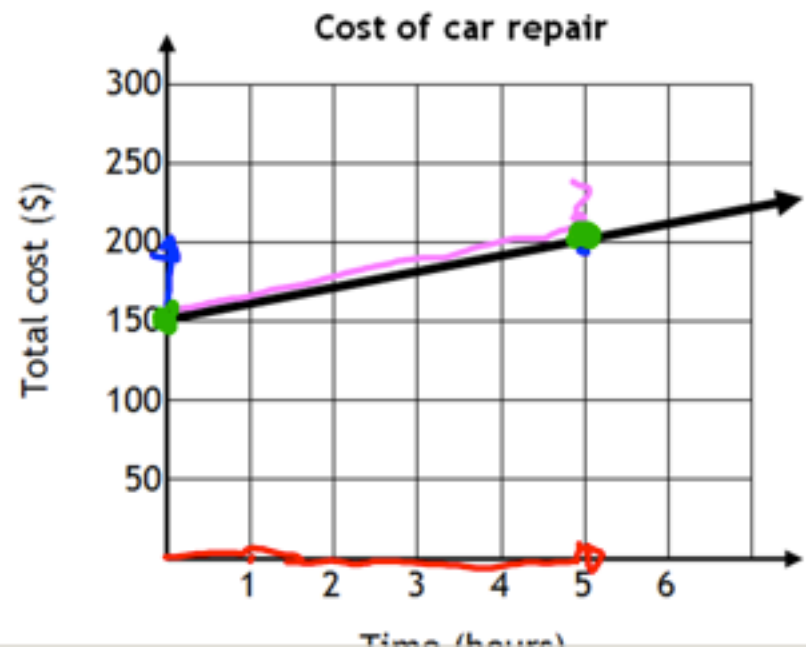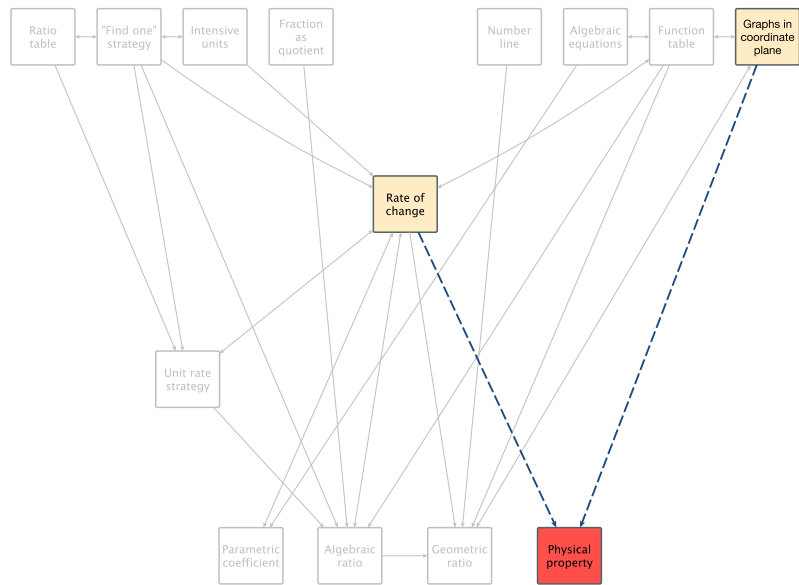Draw an arrow on the number line below to show this change.

Temperature F°

30°
24°
18°
12°
6°
0°
-6°
-12°
-18°

...w change on number-line diagrams

...e predictions i

...ph.

...Q point
...racted
...to 2) and
...which
...out 1.

Number of Bluefin Tuna (Atlantic, east stock)

600000
500000
400000
300000
200000
100000

1 2 3 4 5 6 7 8 x

Years past 2000

• What are two variables that are changing in the graph?
• Write a sentence that describes how these two variables are changing
• Find the rate of change in the graph
• At this rate, when will the stock of Bluefin tuna be depleted?

1. The temperature in Alamosa, Colora...
12° to 24°.

Draw an arrow on the number line below to show this
chang...

Frederick Peck

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www.RMEInTheClassroom.com

NCTM 2014

Cost of car repair

Total cost ($)
300
250
200
150
100
50

1 2 3 4 5 6
Time (hours)

Cost of car repair

Total cost ($)
300
250
200
150
100
50

1 2 3 4 5 6
Time (hours)

"RISE"
change in
cost
(output)

5 hours
Change in
"RUN" input

210
160

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Thursday, April 10, 14

stage 6

Ratio table

"Find one" strategy

Intensive units

Fraction as quotient

Number line

Algebraic equations

Function table

Graphs in coordinate plane

Rate of change

Unit rate strategy

Parametric coefficient
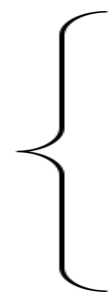
Algebraic ratio

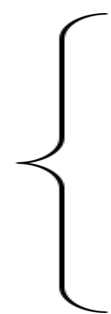Geometric ratio

Physical property

stage 6

Reinvented & objectified {
- physical property

Assembled & coordinated {
- rate of change
- graphs in coordinate plane

Activities {
- compare rates given graph of two intersecting linear functions
- measure steepness of objects

Stage	Artifacts	Characteristics of tasks
6	Assembled and coordinated: • Rate of change • Graphs in coord. plane Reinvented & objectified • Physical property	• Compare rates given two intersecting linear functions graphed in a coordinate plane. • Measure and compare the steepness of objects

Example activities

Charlie and Linus are running along a straight track. A position vs. time graph for both runners is shown below.
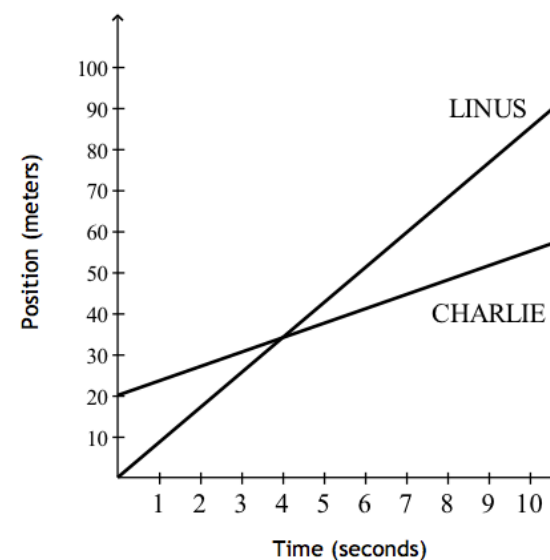
• At the instant, $t = 2$ sec, who is running faster, Charlie or Linus?

• Do Linus and Charlie ever have the same speed? If so, at what time?

8. Charlie and Linus are running along a straight track. A position vs. time graph for both runners is shown below.
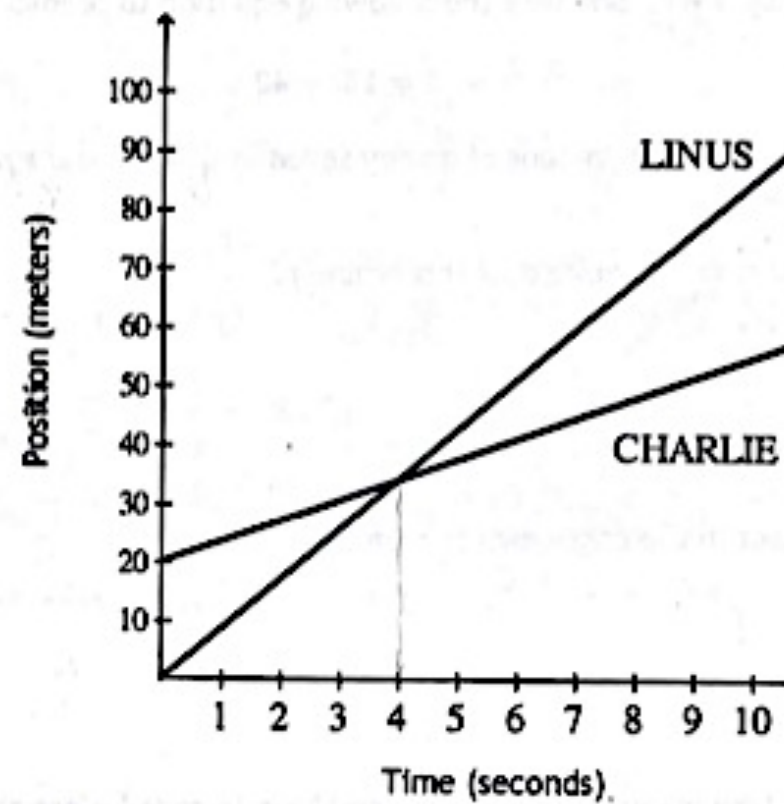
a. At the instant, $t = 2$ sec, who is running faster, Charlie or Linus? ___linus___

Explain your reasoning

linus es line is steeper so he is running faster

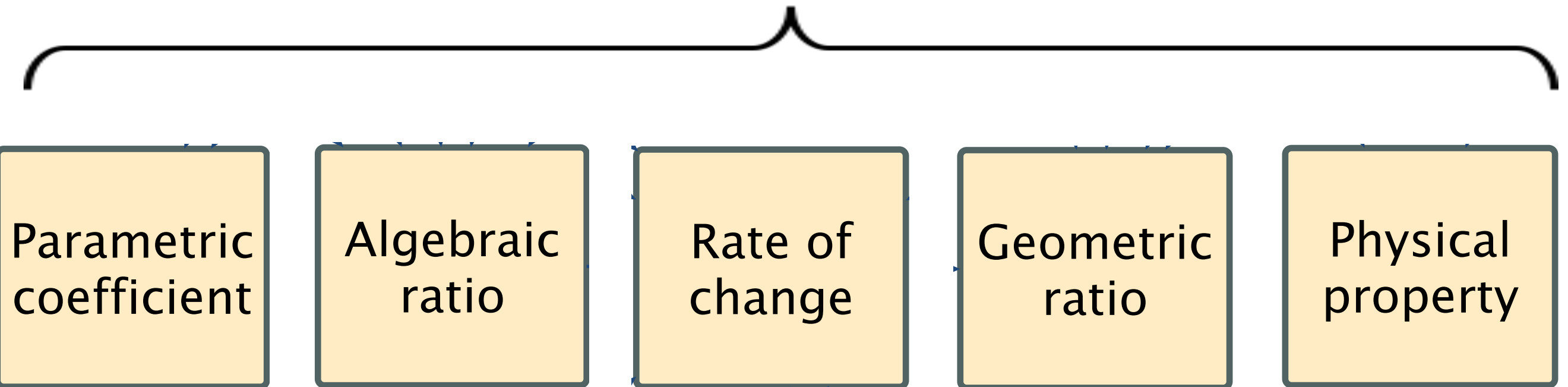b. Do Linus and Charlie ever have the same speed? If so, at what time?

Explain your reasoning.
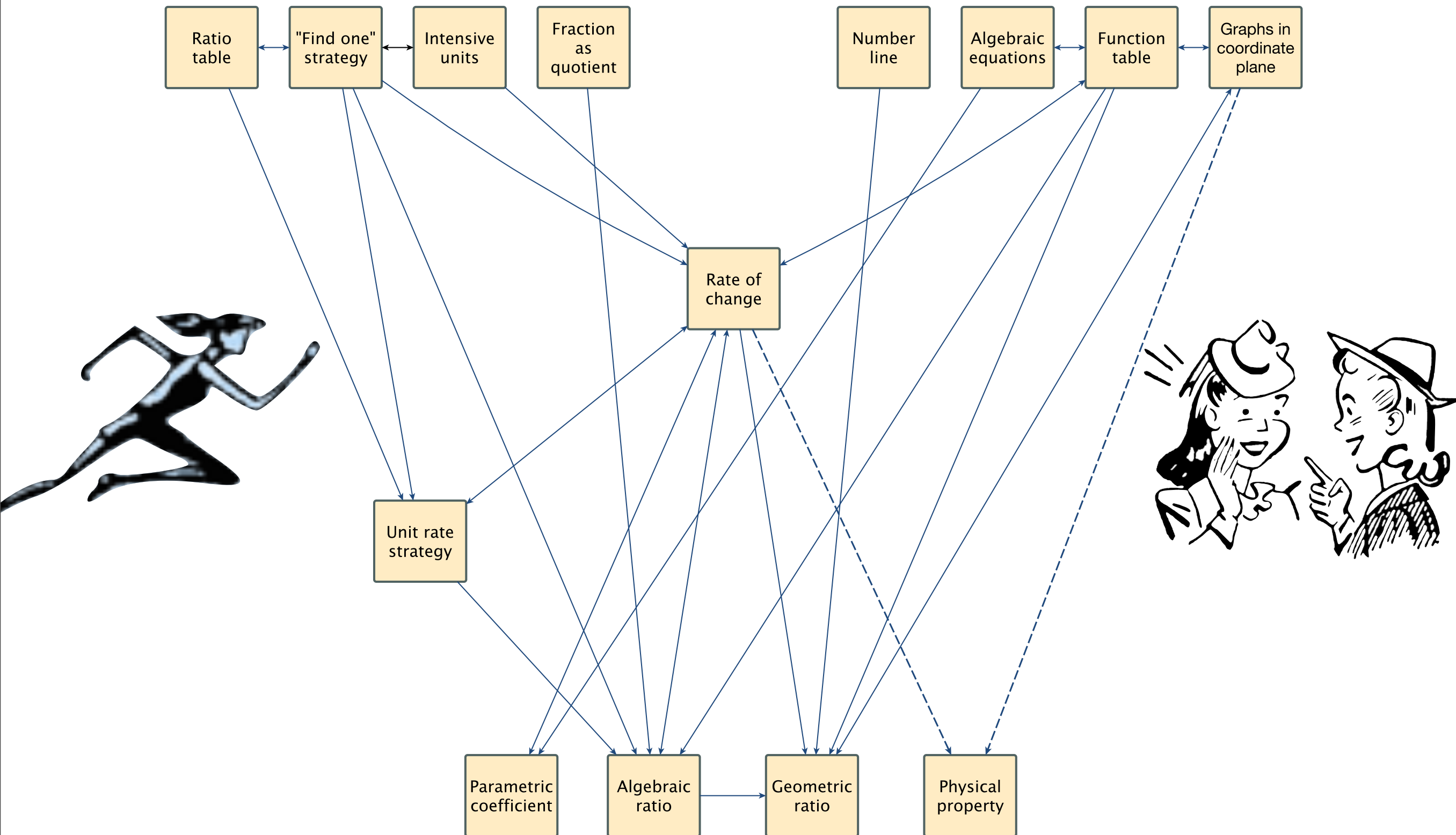
yes at a seconds they are going at the same speed

summary
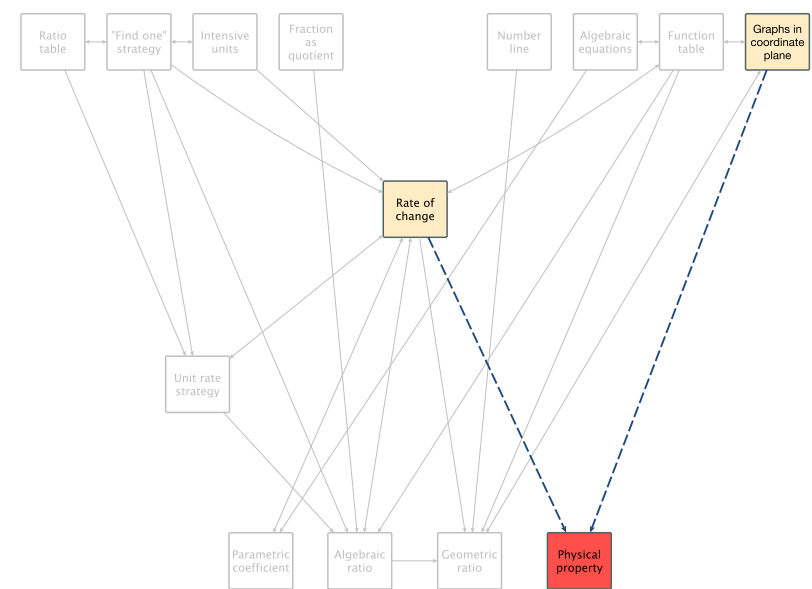
How do students make slope meaningful?

slope

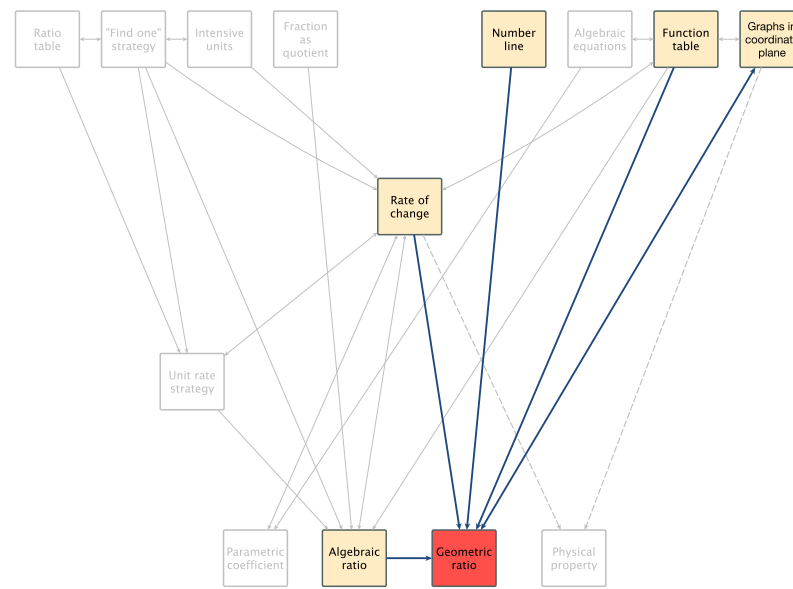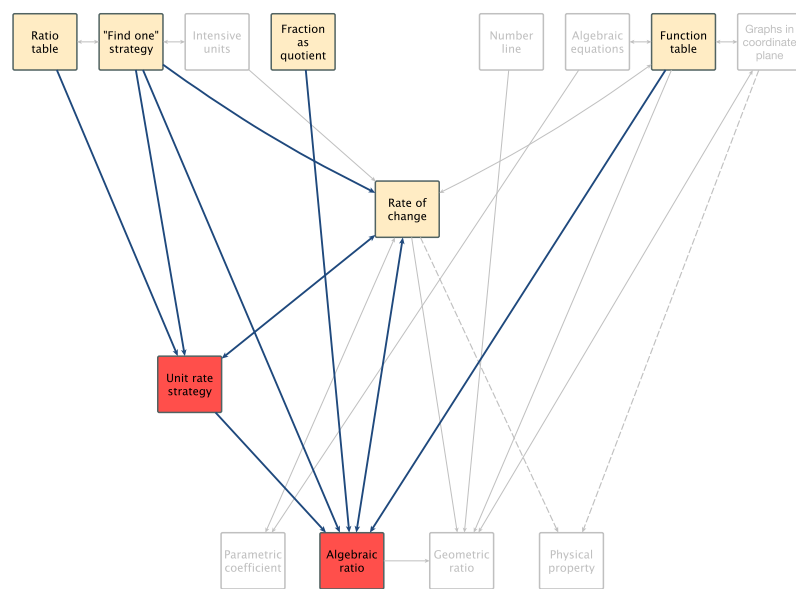Parametric coefficient

Algebraic ratio

Rate of change

Geometric ratio

Physical property

cascade of artifacts

Ratio table

"Find one" strategy

Intensive units

Fraction as quotient

Number line

Algebraic equations

Function table

Graphs in coordinate plane

Rate of change

Unit rate strategy

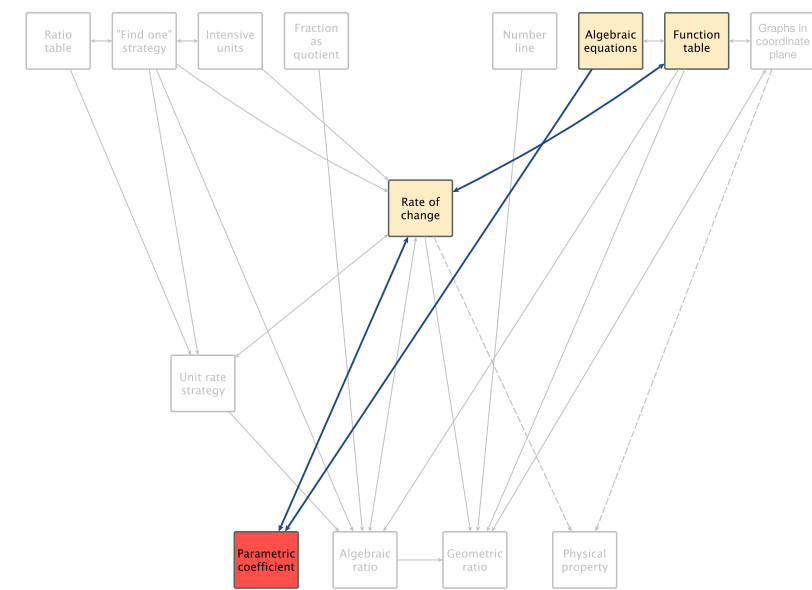Parametric coefficient

Algebraic ratio

Geometric ratio

Physical property
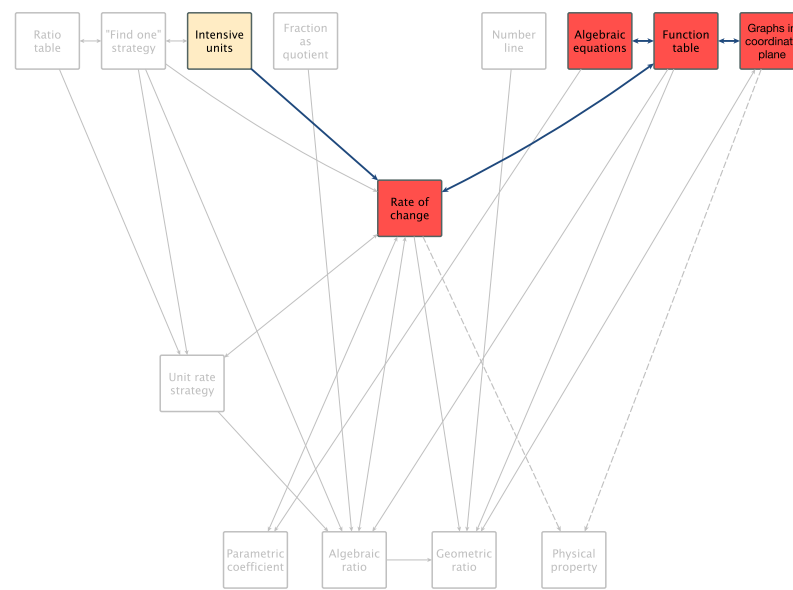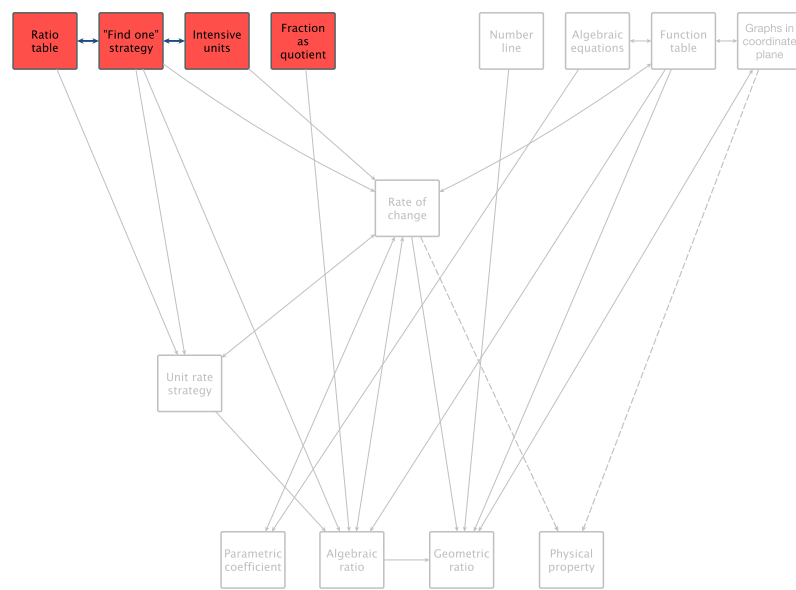
progression of learning

Questions and discussion

Questions and discussion

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